



# **iJRASET**

International Journal For Research in  
Applied Science and Engineering Technology



---

# **INTERNATIONAL JOURNAL FOR RESEARCH**

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume:** 12    **Issue:** XII    **Month of publication:** December 2024

**DOI:** <https://doi.org/10.22214/ijraset.2024.66027>

**[www.ijraset.com](http://www.ijraset.com)**

**Call:** ☎ 08813907089

**E-mail ID:** [ijraset@gmail.com](mailto:ijraset@gmail.com)

# Hydraulic Modeling of the Heterogeneous Mixture Transfer Process in Hydromorphic Environment with Unsteady Filtration

Azizbek Ernazarov<sup>1</sup>, Umidjon Sadiyev<sup>2</sup>, Daniyar Paluanov<sup>3</sup>

<sup>1,2</sup>Scientific Research Institute of Irrigation and Water Problem, Tashkent, Uzbekistan

<sup>3</sup>Tashkent state technical university named after Islam Karimov, Tashkent, Uzbekistan

**Abstract:** *The development of water management and land reclamation require qualitatively new theoretical and methodological levels of research, in particular, the search for effective mechanisms for solving the associated mass transfer by interconnected flows of surface, groundwater and infiltration moisture of the soil, which causes the need to develop more advanced hydrodynamic models of the dynamics of groundwater and surface water, allowing to describe the processes of mass transfer taking into account hydraulic, hydrogeological, physico-chemical processes in systems – open watercourse and upper layers of soil.*

*By solving scientific and technical problems of water management and reclamation practice, special attention should be paid to the interrelated processes of water movement and mass transfer of various physical nature, namely along furrows, in the humidification zone - infiltration. The article describes comprehensively study the processes of interconnected surface and groundwater. The solution of these problems can be obtained by creating mathematical models of interconnected surface water and moisture changes in the soil moisture zone*

**Keywords:** *mass transfer, hydrodynamic models, interconnected surface water and moisture, soil moisture, mass transfer coefficient, Reynolds criterion, Peclet criterion.*

## I. INTRODUCTION

The development of hydrodynamic models of the movement of surface and groundwater requires taking into account the complex nature of the phenomena under study, the real spatial and temporal scales of the considered hydrological reclamation objects and processes. By using the hydraulic approximation in the hydrodynamic description of the movement of both groundwater and surface water, it is necessary to take into account the ratio of the vertical dimensions of currents with horizontal measurements of hydrological and reclamation facilities.

The creation of a hydrodynamic model of interconnected surface and groundwater is based on the equations of Richards, Boussinesq and Saint-Venant. That is, to describe the movement of vertical infiltration water in the zone of incomplete saturation of soil, the one-dimensional Richards model is used and the change in the groundwater level.

Despite this, to date, the problems of mass transfer in interconnected groundwater and surface water flows using spatial formulation have not been fully studied. In this regard, a joint approach to constructing such models is usually used, based on the coupling of surface and groundwater models, which are complex hydraulic and geohydrodynamic processes.

By studying the dynamics of soil moisture, in which a complex geo-hydro dynamic process of transformation and transpiration of moisture by plant roots takes place, it is especially necessary to pay attention to modeling infiltration runoff based on hydraulic methods.

## II. GENERAL PROVISIONS

For modeling the process of mass transfer by interacting currents of surface and groundwater, taking into account the migration of moisture in the humidification zone, the dimensional analysis method is used.

To carry out the basic analysis of dimensions, equations are used that express the relationship between the parameters and variables of the process and the quantities included in them must be adequate and commensurate. Also, all terms of the equations must have the same dimensions. When solving the problems of mass transfer by interacting surface water currents of moisture dynamics in the humidification zone, the method of dimensional analysis is of particular importance.

The main content of the theory of dimension is the  $\pi$ -theorem, when there is some functional dependence between various quantities  $f(X_1, X_2, \dots, X_n) = 0$ , and it is assumed that  $m$  will be equal to the maximum number of these dimensional quantities with independent dimensions. In this case, the relationship between the dimensional quantities can be represented as  $(n-m)$  and, each of them will have the form of a power-law monomial. Then, the numbers of the basic units of measurement used to measure all variables will look like this:

$$\begin{aligned}\Delta P &= ML^{-1}T^{-2} = H/m^2 = kg/s^2m, \\ V &= LT^{-1} = m^2/s, \\ \rho &= ML^{-3} = H/m^3, \\ v &= L^2T^{-1} = m^2/s.\end{aligned}$$

