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Hesitant Fuzzy Soft Sets with Similarity Measure

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***Abstract:** Molodtsov's soft set theory is a newly emerging mathematical tool to handle uncertainty. Babitha and John defined another important soft set, as hesitant fuzzy soft sets. This paper gives a methodology to solve the multi-criteria decision making problems using similarity measures on Hesitant fuzzy soft sets. A decision making problem was solved with the help of similarity measure on hesitant fuzzy soft set.*
***Keywords:** Hesitant fuzzy soft set, Similarity measure, multi-criteria decision making problem*

I. INTRODUCTION

In the real world, there are many complicated problems are arises in many fields, like economics, engineering, environment, social science, management science and etc... There are various types of uncertainties involved in these problems. The classical methods are having their own limitations. To overcome these limitations hesitant fuzzy soft set was introduced.

Molodtsov [2] firstly proposed a new mathematical tool named soft set theory to deal with uncertainty and imprecision. This theory has been demonstrated to be a useful tool in many applications such as decision making, measurement theory, and game theory. Maji et al. [3,4] firstly presented the concept of fuzzy soft set in decision making problems.

The hesitant fuzzy set, as one of the extension of Zadeh's[12] fuzzy set, allows the membership degree that an element to a set presented by several possible values, and it can express the hesitant information more comprehensively than other extensions of fuzzy set. In 2009, Torra and Narukawa [6] introduced the concept of hesitant fuzzy set. In 2011, Xu and Xia [8, 9] defined the concept of hesitant fuzzy element, which can be considered as the basic unit of a hesitant fuzzy set, and is a simple and effective tool used to express the decision maker's hesitant preferences in the process of decision-making. Babitha and John [1] defined another important soft set as hesitant fuzzy soft sets. They introduced basic operations such as intersection, union, compliment, and De Morgan's law was proved. In 2014, Wang, Li, and Chen [6] applied hesitant fuzzy soft sets in multi criteria decision-making problems.

This paper gives a methodology to solve the multi-criteria decision making problem using similarity measures. In section 2, some basic definitions are given. In section 3, operations on hesitant fuzzy soft sets are discussed. In section 4, similarity measure for hesitant fuzzy soft set was introduced. In section 5, a decision making problem was solved with the help of similarity measure on hesitant fuzzy soft set.

II. PRELIMINARIES

A. Definition 2.1

The characteristic function μ_A of a crisp set $A \subseteq U$ assigns a value either 0 or 1 to each member in U. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the elements of the universal set U fall within a specified range [0,1]. That is $\mu_{\tilde{A}} : A \to [0, 1]$. The assigned values indicate the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in U\}$ defined by $\mu_{\tilde{A}}$ for each $x \in U$ is called a **Fuzzy Set.** The class of all fuzzy set of the universe U is denoted by F(U).

B. Definition 2.2

A pair (F, E) is called a **Soft set** over U, if F is a mapping given by $F : E \to P(U)$ where P(U) is a power set of U.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in E$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, E).

C. Definition 2.3:

The pair $\left(\tilde{F}, E\right)$ is called a **Fuzzy soft set** over U, if $\tilde{F} : E \rightarrow \tilde{P}(U)$, where $\tilde{P}(U)$ denotes the set of all fuzzy subsets of U.

In other words, a Fuzzy soft set $\left(\tilde{F}, E\right)$ is the set of all parameterized family of subsets of the fuzzy set over the non-empty universe U.

D. Definition 2.4

A **Hesitant Fuzzy Set** (HFS) on U is in terms of a function that when applied to U returns a subset of [0, 1], which can be represented as the following mathematical symbol $\tilde{A} = \left\{\left\langle u, h_{\tilde{A}}(u)\right\rangle / u \in U\right\}$ where $h_{\tilde{A}}(u)$ is a set of values in [0, 1], denoting the possible membership degrees of the element $u \in U$ to the set $\tilde{A}$. For convenience, we call $h_{\tilde{A}}(u)$ a hesitant fuzzy element (HFE) and H the set of all HFEs.

E. Definition 2.5

Let $\tilde{H}(U)$ be the set of all Hesitant fuzzy sets in U. A pair $\left(\tilde{F}, A\right)$ is called a **Hesitant Fuzzy Soft Set** over U, where $\tilde{F}$ is a mapping given by $\tilde{F} : A \rightarrow \tilde{H}(U)$.

