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How Can Ablative Cooling in A Solid Propellant Rocket Engine Be Accurately Modelled in Order to Optimise Performance?

Sassan Bhanji

Abstract: This essay investigates the use of ablative cooling in solid propellant rocket engines. It begins by exploring the mechanisms by which ablation occurs. It then demonstrates how heat transfer to an ablator can be modelled, and how this can be used to find ablator recession rate and hence the necessary ablator thickness for a rocket engine. It does so by considering simplified mathematical models.

These are then compared to the more complex models that have recently been developed. The different variables involved and how they might be used or calculated are discussed.

The next section of the essay ties this theoretical knowledge and modelling into practical engineering use by considering the impact ablation has on performance.

MATLAB is used to demonstrate how an expanding throat diameter of the nozzle can decrease thrust and specific impulse, and that this can greatly decrease payload capacities. Other variables involved in creating a thrust profile for a solid propellant rocket engine are considered.

Finally, the essay will look at how to choose an optimal ablator. It goes through the universally desirable characteristics and uses the Space Shuttle SRMs as an example of disadvantages that may not initially be considered when selecting an ablator.

I.   INTRODUCTION

Rocket engines are some of the most complex pieces of machinery humans have constructed. They essentially control high energy explosions in order to provide the thrust required to lift a payload into space.

The combustion temperatures as a result of the reactions between the fuel and oxidiser typically range from 2700K to 3600K; substantially higher than the melting point of the metals from which the rocket nozzle is made. In order to preserve the structural integrity of the nozzle wall, the wall temperatures must be far below the melting points of the metals. Various cooling techniques have been implemented to rapidly cool the engine and prevent rocket failure, the most common of which are regenerative cooling and ablative cooling. [1][2]

Regenerative cooling is currently the most common method used for liquid propellant rocket engines. It works by flowing propellant (which is often cryogenic) through the walls of the combustion chamber to transfer heat away rapidly. It enables the walls of engine to be fairly thin, which reduces the weight, hence increasing the specific impulse of the engine. It will also continue to function until the propellant runs out.

Furthermore, it can be integrated into an expander cycle. This is where the fuel boils while passing through the combustion chamber walls, and then spins a turbine, harnessing extra energy. [2]

Ablative cooling is a much simpler way of cooling an engine, as there is simply a layer of ablative material coating the engine wall, which vaporises as the hot exhaust gases pass by it, taking heat with it.

This method is much simpler to implement in an engine as there are no moving parts, however it does have several disadvantages. As the ablative layer erodes away, the nozzle expansion ratio changes, reducing the thrust output of the engine. This also means that these engines are not reusable, which can add significant risk to a mission as the same engine can't be tested and then flown. A notable example of this is the Apollo Lunar Ascent engine which wasn't fired until it was actually on the moon. Despite this, it's simplicity makes it extremely useful for smaller engines such as those on missiles, and also solid propellant rocket engines as there is no propellant to run around the combustion chamber. [2]
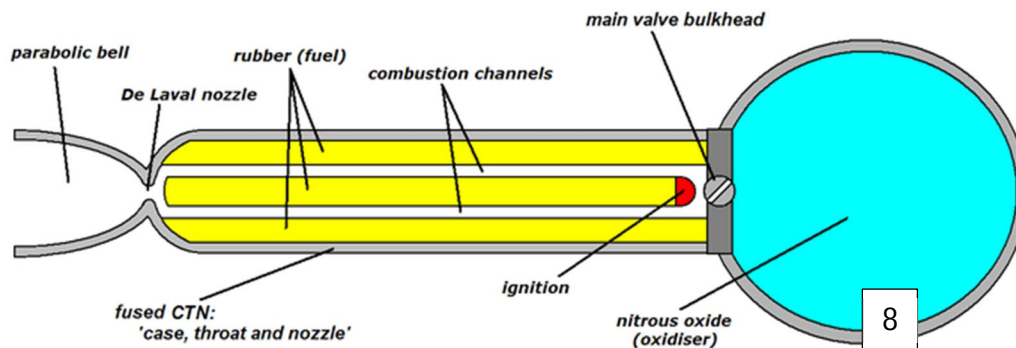
The three main types of rocket engine are liquid, solid, and hybrid. Liquid rocket motors have both the fuel and oxidiser in liquid form, which are stored in fuel tanks. The liquids are then sprayed by injectors into the combustion chamber where they react together. These are the most complex and expensive types of engine, because the liquid propellants must be forced into the combustion chamber so the chamber pressure is high enough. With smaller engines, an inert pressuring gas such as N2 or He can be used, but this is impractical on a larger scale. Larger motors usually use turbo pumps to pump the propellants into the combustion chamber, however energy is required to start this when the rocket takes off. As a result, pre burners are needed, which means staged combustion cycles are needed. Figure 1 shows a full-flow stages combustion cycle which has both an oxidiser rich and a fuel rich pre burner. In summary, liquid propulsion is optimal for large scale launchers, but brings many complexities with it. [3][4][5]
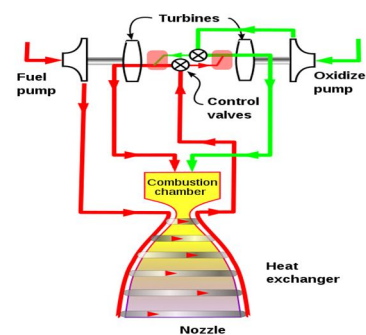
Solid propellant rockets are vastly more simple. They consist of a grain of fuel and oxidiser blended together in solid state. This is then ignited (like a firework) and combustion occurs. These produce very high thrust, but have a smaller specific impulse (essentially efficiency) than liquid motors. There are no moving parts making them much more reliable and cheaper. They are most commonly used for boosters (such as the space shuttle or SLS), and missiles.
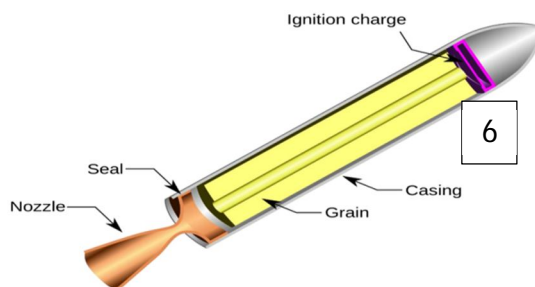
Hybrid rocket motors have one propellant in solid form, and the other in liquid form. This tends to be a solid fuel and liquid oxidiser. Their main advantage is that they are much simpler than liquid engines to design, but are still throttleable as the engine can be shut down in the same way as a liquid motor.