Let's assume that some parameters of the mass transfer process  $\Pi$  are related to other process parameters  $A, B, C, D$  by the following dependence:

$$\Pi = f(A, B, C, D) = kA^a B^b C^c D^d \quad (1)$$

Where  $k, f, c, d$ - unknown coefficients that we can determine based on experimental studies. It is assumed that the process parameters  $A, B, C, D$  depend on the hydraulic parameters of the water flow through the irrigation channel, viscosity, density, velocity, temperature, which in the notation of dimensions are expressed as:

$$\begin{aligned}\Pi &= L^{a_0} T^{m_0} \theta^{k_0} M^{n_0} \\ A &= L^{a_1} T^{m_1} \theta^{k_1} M^{n_1} \\ B &= L^{a_2} T^{m_2} \theta^{k_2} M^{n_2} \\ C &= L^{a_3} T^{m_3} \theta^{k_3} M^{n_3} \\ D &= L^{a_4} T^{m_4} \theta^{k_4} M^{n_4}\end{aligned}$$

Then equation (1) takes the following form:

$$L^{a_0} T^{m_0} \theta^{k_0} M^{n_0} = k(L^{a_1} T^{m_1} \theta^{k_1} M^{n_1})(L^{a_2} T^{m_2} \theta^{k_2} M^{n_2})(L^{a_3} T^{m_3} \theta^{k_3} M^{n_3})(L^{a_4} T^{m_4} \theta^{k_4} M^{n_4})^d \quad (2)$$

Comparing the degrees at the same dimensions, we get:

$$\begin{aligned}a_0 &= a\alpha_1 + b\alpha_2 + c\alpha_3 + d\alpha_4 \\ m_0 &= am_1 + bm_2 + cm_3 + dm_4 \\ k_0 &= ak_1 + bk_2 + ck_3 + dk_4 \\ n_0 &= an_1 + bn_2 + cn_3 + dn_4\end{aligned}$$

In the numerical implementation of the equation system with regarding to  $a, b, c, d$  and  $k$ , we accept the coefficient  $n-m$  ( $n$ - the number of unknown coefficients,  $m$ - the number of equations). To solve the equation (2) it is necessary to determine the mass transfer coefficient. By using the dimension method, we define an empirical expression for the mass transfer coefficient  $\beta$  [ $LT^{-1}$ ], for the function of the characteristic body size  $r[L]$ , flow rates  $V[LT^{-1}]$ , viscosity  $\eta[MT^{-1}L^{-1}]$ , flow density  $\rho[ML^{-3}]$  and the diffusion coefficient  $D[L^2T^{-1}]$ .

The mass transfer coefficient takes the following form:

$$\beta_L = kr^a V^b \rho^c \eta^d D^e$$

Substituting the dimension values, we get:

$$LT^{-1} = kL^a (LT^{-1})^b (ML^{-3})^c (MT^{-1}L^{-1})^d (L^2T^{-1})^e$$

Comparing the degrees for the corresponding dimensions, we obtain:

$$\begin{aligned}L: 1 &= a + b - 3c - d + 2e \\ T: -1 &= -b - d - e \\ M: 0 &= c + d\end{aligned} \quad (3)$$

Here the number of equations is equal to  $m=3$ , and the number of unknown coefficients is equal to  $n=5$ . The number of key parameters is equal to  $n-m=2$ . The key parameters are used  $b, d$ , and the rest of the coefficients are expressed in terms of these coefficients. From the last equation (3) there is  $c=-d$ , from the second equation  $-e=1-b-d$ . Then, from the first equation it will take the form  $a=b-1$ . Taking into account these values, we get:

$$\beta_L = kr^{b-1} V^b \rho^{-d} \eta^d D^{1-b-d}$$

After that, this equation is transformed into:

$$\frac{\beta_L r}{D} = k \frac{r^b V^b \eta^d}{D^b \rho^d D^d}$$

$$Sh = \frac{\beta_L r}{D}, Pe = \frac{Vr}{D}, Sc = \frac{\eta}{\rho D}$$

By entering the criteria into the equation for calculating the mass transfer coefficient, we obtain the following:

$$Sh = kPe^b Se^d$$

If  $Pe=ReSc$ , then we may write it down:

$$Sh = kPe^b Sc^{b+d}$$

Coefficients  $(k, b, d)$  for any mass transfer process, we can determine using experimental data.

