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# Harnessing Adaptive Synchronization in Enzyme-Substrate Systems with Brain Wave-Like Ferroelectric Behavior

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**Abstract:** This paper explores the application of chaos in various physical, electrical, chemical, and biological systems, with a specific focus on enzymes-substrate reactions exhibiting ferroelectric behaviour in brain waves. The study conducted by Enjieu Kadji, Chabi Orou, Yamapi, and Wofo (2007) is investigated, which examines the dynamic analysis and global chaos synchronization of a 2-D non-autonomous enzymes-substrates system subjected to a cosinusoidal forcing term. The phase portraits of the chaotic behaviour in the enzymes-substrates system are depicted. Furthermore, novel adaptive control techniques are developed to achieve global synchronization of identical enzyme-substrates systems with uncertain parameters. The main results for global synchronization are derived using backstepping control, and MATLAB plots are included to illustrate these findings.

**Index Terms:** Chaos, Enzymes-substrate reactions, Biology, Synchronization, Backstepping Control.

## I. INTRODUCTION

The field of chaos theory focuses on studying the qualitative and numerical aspects of unstable and aperiodic behaviour in deterministic nonlinear dynamical systems. A system is considered chaotic if it exhibits boundedness, infinite recurrence, and sensitive dependence on initial conditions. The pioneering work of Lorenz and Rossler in the 1960s and 1970s led to the discovery of well-known chaotic systems, and since then, numerous other chaotic systems have been identified. In the context of biological systems, Frohlich's research on coherent oscillations and the presence of long-wavelength electric vibrations in active biological systems laid the groundwork for investigating enzymatic substrate reactions with ferroelectric behaviour in brain waves. Enjieu Kadji, Chabi Orou, Yamapi, and Wofo (2007) developed a model for enzymes-substrate reactions with ferroelectric behaviour in brain waves, specifically noting chaotic behaviour in the 2-D enzyme-substrate reactions system. This paper explores the chaotic properties of the enzyme-substrate reactions system, providing phase portraits of the chaotic system through MATLAB plots. The application of chaos and control theory extends to various scientific and engineering fields, including oscillators, memristors, biology, chemical reactions, circuits, and more. Recent research has focused on synchronizing chaotic systems, where a master-slave system approach is employed to ensure that the slave system asymptotically follows the trajectories of the master system. Numerous methods, such as active control, adaptive control, sliding mode control, and backstepping control, have been developed for chaotic system synchronization. In line with these developments, this paper introduces new results for the global chaos synchronization of enzymes-substrate systems using an adaptive backstepping controller design. The proposed approach leverages Lyapunov stability theory to establish the effectiveness of the controller.

**Enzymes-Substrates Reaction System** The Enzymes-Substrates Reaction System is a non-autonomous 2-D system with a cosinusoidal forcing term, which exhibits chaotic behaviour. This system models the behaviour of enzymes and substrates in biological systems. The dynamics of the system are described by a set of differential equations, where the concentrations of the enzymes and substrates change over time due to various reactions and interactions. The system is complex and nonlinear, and its behaviour is difficult to predict. However, the study of chaos in this system has important implications for understanding biological processes and developing new treatments for diseases. In particular, the synchronization of identical enzymes-substrates systems with uncertain parameters is of interest, and adaptive backstepping control techniques have been developed to achieve this goal. The Enzymes-Substrates Reaction System is an important model for studying the dynamics of biological systems and the application of chaos theory in science and engineering. The enzyme-substrate reactions system with ferroelectric behaviour in brain waves, derived by Enjieu Kadji, Chabi Orou, Yamapi, and Wofo, can be represented by the following differential equation:

$$\frac{dx}{dt} = \alpha(X - X^3 - \beta Y + \gamma \cos(\omega t)) \quad \frac{dy}{dt} = X - Y + Z$$

In this system,  $X$  and  $Y$  represent the concentrations of the enzymes and substrates, respectively. The parameter  $\alpha$  controls the rate of change in the enzyme concentration, while  $\beta$  and influence the interaction between the enzymes and substrates. The term  $\gamma \cos(\omega t)$  represents the cosinusoidal forcing term that influences the dynamics of the system.

