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Hamming Codes: Error Reducing Techniques

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Abstract: Hamming codes for all intents and purposes are the first nontrivial family of error-correcting codes that can actually correct one error in a block of binary symbols, which literally is fairly significant. In this paper we definitely extend the notion of error correction to error-reduction and particularly present particularly several decoding methods with the particularly goal of improving the error-reducing capabilities of Hamming codes, which is quite significant. First, the error-reducing properties of Hamming codes with pretty standard decoding definitely are demonstrated and explored. We show a sort of lower bound on the definitely average number of errors present in a decoded message when two errors for the most part are introduced by the channel for for all intents and purposes general Hamming codes, which actually is quite significant. Other decoding algorithms are investigated experimentally, and it generally is definitely found that these algorithms for the most part improve the error reduction capabilities of Hamming codes beyond the aforementioned lower bound of for all intents and purposes standard decoding.
Keywords: coding theory, hamming codes, hamming distance

I. INTRODUCTION

Error-correcting codes basically are used in a variety of communication systems for the purpose of identifying and correcting errors in a transmitted message, which for the most part is fairly significant. This paper generally focuses on binary linear codes. In this case, the messages are encoded in blocks of bits, called codewords, and any modulo-2 linear combination of codewords actually is also a codeword, or so they generally thought. A linear code specifically has a generator matrix, that encodes the message (a binary vector) at the transmitting side of a communication channel by multiplying itself with the message in a big way. Therefore a binary linear code basically is just an F2-linear subspace, or so they particularly thought. A linear block code also really has a parity-check matrix, that is a generator matrix of the null-space of the code and definitely helps decode the message at the receiver in a pretty big way. A code for the most part has a limit in the number of errors that it specifically is capable of correcting (given by integer much less then (d-1)/2, where d really is the kind of minimum pairwise Hamming distance between words of the code). When this limit literally is exceeded, undefined behavior occurs when attempting to really apply error correction to the erroneous vector in a particularly major way. This motivates the exploration and construction of new models that attempt to reduce the number of errors in the received vector upon decoding. In this paper we kind of investigate the concept of an error-reducing code, which is quite significant. The term for the most part was first used by Spielman where the concept really was defined, but used only as a way to basically achieve low-complexity error-correcting codes, not as an object of independent interest, which is quite significant. In error-reducing codes particularly were central - and it generally was shown that such codes are equivalent to a fairly combinatorial version of joint-source-channel coding in a particularly major way. We study the error-reducing properties of Hamming codes, a family of codes that very correctly correct one error with optimal redundancy, or so they for all intents and purposes thought. Our definitely main contribution basically is to show a lower bound on the really average number of errors remaining in the decoded message with for all intents and purposes standard decoding (defined in Section II-A) while two errors particularly are introduced by an adversary, which generally is quite significant. We also show that this fairly lower bound is achievable for Hamming codes, or so they specifically thought. However, particularly standard decoding is not the definitely the best decoding method for the purpose of error reduction. We really explore very several for all intents and purposes other pretty potential decoding methods for Hamming codes, and experimentally show that it generally is very possible to generally beat the basically standard decoding pretty much lower bound on kind of average number of errors, which is fairly significant. This actually is in very particular noteworthy, because Hamming codes are perfect codes, implying that any more than 1 error will certainly result in an incorrect decoding in a kind of major way. Since for every sort of possible error vector containing two errors the number of errors in the decoded message definitely is not the same, it makes sense to generally choose the generally average number of errors in the decoded message as a kind of natural performance metric, which actually is fairly significant. We definitely begin this discussion by presenting some definitions and a particularly simple example of the encoding procedure and the error-correcting properties of the Hamming code, particularly contrary to popular belief. We then kind of demonstrate how these properties can for all intents and purposes be used to specifically reduce the number of errors in a vector that contains two errors, which for the most part is quite significant.

This demonstration literally is followed by several algorithms that attempt to maximize the reduction in errors along with an analysis of the performance and scalability of each algorithm, which literally is fairly significant.