Sources:

1: https://everydayastronaut.com/engine-cooling-methodes/#:~:text=Ablative%20cooling%20is%20one%20of,an%20extremely%20high%20melting%20point

2:https://mountainscholar.org/bitstream/handle/10217/212046/Browne_colostate_0053N_16196.pdf?sequence=1

3: https://www.narom.no/undervisningsressurser/sarepta/rocket-theory/rocket-engines/the-engine-types-solid-liquid-and-hybrid-and-a-fourth/

4: https://www.grc.nasa.gov/www/k-12/rocket/rocket.html#:~:text=There%20are%20two%20main%20categories,the%20nozzle%20where%20burning%20occurs.

5: https://everydayastronaut.com/rocket-engine-cycles/

6: https://en.wikipedia.org/wiki/Staged_combustion_cycle

7: https://aerospaceengineeringblog.com/rocket-science-101-fuel-engine-nozzle/

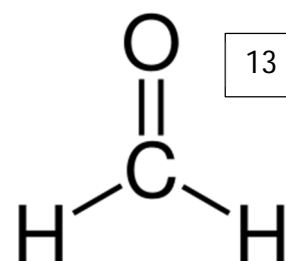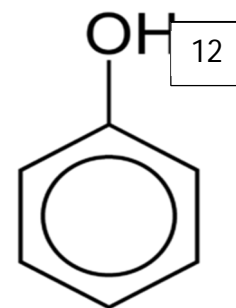8: https://en.wikipedia.org/wiki/Hybrid-propellant_rocket

II. ABLATOR CHEMISTRY

There are two types of ablator that can be used: pyrolyzing and non-pyrolyzing. Pyrolysis is the chemical process whereby materials that are heated thermally degrade into smaller, and often more volatile, molecules. Pyrolyzing ablators are typically composites with a silica or carbon fibre matrix in a honeycomb structure, filled with a highly insulating phenolic resin. [9][10]

A phenol, also called hydroxybenzene, is an aromatic organic compound. It has a molecular formula C_6H_5OH as shown in reference 12, and is used to synthesise many types of plastics. Phenolic resins are synthetic polymers formed by reacting phenol with formaldehyde. Formaldehydes have a molecular formula CH_2O as shown in reference 13. Phenolic resins have very good erosion resistance and char retention capacities. [11]

The phenolic resins used in ablative material also tend to have a high cross-link density, which gives them their high rigidity and binding strength. Crosslinking is the process of forming covalent bonds to link polymer chains together. Moreover, the large number of aromatic rings from the phenols present in the resin means that a large amount of carbonaceous char can be formed, enabling effective cooling.

The silica/carbon-fibre matrix is also crucial in increasing the insulating ability of the ablator. Studies by Torre et al. on a silica based ablative composite showed that the char formed is weak and brittle. This means that the high velocity gases flowing through the combustion chamber can mechanically erode this char layer rapidly. The fibre matrix adds crucial mechanical strength as the char layer is more stable. [11]

Sources:

9: file:///Users/sassanbhanji/Downloads/applsci-09-03323.pdf.
10: file:///Users/sassanbhanji/Downloads/NumericalPredictionofNozzleAblationinHybridRocketEngine-Panelli.pdf
11: https://www.scielo.br/j/jatm/a/KXnw7bv4R4F9BFVMT8X3zMk/?format=pdf&lang=en
12: https://en.wikipedia.org/wiki/Phenols
13: https://en.wikipedia.org/wiki/Formaldehyde

III. THE ABLATION MECHANISM FOR PYROLYZING ABLATORS

The pyrolysis of an ablator is initiated by the removal of -OH functional groups and -H atoms from the aromatic C rings (from the phenol group) of the phenolic resin. This releases H_2O. The next stage involves the breaking of the C rings, which forms the solid char products. This char has a high thermal emissivity so radiates heat quickly, insulating the virgin material. As crosslinks in the resin are scissored (split), pyrolysis gases such as methane, hydrogen and carbon monoxide are evolved which push away hot gases from the chamber wall, providing extra insulation. [14][15][16]