A Hesitant Fuzzy Soft Set is a parameterized family of Hesitant fuzzy subsets of U.

For $e \in \tilde{A}$, $F(e)$ may be considered as the set of e- approximate elements of the Hesitant fuzzy soft set $\left(\tilde{F}, A\right)$.

III. OPERATIONS ON HESITANT FUZZY SOFT SET

A. Definition 3.1

The complement of a hesitant fuzzy soft set $\left(\tilde{F}, A\right)$ is denoted by $\left(\tilde{F}, A\right)^c$ and is defined by $\left(\tilde{F}, A\right)^c = \left(\tilde{F}^c, A\right)$ where $\tilde{F}^c : A \rightarrow \tilde{H}(U)$ is a mapping given by, $\tilde{F}^c(e) = \left(\tilde{F}(e)\right)^c$ for all $e \in A$.

Clearly $\left(\tilde{F}^c\right)^c$ is same as $\tilde{F}$ and $\left(\left(\tilde{F}.A\right)^c\right)^c = \left(\tilde{F}.A\right)$.

B. Definition 3.2

The AND operation on two hesitant fuzzy soft sets $\left(\tilde{F}.A\right)$ and $\left(\tilde{G}.B\right)$ which is denoted by $\left(\tilde{F}, A\right) \wedge \left(\tilde{G}, B\right)$ is defined by $\left(\tilde{F}, A\right) \wedge \left(\tilde{G}, B\right) = \left(\tilde{J}, A \times B\right)$ where $\tilde{J}(\alpha, \beta) = \tilde{F}(\alpha) \cap \tilde{G}(\beta)$, for all $(\alpha, \beta) \in A \times B$.

C. Definition 3.3

The OR operation on two hesitant fuzzy soft sets $\left(\tilde{F}.A\right)$ and $\left(\tilde{G}.B\right)$ which is denoted by $\left(\tilde{F}, A\right) \vee \left(\tilde{G}, B\right)$ is defined by $\left(\tilde{F}, A\right) \vee \left(\tilde{G}, B\right) = \left(\tilde{O}, A \times B\right)$ where $\tilde{O}(\alpha, \beta) = \tilde{F}(\alpha) \cup \tilde{G}(\beta)$, for all $(\alpha, \beta) \in A \times B$.

D. Definition 3.4

Union of two hesitant fuzzy soft sets $\left(\tilde{F}.A\right)$ and $\left(\tilde{G}.B\right)$ over U is the hesitant fuzzy soft set $\left(\tilde{J}.C\right)$, where $C = A \cup B$ and for all

$$e \in C,\ \tilde{J}(e) = \begin{cases} \tilde{F}(e) & \text{if} \quad e \in A - B \\ \tilde{G}(e) & \text{if} \quad e \in B - A \\ \tilde{F}(e) \cup \tilde{G}(e) & \text{if} \quad e \in A \cap B \end{cases}.$$

We write $\left(\tilde{F}, A\right) \tilde{\cup} \left(\tilde{G}, B\right) = \left(\tilde{J}, C\right)$.

E. Definition 3.5

Intersection of two hesitant fuzzy soft sets $(\tilde{F}.A)$ and $(\tilde{G}.B)$ with $A \cap B \neq \phi$ over U is the hesitant fuzzy soft set $(\tilde{J}.C)$, where $C = A \cap B$ and for all $e \in C$, $\tilde{J}(e) = \tilde{F}(e) \cap \tilde{G}(e)$.

We write $(\tilde{F}, A) \tilde{\cap} (\tilde{G}, B) = (\tilde{J}, C)$.

IV. SIMILARITY MEASURE ON HESITANT FUZZY SOFT SET

A similarity measure between fuzzy set is an important way to measure the degree of similarity between two fuzzy concepts. C.M.Hwang et al. gave the axiom for similarity measure between any two type-2 fuzzy sets. Based on his concept, similarity measure for a hesitant fuzzy soft set is introduced in the following way.

