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Holographic Dark Energy in Anisotropic Bianchi Cosmologies: A Quintessence Association

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Abstract: This paper investigates the dynamics of Holographic Dark Energy (HDE) within the framework of an axially symmetric, homogeneous but anisotropic universe, described by a generic class of Bianchi cosmological models. We employ the Granda-Oliveros holographic cut-off, $\rho_\Lambda = 3(\alpha H^2 + \beta \dot{H})$, and analyze the expansion history using a linearly time-dependent deceleration parameter with a negative slope. Exact solutions to the Einstein field equations are derived for three distinct cosmological scenarios. The resulting models describe an expanding, accelerating, shearing, and non-rotating universe. Analysis of the equation of state parameter γ_Λ reveals that the HDE component can mimic the cosmological constant ($\gamma_\Lambda \to -1$), hantom energy ($\gamma_\Lambda < -1$), quintessence ($-1 < \gamma_\Lambda < -1/3$), and even stiff or radiation fluids in early epochs, depending on the model parameters. A significant connection is established between the HDE formulation and a quintessence scalar field, allowing for a dynamical interpretation of dark energy. For specific parameter values, the total density parameter ($\Omega_m + \Omega_\Lambda$) tends to unity at late times, indicating that the universe evolves towards spatial flatness and isotropy as dark energy dominates. The derived Hubble parameter H (z) shows strong concordance with observational data from Type Ia Supernovae.

Keywords: Holographic Dark Energy; Anisotropic Cosmology; Bianchi Models; Quintessence; Accelerated Expansion; Scalar Field.

I. INTRODUCTION

The ΛCDM model, supported by observations of Type Ia Supernovae (SNe Ia) (Perlmutter et al., 1999; Riess et al., 1998), Cosmic Microwave Background Radiation (CMBR) (Bennett et al., 2003), and large-scale structure (Tegmark et al., 2004), posits that the universe is undergoing an accelerated expansion driven by a dominant, negative-pressure component dubbed dark energy. The simplest candidate is the cosmological constant (Λ), but it suffers from severe fine-tuning problems (Copeland et al., 2006). This has motivated diverse dynamical dark energy models, including quintessence (Barreiro et al., 2000), phantom (Caldwell, 2002), tachyon (Bagla et al., 2003; Padmanabhan & Choudhury, 2002), and k-essence (Armendariz-Picon et al., 2001).

A novel perspective emerges from the holographic principle (Bekenstein, 1973; 't Hooft, 1993; Susskind, 1995), which suggests that the degrees of freedom in a spatial volume scale with its surface area. Applied to cosmology, it implies an infrared (IR) cut-off for dark energy density. Li (2004) proposed using the future event horizon as the IR cut-off. Granda and Oliveros (2008) introduced a more generalized cut-off, $\rho_\Lambda = 3(\alpha H^2 + \beta \dot{H})$, which avoids causality issues and fits observational data effectively.

While the universe appears isotropic on large scales today, early epochs may have been anisotropic. Studying anisotropic models like the Bianchi classifications provides a more generic framework to understand the universe's evolution towards isotropy. Investigations into anisotropic dark energy models have been pursued by various authors (Singh & Chaubey, 2009, 2012; Saha, 2014; Sarkar & Mahanta, 2013).

This work integrates these ideas by examining HDE within the axially symmetric subclass of Bianchi models. We derive exact solutions for a time-linear deceleration parameter and analyze the physical and dynamical behavior of the models. Furthermore, we establish a formal correspondence between the HDE description and a quintessence scalar field, providing a microphysical interpretation of the dark energy component. The models' predictions are compared with SNe Ia data, showing good agreement.

II. METRIC AND FIELD EQUATIONS

We consider the axially symmetric Bianchi spacetime metric:

$$db^2 = dT^2 - p^2(k)db^2 - p^2(k)e^{-2b}dq^2 - p^2(k)e^{-2nb}dr^2$$

1 2 3

Where $p1(k)$, $p2(k)$, $p3(k)$ are directional scale factors, and mm is a constant. Specific values yield standard models: m=0, m=-1(Type III), m=1m=1 (Type V), m=−1m=−1 (Type VI$_0$), and other valuescorrespond to Type VI$_h$ where $h = m + 1h = m + 1$.