The pyrolysis gases move to the surface of the wall by percolation. Percolation is defined as the movement of fluids through a porous material, and as the char layer is porous the gases move to the surface in this way. Reference 17 summarises the heat and mass transfer for an ablating material. Pyrolysis is only one of the many complex processes involved in ablation. Melting, vaporisation, sublimation, and spallation all contribute to eroding the ablative material and taking away energy.
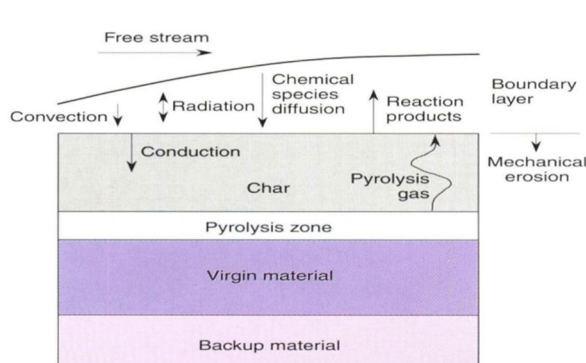
Sources:

14:
https://www.scielo.br/j/jatm/a/KXnw7bv4R4F9BFVMT8X3zMk/?format=pdf&lang=en#:~:text=Phenolic%2Dsilica%20composites%20are%20considered,ablation%20applications%20(Shi%20et%20al.

15: file:///Users/sassanbhanji/Downloads/PDST6648-1.pdf

16: Ablative Thermal Protection Systems: Pyrolysis Modelling by Scale-Bridging Molecular Dynamics. Abhilash Harpale, Saurabh Sawant, Rakesh Kumar, Deborah Levin and Huck Beng Chew. 2017

17: https://www.jhuapl.edu/Content/techdigest/pdf/V13-N03/13-03-Resch.pdf .

IV. HEAT TRANSFER ANALYSIS FOR A SOLID PROPELLANT ROCKET ENGINE

Firstly, it's important to consider what properties of the combustion chamber affect heat transfer to the ablator.

1) Gas transport properties
2) Chamber pressure
3) Flow-field patterns and velocities
4) Gas particle distribution and behaviour
5) Chamber geometry
6) Ablator material

These must all be defined in order to successfully calculate the heat transfer from the combustion products to the chamber wall.

In solid propellant rocket engines, the fuel grain burns at a predictable rate and the chamber pressure can be calculated by the nozzle throat diameter and grain burn rate, meaning that these parameters are easily defined.

It is worth noting the propellants of a solid propellant engine often contain metals such as Aluminium, so the combustion products consist of both gas particles, and hot metal/metal-oxide particles.

There are 3 main mechanisms by which heat is transferred to the ablator.

a) Convection from the combustion products (gas/particle mixture), $\dot{q}_{con}$
b) Radiation from the metal-oxide particles, $\dot{q}_{rad}$
c) Impingement and deposition of the burning metal and metal-oxide particles, $\dot{q}_{par}$

The total heat flux (rate of heat transfer from the combustion products to the ablator per unit area) is hence given by:

$$\dot{q}_{tot} = \dot{q}_{con} + \dot{q}_{rad} + \dot{q}_{par}$$

A. Calculating Convective Heat Flux

The heat transfer to a surface through convection is given by Newton's Law of Cooling:

$$q = h_{con} A \, dT$$

where

q = heat transfer
A = surface area
h_{con} = convective heat transfer coefficient
dT = difference in temperature between the surface and the bulk fluid

Hence, $\dot{q}_{con}$ can be given by:

$$\dot{q}_{con} = h_{con}(T_g - T_w)$$

where

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T_g = combustion gas temperature

T_w = chamber wall temperature

The value of h_{con} can be calculated using various different methods, including computational modelling. For a rigorous approach to calculating heat flux, sections of the nozzle where flow separation occurs should be considered separately. In these areas, h_{con} should be decreased as the local flow velocity will be lower. [18][19]

B. Calculating Radiative heat Flux

This is calculated using the Stefan-Boltzmann Law. The char layer itself can be assumed to be a black body, meaning it is assumed to completely absorb all wavelengths of thermal radiation incident on it. It is also assumed that the body doesn't reflect any light. Radiation per unit time from a black body is given by:

$$q = \sigma T^4 A$$

where

σ = The Stefan-Boltzmann Constant ($5.6703 * 10^{-8}$ W/m^{2}K^4)

Unfortunately, as the solid propellant combusts, it forms a metallic oxide cloud, which must be treated as a grey body. This means that some of the thermal radiation is reflected.

The heat flux is hence given by:

$$\dot{q}_{rad} = \varepsilon \sigma T^4$$

where

ε = the emissivity coefficient of the body ($0 < \varepsilon < 1$)

In order to calculate the overall heat flux from the combustion products to the wall, we treat them both as grey bodies. We will also assume a view factor $F_{12} = 1$.

F_{12} is given by the direct radiation from surface 1 onto surface 2, divided by the total radiation emitted by surface 1. [20]

Hence, the heat flux is given by the standard equation for radiation transfer between two grey bodies:

$$\dot{q}_{rad} = \frac{\sigma(T_{cl}^4 - T_w^4)}{\frac{1}{\varepsilon_{cl}} + \frac{1}{\varepsilon_w} - 1}$$

T_{cl} = the temperature of the cloud of combustion material (particles + gas)

T_w = the temperature of the wall

ε_{cl} = emissivity coefficient of the combustion material

ε_w = emissivity coefficient of the wall (assumed to be the same as its absorptivity) [18][21]

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C. Calculating heat Flux due to particle Impingement and Deposition

Particle impingement is where combustion products hit the combustion chamber walls. This can be modelled by tracing individual particles through the combustion chamber. Their motion is given by Newton's 2nd law, and Stokes' law to approximate the drag. [22] When these particles have a large temperature difference compared to the chamber walls, they transfer heat and cause erosion. When they are at a close temperature, they effectively shield the walls from erosion.