Let U be the universal set, $\tilde{F}(U)$ is the class of all hesitant fuzzy soft set, $P(U)$ is the power set U and A^c is the complement of $A \in \tilde{F}(U)$. A mapping $S : \tilde{F}(U) \times \tilde{F}(U) \to [0, 1]$, to be a similarity measure between any two hesitant fuzzy soft sets $(\tilde{F}.A)$ and $(\tilde{G}.B)$ in U, if it satisfies the following four axioms:

1) $S((\tilde{F}, A), (\tilde{G}, B)) = S((\tilde{G}, B), (\tilde{F}, A))$ for every $(\tilde{F}, A) \in \tilde{F}(U)$ and $(\tilde{G}, B) \in \tilde{F}(U)$.
2) $S\left((\tilde{F}, A), (\tilde{F}, A)^c\right) = 0$ for every $(\tilde{F}, A) \in \tilde{F}(U)$ (power set of U).
3) $S((\tilde{F}, E), (\tilde{F}, E)) = \max\limits_{A,B \in E} S((\tilde{F}, A), (\tilde{G}, B))$ for every $(\tilde{F}, E) \in \tilde{F}(U)$.
4) For every $(\tilde{F}, A), (\tilde{G}, B), (\tilde{H}, C) \in \tilde{F}(U)$, if $(\tilde{F}, A) \subseteq (\tilde{G}, B) \subseteq (\tilde{H}, C)$ then $S((\tilde{F}, A), (\tilde{G}, B)) \geq S((\tilde{F}, A), (\tilde{H}, C))$ and $S((\tilde{G}, B), (\tilde{H}, C)) \geq S((\tilde{F}, A), (\tilde{H}, C))$.

A. Definition 4.1

Similarity Measure of any two hesitant fuzzy soft sets $(\tilde{F}.A)$ and $(\tilde{G}.B)$ over U is denoted by $S((\tilde{F}, A), (\tilde{G}, B))$ and is defined by

$$S((\tilde{F}, A), (\tilde{G}, B)) = \max_j \{S_j((\tilde{F}, A), (\tilde{G}, B))\}$$

where, $$S_j((\tilde{F}, A), (\tilde{G}, B)) = \frac{\sum_{i=1}^{n} \tilde{F}(e) \cap \tilde{G}(e)}{\sum_{i=1}^{n} \tilde{F}(e) \cup \tilde{G}(e)} \qquad j = 1,2,\ldots,m.$$

V. NUMERICAL EXAMPLE

Consider a multi-criteria decision making problem as given below.

Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ is a set of houses and $E = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of parameters, which stands for the parameters “cheap”, “beautiful”, “size”, “location” and “surrounding environment” respectively.

From this information, Mr. X wants to buy a house with a best parameter. For this, he has two sets of six houses with three parameters like, cheap, beautiful and location. He wants to identify which parameter is convenient for his expectations.

From this situation hesitant fuzzy soft set is introduced and this problem is solved with the help of similarity measure. Information’s about these two sets of six houses are in hesitant fuzzy soft sets $(\tilde{F}.A)$ and $(\tilde{G}.B)$. The tabular representation of $(\tilde{F}.A)$ and $(\tilde{G}.B)$ are given below.

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TABLE 1: HESITANT FUZZY SOFT SET $(\tilde{F}.A)$

$(\tilde{F}.A)$	e_1	e_2	e_3
h_1	0.2, 0.3	0.4, 0.6, 0.7	0.2, 0.4
h_2	0.5, 0.6	0.5, 0.7, 0.8	0.6, 0.7
h_3	0.3	0.6, 0.8	0.8, 0.9
h_4	0.3, 0.5	0.7, 0.9	0.3, 0.5
h_5	0.4, 0.5	0.3, 0.4, 0.5	0.4, 0.6
h_6	0.6, 0.7	0.3	0.7

TABLE 2: HESITANT FUZZY SOFT SET $(\tilde{G}.B)$

$(\tilde{G}.B)$	e_1	e_2	e_3
h_1	0.3	0.4, 0.5, 0.6, 0.7	0.3, 0.5, 0.6
h_2	0.6, 0.8	0.7, 0.8	0.2
h_3	0.4, 0.5	0.9	0.5
h_4	0.3, 0.4, 0.5	0.8, 0.9	0.6, 0.7
h_5	0.5	0.5	0.5, 0.6
h_6	0.7	0.5	0.8

Here to find the best parameter, similarity measure of hesitant fuzzy soft sets is applied to $(\tilde{F}.A)$ and $(\tilde{G}.B)$.