In the sequel, the dimensionality method is used to calculate the heat transfer coefficient  $a = \frac{Bm}{m^2 K} = \frac{kr}{cek^3 K} = [MT^{-3}\theta^{-1}]$ , depending on the flow rate  $V[LT^{-1}]$ , body size  $r[L]$ , viscosity of the medium  $\eta[MT^{-1}L^{-1}]$ , the density of the medium  $\rho[ML^{-3}]$ , thermal conductivity of the medium and heat capacity of the flow  $\lambda[LMT^{-3}\theta^{-1}]$ , t.e.

$$a = kr^a V^b \rho^c \eta^d \lambda^e C_p^f \quad (4)$$

Taking into account the dimensions of the corresponding quantities when comparing the degrees of the same dimensions, we obtain:

$$MT^{-3}\theta^{-1} = kL^a(LT^{-1})^b(ML^{-3})^c(MT^{-1}L^{-1})^d(MLT^{-3}\theta^{-1})^e(L^2T^{-2}\theta^{-1})^f$$

$$L: 0 = a + b - 3c + d + e + 2f$$

$$T: -3 = -b - d - 3e - 2f$$

$$M: 1 = c + d + e$$

$$\theta: -1 = -e - f$$

In this system  $m=4$  equations and  $n=6$  unknown coefficients. Taking as key coefficients  $c$  and  $f$ , we express the remaining coefficients in terms of them. From the last equation we have  $e=1-f$ , from the second equation  $-b=c$ , from the third equation  $d=f-c$ , then from the first equation we have  $a=c-1$ . Substituting these coefficient values into the above equation and grouping the corresponding variables, we obtain:

$$\begin{aligned} \frac{ar}{\lambda} &= k \left( \frac{rV\rho}{\eta} \right)^c \left( \frac{\eta C_p}{\lambda} \right)^f \\ Nu = \frac{ar}{\lambda}, Re &= \frac{rV\rho}{\eta} = \frac{rV}{\nu}, Pr = \frac{\eta C_p}{\lambda} \\ Nu &= K(Re)^c (Pr)^f \end{aligned}$$

In this expression, the coefficients  $c, k$  and  $f$  defined based on the experimental values of the measured values. From the modeling conditions, it becomes necessary to determine the coefficient of resistance of solid particles. At the same time, the resistance force of solid particles  $F [MLT^{-2}]$  depends on: particle diameter  $a[L]$ , flow rates  $V [LT^{-1}]$ , densities  $\rho[ML^{-3}]$  and dynamic viscosity  $\eta[ML^{-1}T^{-1}]$ . The general expression for the resistance forces we may write as:

$$F_D = kd_p^a V^b \rho^c \eta^d$$

Moving on to the dimensional values and comparing the same degrees, we have:

$$MLT^{-2} = kL^a(LT^{-1})^b(ML^{-3})^c(ML^{-1}T^{-1})^d$$

$$L: 1 = a + b - 3c - d$$

$$T: -2 = -b - d$$

$$M: 1 = c + d$$

Taking the coefficient  $d$  as the key, we write down the remaining coefficients as  $a = 2 - d, b = 2 - d, c = 1 - d$ .

Then we may write:

$$\begin{aligned} F_D &= kd_p^{2-d} V^{2-d} \rho^{1-d} \eta^d \\ Re_d &= \frac{Va\rho}{\eta} \\ F_D &= kd_p^2 V^2 Re_d^{-d} \\ F_D &= C_d S \frac{\rho V^2}{2} \end{aligned}$$

By entering a dimensionless number, we get:  $S = \frac{\pi d_p^2}{4}$



Expressing the strength of resistance as  $C_D = \frac{24}{Re_d}$ , we will get:

$$C_D = \frac{8}{\pi} \left( \frac{F_D}{d_p^2 V^2} \right) = \frac{8K}{\pi Re_d^{-d}} = A Re_d^{-d} = f(Re_d)$$

where  $A$ - some experimentally determined coefficient. As noted above, for small values  $Re \ll 1$ , value  $d=1$  and  $A=24$ . Similarly, the resistance force for a particle in a non-Newtonian fluid is determined by assuming  $F = f(\rho, k, d, V)$ ,  $n$ - the indicator of the degree and  $k$ - the coefficient of consistency, as well as the following dimensions will be introduced:  $k = [ML^{-1} T^{n-2}]$ ,  $n = [M^0 L^0 T^0]$ . Further, similar to the above calculation for the coefficient of resistance force, we obtain:

$$C_D = \frac{F_d}{\rho V^2 d_p^2} = f \left( \frac{\rho V^{2-n} d_p^n}{k}, n \right) = f(Re_d, n)$$