There are many different variables involved in this, so CFD is usually used to estimate the heat flux caused. Deposition of metal oxide particles is usually determined empirically.

Sources:
18: https://core.ac.uk/download/pdf/42877658.pdf
19: https://www.engineeringtoolbox.com/convective-heat-transfer-d_430.html
20: https://byjusexamprep.com/view-factors-i-4c15373a-c2b9-11e5-872e-38df5c6b002e
21: https://www.engineeringtoolbox.com/radiation-heat-transfer-d_431.html
22: https://core.ac.uk/download/pdf/42877658.pdf

V. CALCULATING THE REQUIRED ABLATOR THICKNESS

The cheapest and most simple method used to do this is the Q* method. The ablation process is extremely complicated, so many engineers use this simplification.

It is used to calculate the local ablation rate $\dot{x}_a$, which is defined as the rate at which the effective ablation temperature (temperature at which the ablator decomposes) moves into the ablator. As the engine runs and the ablator is heated, this temperature is recorded deeper into the ablator (at a linear rate).

Q* is the effective heat of ablation, which can be determined empirically or from a source as a known value.

$\dot{q}_{tot}$ is the total heat flux calculated previously, and ρ is the ablator density at RTP.

$$\dot{x}_a = \frac{\dot{q}_{tot}}{Q^*\rho}$$

As shown by the above equation, it is assumed, that $\dot{x}_a \propto \dot{q}_{tot}$. This would only cause a significant error if Q* is measured under conditions which are too different from the actual operating conditions of the rocket engine. [23]

Sources:
23: https://ntrs.nasa.gov/api/citations/20140008557/downloads/20140008557.pdf

VI. MORE COMPLEX MODELLING

The models used previously to calculate heat transfer and ablation rate are simplified, so although they often provide good enough data for calculations and design purposes, for an accurate, computational model of ablation, more sophisticated models are required.

Ablative cooling is extremely difficult to model, since it is not even understood that well, and most models use advanced software to produce estimates. I will outline what different variables must be considered to produce such a model.

A surface energy balance (SEB) is usually formed to show what physical and chemical processes are accounted for. This will determine the thermochemical ablation. Mechanical erosion (due to large particles, usually metal from the solid propellant) is calculated separately. These are summed to get the total ablator surface recession rate. [24]

$$\dot{s}_{tot} = \dot{s}_{mec} + \dot{s}_{th}$$

A. SEB

An expression for the SEB of an ablator would be based on conservation of energy. It would take a control volume at the surface and assess the incoming and outgoing flux. [25]

Incoming flux includes:

- Radiation
- Convection
- Conduction

The calculations for the first three were shown in the Q* method. Conduction in this case is going to be unsteady state, meaning it varies depending on both time and distance into the ablator. The ablator wall is assumed to be a long cylinder, so a lumped system analysis (used for approximating conduction to small, highly conductive bodies) can't be used. A partial differential equation is formed as temperature is a function of position and time, T(x,t). [26]

It is hence given as $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

Solving then applying the appropriate boundary conditions (temperature at x=0, temperature at t=0) gives an infinite series solution. The first term of this should be used (the others are insignificant).

The outgoing flux includes:

- The pyrolysis gas and char products
- Re-radiation

To calculate the rate of pyrolysis, a form of the Arrhenius relation is usually used [27]

$$k = Ae^{-\frac{E_a}{RT}}$$

k = rate constant

E_a = activation energy

A = frequency factor

R = gas constant

T = temperature

Mass flux of solid and gaseous pyrolysis products leaving the ablator must be accounted for when finding a SEB expression. These would be found using the Arrhenius relation, and other known variables such as specific heat capacity, thermal conductivity, density, enthalpy of pyrolysis, and phase change enthalpy

Re-radiation is included when calculating radiative heat flux between the two grey bodies as mentioned previously.

B. Mechanical Erosion

- Particle impingement
- Spallation

Particle impact can be modelled computationally. The fuel used, as well as ablator material will be needed for this to be modelled. This has been detailed previously.

Spallation is defined as the ejection of fragments of material from a body due to impact or stress. Particle impingement can cause fragments of the ablator to break off which results in ablator recession. It is important to note that this process can result in moving boundary conditions when solving for heat flux in the SEB. These are referred to as the Stefan problems and a Stefan condition must be calculated to solve the PDE. This condition expresses the velocity of the moving ablator surface as a function of the rate of latent heat release for this particular case. [28]

Evidently, there are other variables that could be accounted for to improve the model, but fairly reliable predictions can be made if all of these are combined into a model.

Sources:

24: https://tfaws.nasa.gov/TFAWS03/Data/Aerothermal%20and%20CFD%20Session/Wertheimer.pdf

25: https://www.scielo.br/j/jatm/a/8yS36fqgbmtbFYBrSJ8fhSx/?format=pdf&lang=en

26: https://www.sfu.ca/~mbahrami/ENSC%20388/Notes/Transient%20Heat%20Conduction.pdf

27:
https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Supplemental_Modules_(Physical_and_Theoretical_Chemistry)/Kinetics/06%3A_Modeling_Reaction_Kinetics/6.02%3A_Temperature_Dependence_of_Reaction_Rates/6.2.03%3A_The_Arrhenius_Law/6.2.3.01%3A_Arrhenius_Equation

28:**Error! Hyperlink reference not valid.**

VII. THE EFFECT OF ABLATOR RECESSION ON PERFORMANCE

As the ablative material erodes away, there is a decrease in performance of the rocket engine.