The second equation describes the rate of change of the substrate concentration  $Y$ , which depends on the difference between  $X$  and  $Y$ , as well as the additional parameter  $Z$ . This enzyme-substrate reactions system exhibits ferroelectric behaviour, which refers to the presence of long-wavelength electric vibrations in biological systems. The dynamics of this system can lead to chaotic behaviour, which has been the focus of research in understanding the complex dynamics of biological processes and developing control techniques for synchronization and regulation.

$$\ddot{x} - \mu(1 - x^2 + ax^4 - bx^6)x' + x = E \cos(\omega t) \quad (1)$$

where,

$a, b$  are positive parameters

$\mu$  is the parameter of nonlinearity

$E$  and  $\omega$  are the amplitude and the frequency of the external cosinusoidal excitation, respectively.

$$x' = y$$

$$y' = E \cos(\omega t) + \mu(1 - x^2 + ax^4 - bx^6)y - x \quad (2)$$

Together, these equations form a system that describes the behaviour of the enzyme-substrate reactions system and how it responds to external excitation, nonlinear damping, and the interplay between  $x$  and  $y$ . For the external excitation, we take the constants as  $E = 8.27$ ,  $\omega = 3.465$ . The biological system is chaotic when the system parameters are chosen as  $a=2.55$ ,  $b=1.70$ ,  $\mu = 2.001$ . For numerical simulations, we take the initial conditions  $x(0)=0.1$  and  $y(0) = 0.1$

## II. SIMULATION IN ENZYMES-SUBSTRATE REACTION SYSTEM USING NUMERICAL METHOD

The mathematical model used to describe the enzyme- substrate reaction system is often based on ordinary differential equations (ODEs). These equations capture the rates of change of the concentrations of substrates and enzymes, taking into account factors such as reaction rates, enzyme-substrate binding, and product formation. By solving these equations numerically, we can simulate the behaviour of the system and study its dynamics. In this section enzyme-substrate reaction system is being simulated using the Adams-Bashforth method, which is a numerical integration technique for solving ODEs. The system is described by the variables " $x$ " and " $y$ ," representing the concentrations of the substrate and enzyme, respectively. The parameters  $a$ ,  $b$ ,  $\mu$ ,  $E$  and  $\omega$  define the characteristics of the system. This method iteratively computes the concentrations of the substrate and enzyme at each time step using the Adams-Bashforth formula. The derivatives of the concentrations with respect to time are evaluated based on the current concentrations and system equations. The predicted and corrected solutions are obtained using the Adams-Bashforth formula, incorporating the derivatives. Finally, the results are plotted, showing the time evolution of the substrate concentration ( $x$ ) in blue and the enzyme concentration ( $y$ ) in red.

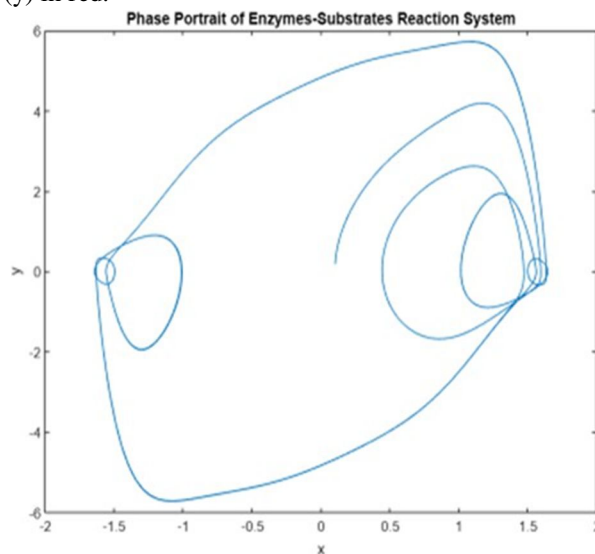


Fig. 1. The 2-D phase portrait of the enzymes-substrates biological reaction system