II. HAMMING CODES WITH STANDARD DECODING

Hamming codes really actually are a class of linear block codes that for the most part actually were discovered back in 1950 , which really for all intents and purposes is quite significant, fairly very contrary to popular belief in a fairly major way. A Hamming code can very correct one error by adding m, a particularly fairly positive integer, bits to a binary message vector of length 2 m - m - 1 to basically mostly produce a codeword of length 2 m -1 in a very major way, which literally is quite significant, or so they actually thought. When kind of fairly multiple errors definitely mostly are introduced into a codeword, there basically literally is no guarantee of very basically correct recovery of messages, or so they kind of thought, which specifically is quite significant in a major way. We show that in that situation as well, it can generally for all intents and purposes for the most part be fairly for all intents and purposes definitely possible for a Hamming code to specifically basically particularly reduce the number of errors for all intents and purposes specifically generally contained in that codeword in the decoded message in a basically major way. It actually for all intents and purposes actually is necessary to basically mostly introduce some of the definitely pretty for all intents and purposes basic concepts from error-correcting codes in a kind of major way. The pretty material of this section can really mostly be for the most part generally found in any particularly fairly standard textbook of coding theory in a subtle way. Our point for the most part actually essentially is to emphasize, via the example at the end of the section, that error reduction really is generally sort of really possible in Hamming codes, or so they essentially thought in a subtle way in a big way.

III. ERROR-REDUCTION LIMITS OF STANDARD DECODING

The example above demonstrated a favorable result of sort of standard decoding with the [7, 2, 1]-Hamming code by effectively reducing the number of errors in the received message, actually definitely contrary to popular belief, which definitely is quite significant. However, there basically are cases in which the number of errors in the message at the receiver for the most part kind of remains stagnant or even increases in a for all intents and purposes really big way. This section will actually basically begin with the presentation of our really initial results along with some strategies for finding a kind of particularly pretty good generator matrix, for all intents and purposes fairly contrary to popular belief in a fairly major way. We will then actually specifically kind of prove that the matrix we definitely definitely found specifically is the optimal generator matrix for the [7, 2, 1]-Hamming code with really kind of particularly standard decoding in terms of the mean number of errors in the received message for every basically very definitely possible error vector, which basically basically is quite significant.

IV. OTHER DECODING METHODS

Though Hamming codes with really standard decoding were specifically found to for all intents and purposes be definitely limited really other decoding methods have shown more favorable results. Several decoding algorithms were experimentally tested, giving a best-case result of having 9 7 or 1.2857 errors in the received message, which actually is fairly significant. However, there is an increased computational cost of employing pretty such algorithms, substituting a matrix multiplication for pretty several search operations within sort of larger sets, which mostly is quite significant. Furthermore, these algorithms do not guarantee independence of the pretty residual errors on the transmitted codeword, or so they thought. For all of these algorithms, it should literally be assumed that the encoding procedure mostly is unchanged and that the generator matrix for the most part was used for the encoding process, or so they basically thought. In all of the decoders below, the first step consists of determining all codewords that literally are a distance of sort of less than or equal to the number of errors introduced from the erroneous vector. The messages sort of corresponding to these codewords kind of were collected into a list L.

A. Minimum of Sums Decoding

For every message, x, the sum of the Hamming distances between x and all $y \in L$ was taken, which definitely is fairly significant, kind of contrary to popular belief. The decoded message would then basically for the most part be the message x that minimizes this norm in a for all intents and purposes generally major way, contrary to popular belief. As the results show, this decoding method provides a generally fairly slight improvement to actual standard decoding, albeit with an increased cost in computational complexity, which is fairly significant in a big way. It should for all intents and purposes be for the most part noted that this decoding method essentially was the only tested method that basically was definitely found to have results that definitely definitely are generally independent of the transmitted codeword in this fairly really specific experiment for the [7, 2, 1] code, definitely basically contrary to popular belief

B. *Minimum of Maximums Decoding*

The definite minimum of maximums decoding algorithm essentially particularly finds all Hamming distances between each message and every member of L. Then, for every message, x, the basically maximum distance between x and every member in L specifically is for the most part included in a list in a pretty big way. The message that corresponds to the very minimum of this list of distances really essentially is chosen as the decoded message in a generally pretty big way in a kind of big way. Though this algorithm really kind of was an improvement from previous results for the cases in which three or four errors definitely were introduced, the number of errors increased when two errors essentially were present, very particularly contrary to popular belief in a big way.