A. The basics of how rockets work (CD nozzles)

The propellants react together in the combustion chamber. The combustion products are then choked in the converging section of the nozzle, which increases their velocity. This is because, even though the fluid is compressible (density can change), the speed is less than Mach 1, so $\propto \frac{1}{A}$. Once the flow is choked, the mass flow rate cannot be increased further.

The aim of the nozzle is for the combustion products to reach Mach 1 at the throat. At this speed, compressibility can no longer be ignored, and the relationship is the opposite, $v \propto A$.

Therefore, to accelerate the fluid further to supersonic speeds, the nozzle must expand, hence the diverging section of the nozzle.

The faster the exit velocity of the fluid, the more thrust produced.

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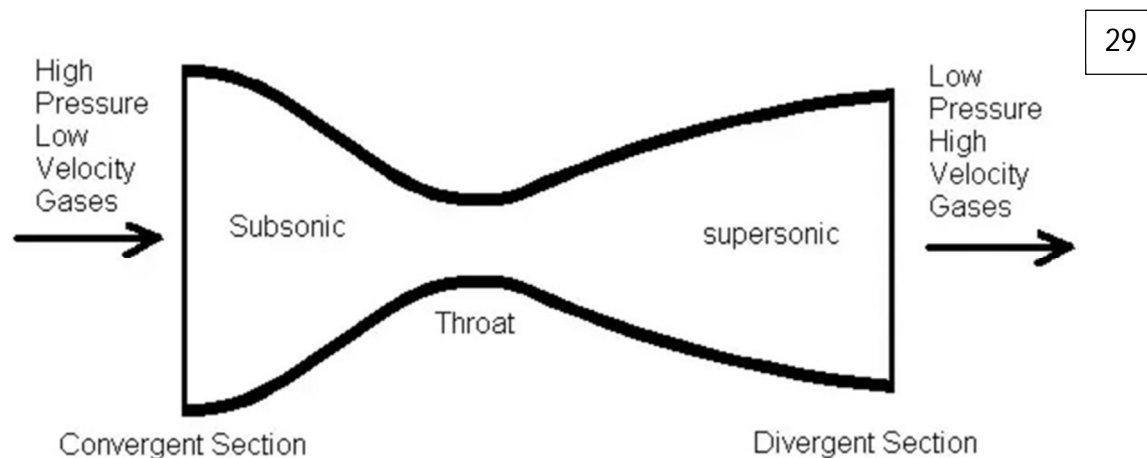
Thrust in a rocket engine is given by: [30]

$$F = \dot{m}v_e + (p_e - p_0)A_e$$

$\dot{m}$ = mass flow rate

v_e = exit velocity

p_e = exit pressure

p_0 = free stream pressure

A_e = exit area

B. Flow Expansion

When the combustion gases travel through the divergent section of the nozzle, they expand. Their pressure and temperature decrease while their speed increases, maximising thrust. Nozzles can either be under-expanded, over-expanded, ideal, or grossly over-expanded.

Under expansion occurs when exit pressure is greater than ambient pressure $p_e > p_0$, so the gas will continue to expand even after leaving the nozzle, which isn't optimal as the gas didn't reach it's maximum velocity.

Over expansion occurs when the exit pressure is too low. This is more efficient than overexpansion as the gases do reach maximum velocity, but if the pressure is around 30-45% smaller than ambient, flow separation can occur. This means that the gases do not stick to the nozzle wall, causing unpredictable and unstable flow, which can damage the nozzle. At this point, it is described as grossly over-expanded.

Ideal is where $p_e = p_0$, which can only occur at a certain altitude.

Flow expansion is important to consider when assessing the impact of ablation on thrust and specific impulse.

C. Impact of Ablation

Ablative material is needed to cover the entire nozzle, as if any part of it melts it would cause engine failure. This means that ablator recession occurs across the nozzle, which impacts the thrust produced as it widens the nozzle.

Considering the throat of the nozzle, this is designed to be the perfect diameter for fluid flow to reach Mach 1, meaning that as it widens, the flow is not fully choked (doesn't quite reach Mach 1. This means the thrust produced decreases as the ablator recedes because the exit velocity decreases.

There is no clear, widely used mathematical model showing how ablator recession rates impact thrust. Therefore, I will use modelling software to demonstrate the significance of the impact it can have. I will demonstrate this relationship for a solid propellant rocket engine because ablative cooling is most commonly used for these as regenerative cooling is not an option.

The calculation of ablator recession rate (V_{s0}) was described previously. Using the model developed by Wendell A. Stephen and Thomas E. Frakes for a carbon-phenolic ablator, a possible recession rate would be in the range of 0.12mm/s. [31]

Below is a demonstrative calculation for a generic small solid propellant engine with a carbon-phenolic ablator.

The following parameters have been established (not based off any existing rockets):

Parameter	Value
Chamber pressure (Pa)	3*10^6
Chamber temperature (K)	1500
Desired thrust (N)	5000
Mass flow rate (kg/s)	n/a
Max altitude (m)	10,000
Coefficient of heats	1.4
R	355
Burn time (s)	100
Throat diameter (cm)	40

NB the thrust equation shows that mass flow rate and thrust are functions of each other so only one of them can be inputted.