The Einstein field equations (with 8πG=c=18πG=c=1) are:

1

$V_{ij} - ()\, u_{ij} V = -(Qij(m) + Qij(\Lambda)]$ 2

where $Qij(m)$ and Q$ij(\Lambda)$ are the energy-momentum tensors for matter and HDE, respectively.

The HDE density is given by the Granda-Oliveros cut-off:

$$\rho\Lambda = 4(xQ2 + y\dot{Q})$$

with x and y being dimensionless constants.

The mean scale factor is a=(a1a2a3)1/3a=($a1a2a3$)1/3

III. SOLUTION WITH LINEAR DECELERATION PARAMETER

To solve the system, we assume a linearly time-varying deceleration parameter (Saha, 2014):

$$q = -\frac{\ddot{q}}{\dot{q}^2} = -1 + \frac{k}{qt}$$

Initial condition ($k > 0, n > 1$).

This form allows for a transition from decelerated (q>0) to accelerated (q<0) expansion. We analyze three distinct cases derived from the integration of the field equations under this ansatz, labeled Case 1, Case 2, and Case 3.

A. *Physical and Dynamical Parameters*

For each case, we derive:

- The directional Hubble parameters $H1$,$H2$,$H3$ and the mean H.
- The expansion scalar θ=3h.
- The shear scalar σ2σ2 and anisotropy parameter Qm, which measure the deviation from isotropy.
- The energy densities ρmρm and ρΛ$\rho\Lambda$, and the corresponding density parameters $\Omega m = \rho m/(3H2)\Omega m = \rho m/(3H2)$ *and* $\Omega\Lambda = \rho\Lambda/(3H2)\Omega\Lambda = \rho\Lambda/(3H2)$.
- The HDE equation of state parameter γΛ= $p\Lambda/\rho\Lambda$.

A key result across all models is that for specific parameter choices

$$\Omega m + \Omega\Lambda \to 1\, as\ t \to \infty t \to \infty.$$

This indicates the universe evolves towards a spatially flat, isotropic FRW geometry at late times, driven by the dominance of HDE.

B. *Equation of State and Quintessence Correspondence*

The derived EoS parameter γΛ$\gamma\Lambda$ exhibits rich behavior:

$\gamma\Lambda = -1\gamma\Lambda = -1$ identically, representing a pure cosmological constant.

$\gamma\Lambda$ has a more complex time-dependent form but can cross the phantom divide. To link HDE to a fundamental scalar field, we consider a quintessence field ϕ with Lagrangian

$L = 12\dot{\phi}2 - (\phi) = 21\dot{\phi}2 - V(\phi)$

For instance, in Case 1, equating $\gamma\Lambda\gamma\Lambda$ *with* $\gamma\phi\gamma\phi$ yields:

$\dot{\phi}2 - V(\phi)$

This establishes a concrete mapping, showing that the HDE behavior in our anisotropic setting can be effectively modeled by a dynamical scalar field.

IV. OBSERVATIONAL CONSISTENCY

We compare the theoretical Hubble parameter H(z) derived for each case with observational data from SNe Ia (see Tables 10.1, 10.2, 10.3 in original document). The calculated H(z)$H(z)$ values (e.g., for Case 1: $H(z) = H0(1 + z)nH(z) = H0(1 + z)n$) show close alignment with the observed values and their uncertainties. Furthermore, the computed present-day density parameters $\Omega m \approx 0.14 - 0.39\Omega m \approx 0.14 - 0.39$ *and* $\Omega\Lambda \approx 0.5 - 0.97\Omega\Lambda \approx 0.5 - 0.97$ ($for\ \beta = 0.5$, $\alpha = 1 - 1.46\beta = 0.5$, $\alpha = 1 - 1.46$) are consistent with constraints from joint BAO+SNe Ia analyses (Granda & Oliveros, 2008). Figures plotting (z), $\rho m(z)\rho m(z)$, and $\rho\Lambda(z)$ on a log scale demonstrate the universe's evolution from a denser, anisotropic past to a dark- energy-dominated, isotropic future.

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V. CONCLUSION

We have presented a detailed study of Holographic Dark Energy within a generalized anisotropic Bianchi cosmology. By assuming a linearly decreasing deceleration parameter, we obtained exact cosmological solutions that describe an accelerating universe. The analysis reveals:

1) The model successfully explains the late-time transition to acceleration and the eventual flattening and isotropization of the universe as HDE dominates.
2) The HDE component exhibits remarkable versatility, capable of mimicking the cosmological constant, phantom energy, quintessence, and even early- universe fluid phases based on the parameter nn.
3) A formal correspondence with quintessence scalar field theory is established, providing a dynamical field interpretation for the HDE.
4) The predictions for the Hubble parameter and density parameters show good agreement with current SNe Ia and BAO observational data.