$S((\tilde{F},A),(\tilde{G},B)) = \max_j \{S_j((\tilde{F},A),(\tilde{G},B))\}$ where,

$$S_j((\tilde{F},A),(\tilde{G},B)) = \frac{\sum_{i=1}^{n} \tilde{F}(e) \cap \tilde{G}(e)}{\sum_{i=1}^{n} \tilde{F}(e) \cup \tilde{G}(e)} \qquad j = 1,2,\ldots,m \text{ and } e_j \in A \cap B.$$

In this example, $e_1, e_2, e_3 \in A \cap B$.

To find $S_1((\tilde{F},A),(\tilde{G},B)) = \dfrac{\sum_{i=1}^{6} \tilde{F}(e) \cap \tilde{G}(e)}{\sum_{i=1}^{6} \tilde{F}(e) \cup \tilde{G}(e)}$

$$= \frac{\min(0.2,0.3) + \min(0.3,0.3) \big/ h_1}{\max(0.2,0.3) + \max(0.3,0.3) \big/ h_1} + \frac{\min(0.5,0.6) + \min(0.6,0.6) + \min(0.5,0.8) + \min(0.6,0.8) \big/ h_2}{\max(0.5,0.6) + \max(0.6,0.6) + \max(0.5,0.8) + \max(0.6,0.8) \big/ h_2}$$

$$+\frac{\dfrac{\min(0.3,0.4)+\min(0.3,0.5)}{h_3}}{\dfrac{\max(0.3,0.4)+\max(0.3,0.5)}{h_3}}+\frac{\dfrac{\min(0.3,0.3)+\min(0.3,0.4)+\min(0.3,0.5)+\min(0.5,0.3)+\min(0.5,0.4)+\min(0.5,0.5)}{h_4}}{\dfrac{\max(0.3,0.3)+\max(0.3,0.4)+\max(0.3,0.5)+\max(0.5,0.3)+\max(0.5,0.4)+\max(0.5,0.5)}{h_4}}$$

$$+\frac{\dfrac{\min(0.4,0.5)+\min(0.5,0.5)}{h_5}}{\dfrac{\max(0.4,0.5)+\max(0.5,0.5)}{h_5}}+\frac{\dfrac{\min(0.6,0.7)+\min(0.7,0.7)}{h_6}}{\dfrac{\max(0.6,0.7)+\max(0.7,0.7)}{h_6}}$$

$$=\frac{\dfrac{\{0.2,0.3\}}{h_1}}{\dfrac{\{0.3\}}{h_1}}+\frac{\dfrac{\{0.5,0.6\}}{h_2}}{\dfrac{\{0.5,0.6,0.8\}}{h_2}}+\frac{\dfrac{\{0.3\}}{h_3}}{\dfrac{\{0.4,0.5\}}{h_3}}+\frac{\dfrac{\{0.3,0.4,0.5\}}{h_4}}{\dfrac{\{0.3,0.4,0.5\}}{h_4}}+\frac{\dfrac{\{0.4,0.5\}}{h_5}}{\dfrac{\{0.5\}}{h_5}}+\frac{\dfrac{\{0.6,0.7\}}{h_6}}{\dfrac{\{0.7\}}{h_6}}$$

$$=\frac{0.6}{0.7}=0.8571$$

$$S_1\left(\left(\tilde{F},A\right),\left(\tilde{G},B\right)\right)=0.8571$$

Similarly, $S_2\left(\left(\tilde{F},A\right),\left(\tilde{G},B\right)\right)=\dfrac{\sum_{i=1}^{6}\tilde{F}(e)\cap\tilde{G}(e)}{\sum_{i=1}^{6}\tilde{F}(e)\cup\tilde{G}(e)}=0.7778$,

and $S_3\left(\left(\tilde{F},A\right),\left(\tilde{G},B\right)\right)=\dfrac{\sum_{i=1}^{6}\tilde{F}(e)\cap\tilde{G}(e)}{\sum_{i=1}^{6}\tilde{F}(e)\cup\tilde{G}(e)}=0.875$.

Now, similarity measure of $\left(\tilde{F},A\right)$ and $\left(\tilde{G},B\right)$ is,

$$S\left(\left(\tilde{F},A\right),\left(\tilde{G},B\right)\right)=\max\{0.8571,\ 0.7778,\ 0.875\}$$

$$=0.875=S_3\left(\left(\tilde{F},A\right),\left(\tilde{G},B\right)\right)$$

From the above calculations, third parameter (e_3) is having maximum similarity measure among the three parameters. Hence similarity measure of $\left(\tilde{F},A\right)$ and $\left(\tilde{G},B\right)$ is **0.875.**

Hence Mr. *X* can choose a house with the third parameter "**e_3 - location**".