Derivation of the equation (1) assumes that the probabilistic process under consideration obeys a nonlinear law:

$$\frac{da(t)}{dt} = f(a(t), t) + G\zeta(t) \quad (5)$$

It can be seen from this that the function describes the dynamics of particle sizes depending on convective mass transfer:

$$\begin{aligned} M[\zeta(t)] &= 0 \\ Cov[\zeta(t)\zeta(\tau)] &= B\delta(t-\tau)\delta(t-\tau) \\ f(a(t), t) & \end{aligned} \quad (6)$$

where  $f(a(t), t)$ , describes the growth of particle sizes in moisture transfer processes.  $f(a(t), t) > 0$ ,  $f(a(t), t) < 0$ , describes the dynamics of particle sizes in mass transfer processes.

If we linearize the function in the neighborhood of some mean:

$$f(a(t), t) \approx f(\mu_a(t), t) + \frac{\partial f(\mu_a(t), t)}{\partial \mu_a(t)} (a(t) - \mu_a(t))$$

then the system of differential equations for the normal distribution is represented as:

$$\begin{aligned} \mu_a(a) \\ \frac{d\mu_a(a)}{dt} &= f(\mu_a(t), t) \\ \frac{d\sigma_a^2}{dt} &= \frac{\partial f(\mu_a(t), t)}{\partial \mu_a} \sigma_n^2 + \sigma_n^2 \frac{\partial^2 f(\mu_a(t), t)}{\partial \mu_a^2} + G(\mu_a(t), t) B G^T(\mu_a(t), t) \end{aligned} \quad (7)$$

Decision (7) it is implemented using initial conditions in the form of:

$$(\mu_a(0) = \mu_{a0}, \sigma_n^2(0) = \sigma_{n0}^2.$$

It is assumed that the distribution function obeys the normal law:

$$P(a, t) = \frac{1}{\sigma_n \sqrt{2\pi}} \exp \left[ -\frac{(a - \mu_a)^2}{2\sigma_n^2} \right] \quad (8)$$

and the change of coordinates is described by a linear equation:

$$d(a) = -Aa + \zeta(t) \quad (9)$$

This equation describes changes in the size of capillaries, where the values of particles change monotonously over time.

To determine the law of variation of variance  $\mu_a(a)$  and  $\sigma_n^2(t)$  we get the equation in the form:

$$\frac{\partial P(a)}{\partial t} = \frac{\partial}{\partial a} \left( \frac{da}{dt} P(a) \right) + \frac{B \partial^2 P(a)}{2 \partial a^2} \quad (10)$$

After the appropriate conversion, it turns out  $a_s = \mu_a$ :

$$\begin{aligned} \frac{d\sigma_a^2}{dt} &= -2A\sigma_a^2 + B, \sigma_a^2(t)|_{t=0} = \sigma_{a0}^2 \\ \frac{d\mu_a}{dt} &= -A\mu_a, \mu_a(t)|_{t=0} = \mu_{a0} \end{aligned}$$

By solving this system of equations with initial values, we obtain  $\sigma_a^2 = \sigma_{a0}^2$

$$\mu_a = \mu_{a0} \exp(-At) + \frac{B}{2A} [1 - \exp(-2At)] \quad (11)$$

Taking into account the following assumptions, there is:

- the nature of the distribution function is constant;
- the number of particles per unit volume is determined by the formula:

$$\mu_{a0} = 2; \Pi \int_0^a P(a) da$$

By integrating (10) within the range of 0 to  $r$ , we obtain an expression representing the equation of convective mass transfer:

$$\frac{\partial \Pi}{\partial t} = D \left( \frac{\partial^2 \Pi}{\partial r^2} + \frac{2\partial \Pi}{r\partial r} \right) + \omega(r, t) \quad (12)$$