This work strengthens the case for HDE as a viable dynamical dark energy candidate and demonstrates how anisotropic models can naturally evolve into the isotropic universe we observe today.

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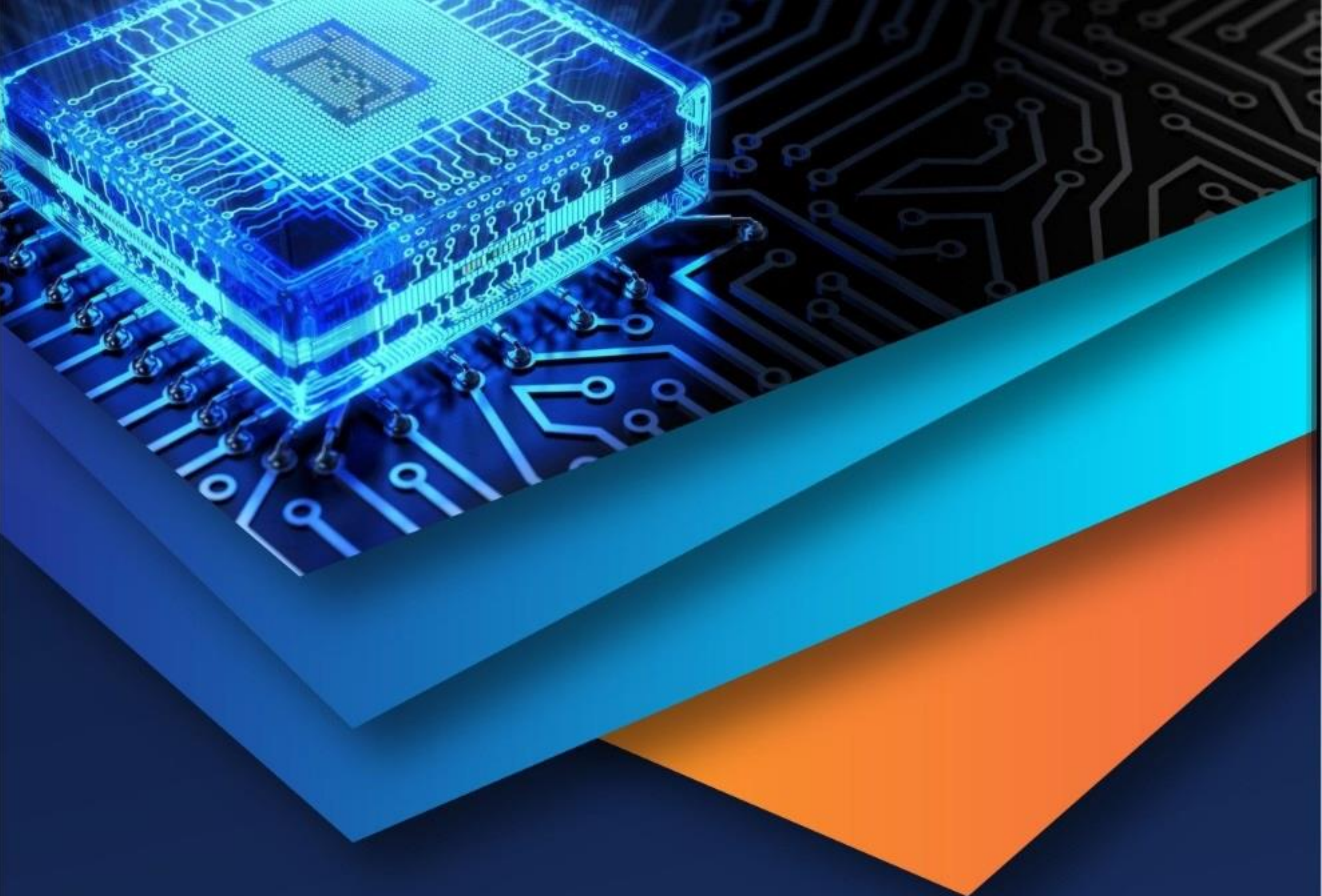

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