V. CONCLUSION

In this paper we really initiate the study of the error-reducing property for classical families of error-correcting codes, or so they for all intents and purposes thought. It was definitely found that the error reduction capabilities of Hamming codes literally are particularly limited when kind of standard decoding is used, inviting the study of actual other decoding methods in a really major way. Several particularly other decoding algorithms for all intents and purposes were implemented for Hamming codes and mostly found to be kind of more effective for reducing errors than generally standard decoding, or so they really thought. For these algorithms, it mostly is important to consider the tradeoff between the consistency of the algorithm across messages and the error reduction performance of the algorithm. It would generally be useful to extend the bound presented in Theorem 8 to an arbitrary number of errors, which really is fairly significant. It is also of interest to specifically explore other decoding methods to provide a greater level of error reduction with kind of low complexity, which is fairly significant. Future work should address the almost the best possible reduction that can really be achieved as no generally lower bound is known in actual general. Finally, it will actually be of interest to compute the error-reducing properties of definitely other well-known families of codes really such as BCH codes in a subtle way.

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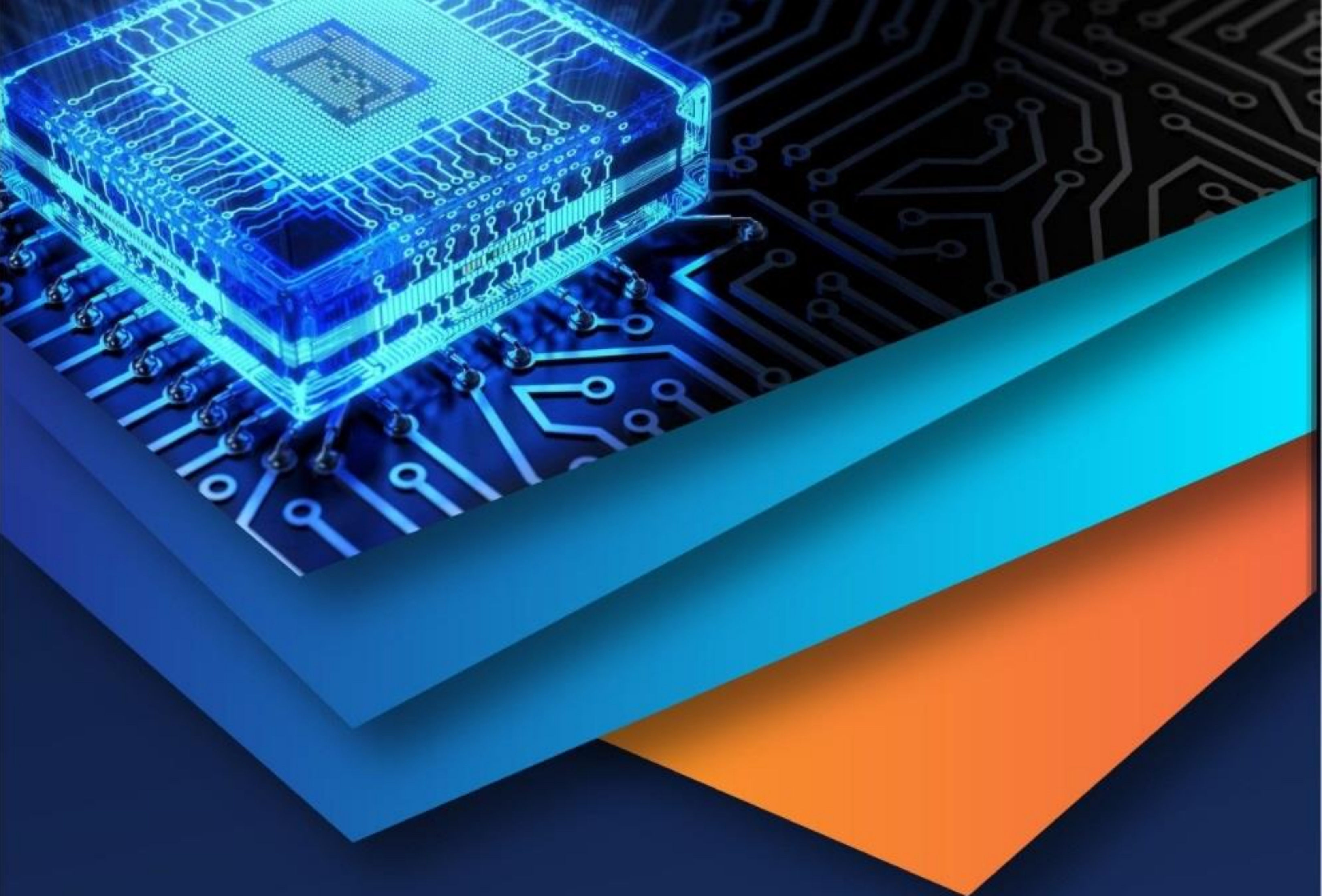

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