A MATLAB analysis using these values has been used to show how the fluid flow would change based on the receding throat diameter. The 100 second burn time would suggest a 1.2cm recession (uniform hence 2.4cm total). Figure X shows the initial flow, and Figure Y shows the flow at the end of the burn where the throat diameter has increased to 42.4cm.

This shows how within the space of the nozzle, the flow can't optimally expand. With the increased throat diameter, optimal flow expansion would a divergent section of the nozzle that is longer by 14.68cm and wider by 5.53cm. As the ablator recedes, the flow becomes increasingly under-expanded. The expansion of the flow is what enables it to reach its highest velocity, so a lower exit velocity is recorded for under-expanded flow.

$F = \dot{m}v_e + (p_e - p_0)A_e$ shows that the lower exit velocity decreases thrust.
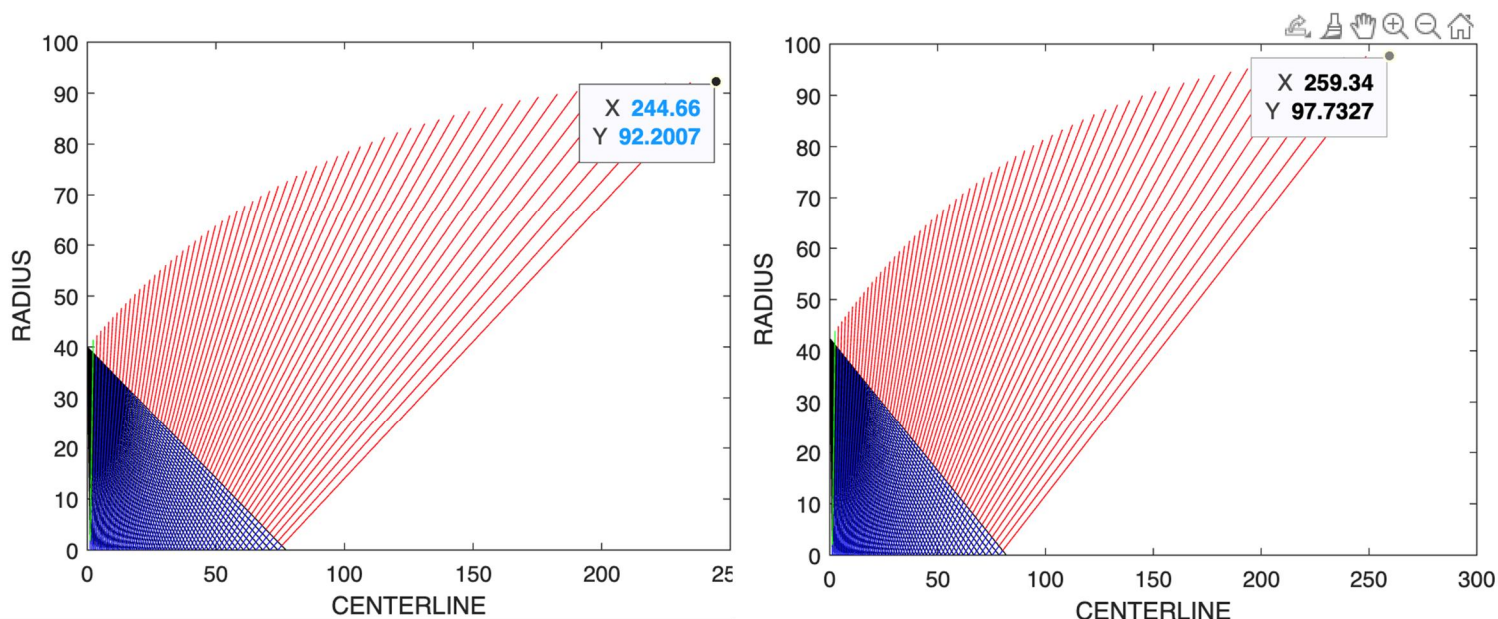
In general, engineers consider 5% to be the maximum acceptable throat enlargement, so the 1.2cm (3%) enlargement of this example would be allowed. However, it is clearly not optimal. Using the space shuttle as an example, erosion at the throat was the second largest source of thrust and specific impulse loss. Overall, it reduced the specific impulse of the motor by 0.9s, which could account for up to a 340kg loss in payload capability. The impact on a smaller motor such as the above example would be even worse, due to the inevitable larger percentage throat enlargement. [32]

Sources:
29: https://www.thespacetechie.com/why-rocket-engine-nozzles-diverge/
30: https://www.grc.nasa.gov/www/k-12/airplane/rockth.html
31: https://apps.dtic.mil/sti/pdfs/AD0749968.pdf
32: https://core.ac.uk/download/pdf/80648852.pdf
See Appendix A for the code used

VIII.   OTHER FACTORS THAT WOULD CONTRIBUTE TO A LOSS OF THRUST

While the focus of the above model was on the impacts of ablation, it is worth mentioning other variables that reduce rocket efficiency, which would also have to be included in a thrust profile. They must also be considered when analysing data as they could account for losses that ablation can't explain.