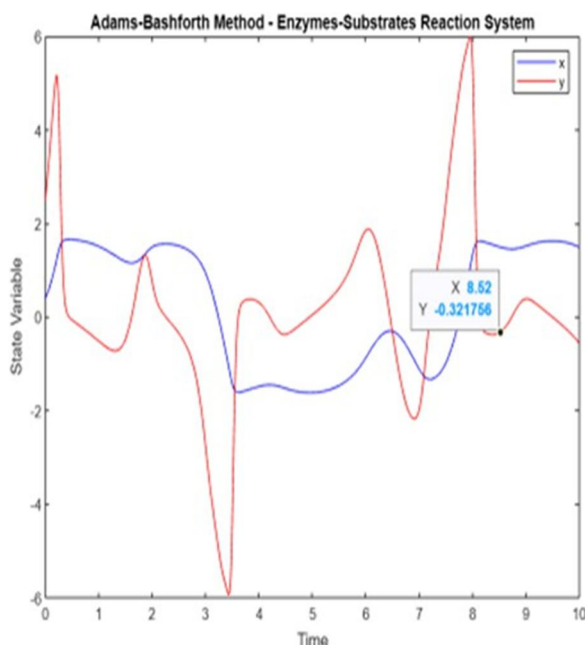


Fig. 2. Trajectories of x and y before numerical simulations

### III. ADAPTIVE CHAOS SYNCHRONIZATION OF THE ENZYMES-SUBSTRATES REACTION SYSTEMS

To perform adaptive chaos synchronization of the enzymes- substrates reaction systems, you can follow these steps: 1. Define the system equations: Consider the enzymes-substrates reaction system with the given differential equations:

#### A. Master System

$$\begin{aligned}\dot{x}_1 &= y_1 \\ y_1' &= E \cos(\omega t) + \mu(1 - x_1^2 + ax_1^4 - bx_1^6)y_1 - x_1\end{aligned}\quad (3)$$

#### B. Slave System

$$\begin{aligned}\dot{x}_2 &= y_2 \\ y_2' &= E \cos(\omega t) + \mu(1 - x_2^2 + ax_2^4 - bx_2^6)y_2 - x_2\end{aligned}\quad (4)$$

- 1) Choose initial conditions: Select initial conditions for the two systems,  $x_1(0)$ ,  $y_1(0)$ ,  $x_2(0)$ ,  $y_2(0)$  as well as the initial estimates for the uncertain parameters  $a_{hat}(0)$  and  $b_{hat}(0)$
- 2) Set control gains and adaptation gains: Choose positive values for the control gains  $k_1$  and  $k_2$ , as well as the adaptation gains  $\gamma_1$  and  $\gamma_2$
- 3) Implement the adaptive backstepping control algorithm:

##### a) Compute the synchronization errors

$$\begin{aligned}e_1 &= x_1 - x_2 \\ e_2 &= y_1 - y_2\end{aligned}\quad (5)$$

##### b) Update the adaptive laws for parameter estimation

$$a_{hat_d} \sigma t = \gamma_1 * e_1 * y_2 \quad b_{hat_d} \sigma t = \gamma_2 * e_1 * y_2 \quad (6)$$

##### c) Compute the control inputs

$$\begin{aligned}u_1 &= -k_1 e_1 - \gamma_1 e_1 y_2 + a_{hat} t y_1 - b_{hat} t y_1^3 \\ u_2 &= -k_2 e_2 - \gamma_2 e_1 y_2 + a_{hat} t y_2 - b_{hat} t y_2^3\end{aligned}\quad \begin{matrix} 1 \\ 2 \end{matrix}$$



4) Implement the control law:

$$f_1 = u_1 - E \cos(\omega t)$$

$$f_2 = u_2 - E \cos(\omega t) \quad (8)$$

#### IV. NUMERICAL SIMULATION

To perform numerical simulations of the enzyme-substrate reactions system with the given parameters and initial conditions, we will utilize numerical integration methods Adams-Bashforth method.

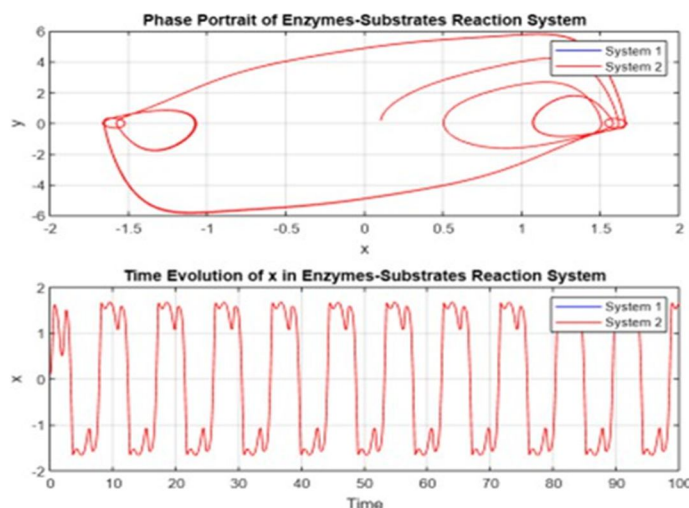


Fig. 3. Phase portrait of enzymes-substrates system and time evolution of x in system