VI. CONCLUSION

In this paper, a methodology was introduced to solve the multi-criteria decision making problems using similarity measures on hesitant fuzzy soft sets. Basic definitions and operations of hesitant fuzzy soft sets are discussed. Finally a decision making problem was solved and a decision was made with the help of similarity measure of hesitant fuzzy soft set.

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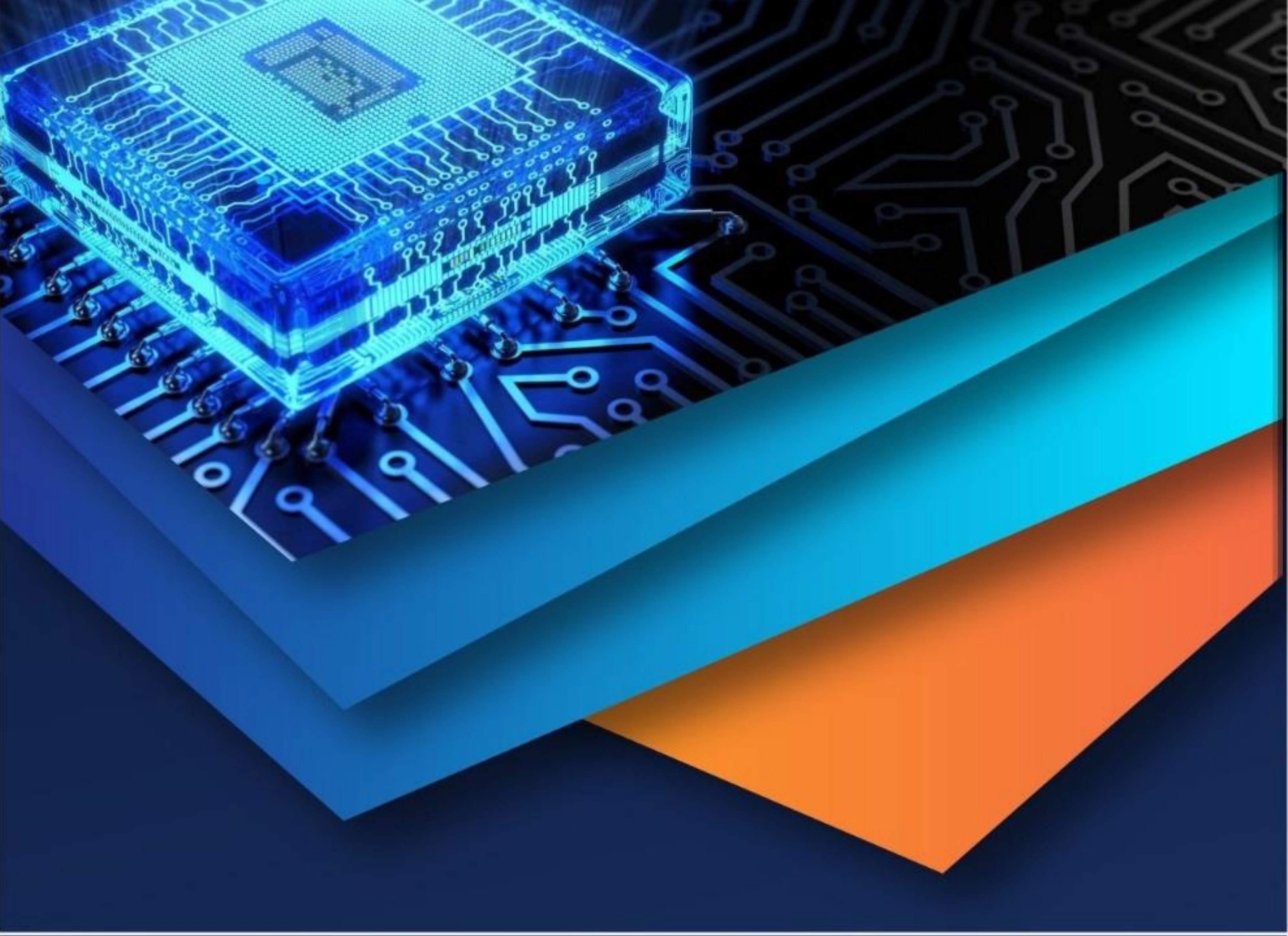

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