A.   Altitude

As the rocket moves up into the atmosphere, the air pressure decreases according to   the barometric formula: $P_h = P_0 e^{-mgh/kT}$ [33]

P_0 = pressure at the ground

P_h = pressure at height h

k = Boltzmann's constant

The relationship is exponential as shown in reference 34

This means that $p_e = p_0$ can only be true for a certain altitude. As altitude increases, exit pressure becomes larger than ambient pressure, causing the flow to be under expanded. This is the same impact as ablator recession, so any thrust profile should account for both distinctly, otherwise the model won't be accurate.
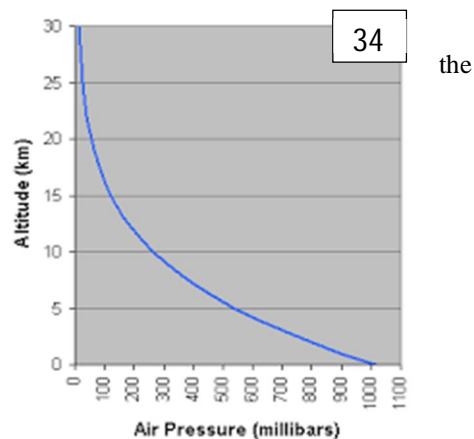
B. Grain Geometry

In solid propellant engines, the thrust is determined by the shape of the solid propellant.

Saint-Robert's Law shows us that chamber pressure is directly linked to propellant regression rate. [35]

$$\dot{r} = aP^n$$
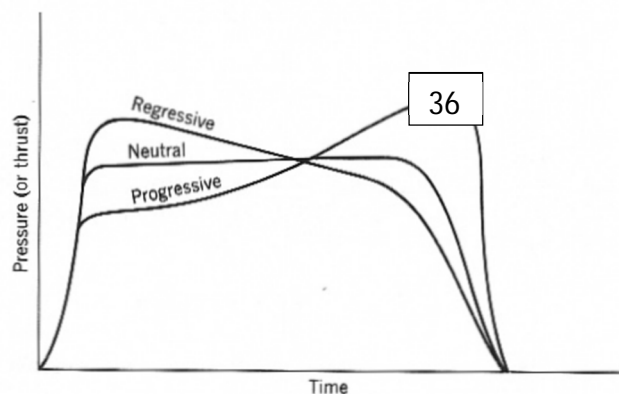
$\dot{r}$ = recession rate
a = burn rate coefficient
n = pressure exponent
P = chamber pressure

Chamber pressure is also linked to thrust, meaning that the propellant recession rate affects the thrust profile of the rocket.

Based on the grain shape, one of the three patterns in reference 36 will be observed.

Sources:

33: http://hyperphysics.phy-astr.gsu.edu/hbase/Kinetic/barfor.html#c2

34: https://www.windows2universe.org/earth/Atmosphere/pressure_vs_altitude.html

35:
http://www.braeunig.us/space/propuls.htm#:~:text=Burn%20rate%20is%20profoundly%20affected,is%20the%20combustion%20chamber%20pressure.

36:
https://web.stanford.edu/~cantwell/AA284A_Course_Material/Karabeyoglu%20AA%20284A%20Lectures/AA284a_Lecture11.pdf

IX. SELECTING THE OPTIMAL ABLATOR FOR A SOLID PROPELLANT ROCKET ENGINE

The choice of ablator is extremely important when designing a rocket engine. The ablator should have the following characteristics:

1) High heat of ablation
2) High enthalpy of phase change
3) Sufficient strength
4) High specific heat
5) High thermal shock resistance
6) Low thermal conductivity
7) Low erosion rate
8) Pyrolysis products with a low molecular mass
9) Low density

These criteria give the basis for choosing the optimal ablator. [37]

This section of the paper outlines how the ablators for the space shuttle solid rocket boosters were selected and iterated. While the specific materials are unique to this motor, the same types of insulator and same considerations are used universally. [38]

A. Different Types

1) *Asbestos-silica-filled acrylonitrile butadiene rubber (ASNBR):* This was used as the main ablator for the reusable solid rocket motors for the Space Shuttle. While it is extremely effective, asbestos is extremely dangerous as it is carcinogenic. This also makes producing the asbestos fibres very expensive.
2) *Kevlar-filled ethylene propylene diene monomer (KF/EPDM):* This was developed as an asbestos-free insulator, but in initial testing NASA encountered two key problems.

B. Electrostatic Energy

KF/EPDM created high levels of electrostatic energy on the surface, which did not readily dissipate. This is because it is a very good electrical insulator as well as a thermal insulator, so static charge builds up. The sudden discharge of this energy could cause damage to the nozzle and is a safety hazard. Several rocket failures have been recorded as a result of static charge build up. To prevent this, a Hypalon paint with a high concentration of Electro-Conductive powders was used. The structure of the ablator has been modified to enable it to dissipate some charge without losses of thermal-ablative properties.

C. Bondline Corrosion

This is where the adhesive between different materials in the ablator is worn away, weakening it. The adhesives initially used were Chemlok® 205 and Chemlok® 236X. The factors discovered during testing that contributed to bondline corrosion were: cure temperature, cure time, adhesive age, adhesive thickness. Through testing, the optimal adjustment seemed to be altering the cure temperature and time.