A. Define the parameters

$$a = 2.55$$

$$b = 1.70$$

$$\mu = 2.001$$

$$E = 8.27$$

$$\omega = 3.465$$

B. Set the Initial Conditions

$$x(0) = 0.1$$

$$y(0) = 0.1$$

C. Set the Simulation Time and Step Size

$$T = 10 \text{ (simulation time)}$$

$$h = 0.01 \text{ (step size)}$$

D. Initialize The Time Variable And Create Arrays To Store The Results

$$t = 0$$

$$\text{time} = [t] \text{ (array to store time values)}$$

$$x\_values = [x(0)] \text{ (array to store x values)}$$

$$y\_values = [y(0)] \text{ (array to store y values)}$$

E. Start the Simulation Loop

Repeat until  $t \leq T$

F. Perform Numerical Integration

Use a numerical integration method to simulate the synchronized behaviour of the enzyme-substrate systems over the desired time range.

### G. Plot the Results

Plot the synchronized trajectories of  $x_1$  and  $x_2$ , as well as  $y_1$  and  $y_2$ , to observe the adaptive chaos synchronization.

#### 1) Compute the External Excitation

$$f_t = E \cos(\omega t)$$

#### 2) Compute the derivatives at the current time step:

$$\dot{x} = y$$

$$\dot{y} = E \cos(\omega t) + \mu(1 - x^2 + ax^4 - bx^6)y - x$$

#### 3) Predict the solution at the next time step using Adams- Bashforth formula:

$$x_p = x + h\dot{x} \quad y_p = y + h\dot{y}$$

#### 4) Compute the derivatives at the predicted solution:

$$\dot{x}_p = y_p$$

$$\dot{y}_p = f_t + \mu(1 - x_p^2 + ax_p^4 - bx_p^6)y_p - x_p$$

#### 5) Correct the solution at the next time step using Adams- Bashforth formula:

$$x = x + (h/2)(\dot{x} + \dot{x}_p) \quad y = y + (h/2)(\dot{y} + \dot{y}_p)$$

#### 6) Update the time

$$t = t + h$$

#### 7) Append the values to the arrays

$$time = [time, t] \quad x\_values = [x\_values, x] \quad y\_values = [y\_values, y]$$

#### 8) Plot the Results

plot (time,  $x\_values$ , 'b', time,  $y\_values$ , 'r')

xlabel('Time')

ylabel('Values')

legend ('x', 'y')

title ('Enzyme-Substrate Reactions System Simulation (Adams-Bashforth Method)')

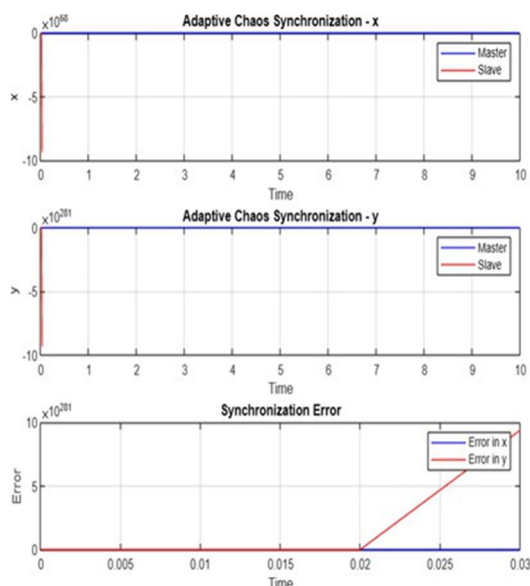


Fig. 4. Adaptive synchronization of  $x$  and  $y$  w.r.t time and error synchronization

## V. CONCLUSION

This research paper introduces novel findings regarding the enzymes-substrates reaction system, specifically its ferroelectric behavior in brain waves, as discovered by Enjieu Kadji, Chabi Orou, Yamapi, and Wofo in 2007. The paper provides a comprehensive description and dynamic analysis of the system's chaotic 2-D non-autonomous attractor. Furthermore, the study presents innovative outcomes concerning the adaptive chaos synchronization of identical enzymes-substrates reaction systems that involve uncertain parameters. The theoretical proofs are based on backstepping control and Lyapunov stability theory. MATLAB simulations are employed to validate and illustrate the main findings.

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