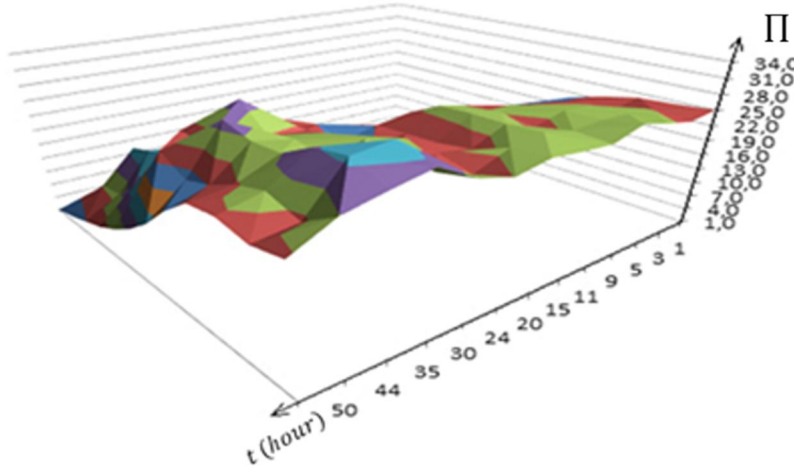


Fig. 1. Results of the numerical experiment of convective mass transfer equation

### III. CONCLUSION

Effective mechanisms have been developed for the numerical implementation of the shallow water and Richard's equations, which allow numerical experiments of the obtained hydrodynamic models with a high degree of accuracy, and an effective hydraulic model of convective mass transfer has been developed, which allows describing the processes of surface runoff and infiltration with varying degrees of accuracy and detail

Based on the developed hydraulic model of convective mass transfer the interacting with surface water flows along a furrow with a non-stationary bottom and between the dynamics of moisture in the soil-humidification zone, it can be used to develop devices and algorithms for controlling the state of the soil humidification zone

### IV. ACKNOWLEDGEMENTS

This publication has been produced within the framework of the Grant «Development of hydraulic technologies for managing of soil moisture during furrow irrigation of agricultural crops» ((REP-24112021/66), funded under the MUNIS Project, supported by the World Bank and the Government of the Republic of Uzbekistan. The statements do not necessarily reflect the official position of the World Bank and the Government of the Republic of Uzbekistan.

### REFERENCES

- [1] Karshiev, Rustam & Urazkeldiyev, Abduvoxid & Rajabov, Adhamjon & Ernazarov, Azizbek. (2021). Hydraulic calculation of reliability and safety parameters of the irrigation network and its hydraulic facilities. E3S Web of Conferences. 264. 04087. doi.org/10.1051/e3sconf/202126404087
- [2] Mahmudov, I. E., et al. "Evaluation of the Management and Use of Water Resources in the Middle Reaches of the Syr Darya Basin (Chirchik-Akhangaran-Keles Irrigation District)." Advanced Engineering Forum, vol. 18, Trans Tech Publications, Ltd., Sept. 2016, pp. 75–79. Crossref, doi:10.4028/www.scientific.net/aef.18.75.
- [3] Ilkhomjon Makhmudov, Umidjon Sadiev, Khurshid Lapasov, Azizbek Ernazarov, Shokhruxh Rustamov; Solution of the filter flow problem by analytical and numerical methods. AIP Conf. Proc. 16 June 2022; 2432 (1): 040006. https://doi.org/10.1063/5.0090359
- [4] Ilkhomjon Makhmudov, Umidjon Sadiev, Shokhruxh Rustamov; Basic conditions for determining the hydraulic resistance to friction in a pipeline when a mixture of water and suspended sediments moves. AIP Conf. Proc. 16 June 2022; 2432 (1): 040005. https://doi.org/10.1063/5.0090349
- [5] Sadiev, Umidjon, et al. "Formation of a geographic information system in the reliable management of water resources of the Southern Mirzachul channel." E3S Web of Conferences. Vol. 410. EDP Sciences, 2023. https://doi.org/10.1051/e3sconf/202341004015
- [6] R. Karshiev et al. Hydraulic calculation of reliability and safety parameters of the irrigation network and its hydraulic facilities, E3S Web of Conferences, EDP Sciences, 264 (2021).
- [7] Gulomov, O. G., A. I. Ernazarov, and M. M. Rustamova. "Hydraulic Modeling Of Water-Air Flow In A Tubular Spillway With Local Hydraulic Resistance." Turkish Online Journal of Qualitative Inquiry 12.9 (2021).
- [8] Makhmudov, I. E., et al. "The Current State Of Irrigation Networks And Their Use In The Water Sector Of The Republic Of Uzbekistan." Journal of Positive School Psychology (2022): 2947-2950.



10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)