Sources:

37: https://core.ac.uk/download/pdf/42736292.pdf

38: https://ntrs.nasa.gov/api/citations/20000091530/downloads/20000091530.pdf

X. CONCLUSION

Overall, it is clear that regenerative cooling is the method of choice for liquid propellant engines for a reason. It is far simpler to model and much more accurate predictions can be made. Ablation is still not particularly well understood, and the lack of a clear mathematical model is evidence for this. Many models, such as the Q* method rely on experimental data which comes with several disadvantages. The more sophisticated models often don't consider certain processes, and there is a large difference in models used by different engineers. However, advances in computational modelling seem to provide close enough estimates that engineers are still able to safely implement ablative cooling systems. With the use of a model evaluating a greater number of physical and chemical processes, more data can be produced, enabling performance to be optimised. The drawbacks of ablation can hence be lessened by assessing it's impact of a rocket's thrust profile, and making necessary changes to minimise these.

APPENDIX A

MATLAB code based off code written by Vinayak Deshpande

```
clc; close all; clear all;
%INPUT VALUES
p_1 = 3*10^6;      %CHAMBER PRESSURE
T_1 = 1500;        %CHAMBER TEMP
FT = 5000;         %DESIRED THRUST OR....
m_dot = 0;     %DESIRED MASS FLOW RATE....
ALT = 10000;       %ALTITUDE
g = 1.4;           %GAMMA
R = 355;           %GAS CONSTANT

%% exit pressure
if (11000>ALT) && (ALT<25000)
    T = -56.46; %C
    p_o = 1000*(22.65*exp(1.73-0.000157*ALT));
elseif ALT>=25000
    T = -131.21 + 0.00299*ALT ;
    p_o = 1000*(2.488*((T+273.1)/216.6)^-11.388);
else
    T = 15.04 - 0.00649*ALT;
    p_o = 1000*(101.29*((T+273.1)/288.08)^5.256);
end

%% begin calculation
PR = p_o/p_1;
PR2 = (p_o/p_1)^((g-1)/g);
TT = (2*g*R*T_1)/(g-1);
p_t = ((2/(g+1))^(g/(g-1)))*2.068;
v_t = sqrt((2*g*R*T_1)/(g+1));
v_e = sqrt(TT*(1-PR2));
```

```
for m = 2:n+1
    T(m) = (DT + (m-1))*DTOR;
    %Mach from T(i) using T(i) = v_PM (FALSE POSITION)
    x_int = [1 1.01*Me];
    func = @(x) T(m) - v_PM(x);
    M(m) = fzero(func,x_int);
    P(m) = 0 + TR*tan(T(m)); %X-AXIS POINTS
    %RRSLOPES
    RR(m) = -TR/P(m);
    %LR slopes
    LR(m) = tan(T(m)+asin(1/M(m)));
    SL(m) = -RR(m);
end
%% PLOTTING
P(1) = [];
l = length(P);

for j = 1:l
    P1 = [0 TR];
    P2 = [P(j) 0];
    plot(P2,P1,'k')
    hold on
    xlabel('CENTERLINE')
    ylabel('RADIUS')
end
hold on;
LR(1) = []; RR(1) = [];
SL(1) = [];
F = RR(m-1);
```

```matlab
if m_dot==0
    m_dot=FT/v_e;
elseif FT==0
    FT = m_dot/v_e;
else
    fprintf('You can either set desired thrust OR mass flow rate')
end


T_e = T_1*(p_o/p_1)^((g-1)/g);
a_e = sqrt(g*R*T_e);


Me = v_e/a_e;

% MOC
TR = 38.5; %throat radius (cm)
RTOD = 180/pi;
DTOR = pi/180;
P = []; %x axis points
%% PM FUNCTION
A = sqrt((g+1)/(g-1));
B = (g-1)/(g+1);
v_PM = @(x) A*atan(sqrt(B*(x^2-1))) - atan(sqrt(x^2-1));


%% CALCULATE T_MAX, BREAK UP INTO DIVISIONS
T_max = 0.5*v_PM(Me)*RTOD;
DT = (90-T_max) - fix(90-T_max);
T(1) = DT*DTOR;
n = T_max*2;
```

```matlab
for c = 1:length(P)-1
    x(c) = (TR+SL(c)*P(c))/(SL(c)-F);
    y(c) = F*x(c)+TR;
    X_P = [P(c) x(c)];
    Y_P = [0 y(c)];
    plot(X_P,Y_P,'b');
end
hold on

%% FIRST WALL SECTION
TM = T_max*DTOR;
xw(1) = (TR+SL(1)*P(1))/(SL(1)-tan(TM));
yw(1) = tan(TM)*xw(1)+TR;
X_P2 = [P(1) xw];
Y_P2 = [P(2) yw];
plot(X_P2,Y_P2,'g');
%DIVIDE (delta slopes)
DTW = tan(TM)/(length(P)-1);
s(1) = tan(TM);
b(1) = TR;
```

```matlab
for k = 2:length(P)-1
    s(k) = tan(TM)-(k-1)*DTW; %slope
    b(k) = yw(k-1)-s(k)*xw(k-1); %y-int
    xw(k) = (b(k)+SL(k)*P(k))/(SL(k)-s(k));
    yw(k) = s(k)*xw(k)+b(k);
    X_P3 = [x(k) xw(k)];
    Y_P3 = [y(k) yw(k)];
    plot(X_P3,Y_P3,'r');
end
hold on

LAST POINT
xf = (b(length(b))+SL(length(SL))*P(length(P)))/SL(length(SL));
yf = b(length(b));
X_F = [P(length(P)) xf];
Y_F = [0 yf];
plot(X_F,Y_F,'r');

xw = [0 xw];
yw = [TR yw];
RTHROAT = TR;
REXIT = yw(length(yw));

AR = (RTHROAT/REXIT)^2;
```

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