# Identical Product of Graphs 

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## Abstract: A new graph product called identical product is introduced in this paper. <br> Keywords: Graph products, Identical Product

## I. INTRODUCTION

A graph[1] is an ordered triple $G=\left(V(G), \boldsymbol{E}(G) I_{G}\right)$ where $\mathrm{V}(\mathrm{G})$ is a nonempty set $\mathrm{E}(\mathrm{G})$ is a set disjoint from $\mathrm{V}(\mathrm{G})$ and $I_{G}$ is an "incidence" relation that associates with each element of $\mathrm{E}(\mathrm{G})$ an unordered pair of elements (same or distinct) of $\mathrm{V}(\mathrm{G})$. Elements of $V(G)$ are called the vertices (or nodes or points) of $G$; and elements of $E(G)$ are called the edges (or lines) of $G$ : $V(G)$ and $E(G)$ are the vertex set and edge set of $G$, respectively. If, for the edge e of $G, I_{G}(e)=\{u, v\}$ Number of vertices and the number of edges in a graph $G$ is called the order $n(G)$ and the size $m(G)$ of $G$ respectively. Number of edges incident on a vertex $v$ of a graph $G$ is called degree of $v$ in $G$ and is denoted by $d_{G}(v)$. A graph $G$ is regular if degree of all vertices in $G$ are equal. Let $G_{1}=\left(V_{1}, E_{1}\right)$ and $G_{2}=\left(V_{2}, E_{2}\right)$ be two simple graphs. Any product [1] $G_{1} * G_{2}$ has its vertex set $V_{1} \times V_{2}$. For any two vertices $\left(u_{1}, v_{1}\right)$ and ( $u_{2}, v_{2}$ ) are adjacent in $G_{1} * G_{2}$, there are various possibilities:
$u_{1}$ adjacent to $v_{1}$ in $G_{1}$ or $u_{1}$ non-adjacent to $v_{1}$ in $G_{1} ; u_{2}$ adjacent to $v_{2}$ in $G_{2}$ or $u_{2}$ non-adjacent to $v_{2}$ in $G_{2}$ and $u_{1}=u_{2}$ and/or $v_{1}=v_{2}$. Two graph products

## II. IDENTICAL PRODUCT

## 1) Definition

Let $G_{1}=\left(V_{1}, E_{1}\right)$ and $G_{2}=\left(V_{2}, E_{2}\right)$ be two simple graphs. The identical product $G_{1} \boldsymbol{I} G_{2}$ has its vertex set $V_{1} \times V_{2}$. Any two vertices $\left(w_{1}, v_{1}\right)$ and $\left(u_{2}, v_{2}\right)$ are adjacent in $G_{1} ■ G_{2}$ if and only if $u_{1}=u_{2}$ or $v_{1}=v_{2}$.

## Example:



## 2) Theorem

The identical product of any two graphs $G$ and $H$ with $n_{1}$ and $n_{2}$ vertices respectively
Proof: From the definition of the identical product, it is clear that the adjacency of two vertices in $G \boldsymbol{m} H$ will not depend on the adjacency of vertices in $G$ or $\mathrm{H}\left(\right.$ since $\left(u_{1}, v_{1}\right)$ and $\left(u_{2}, v_{2}\right)$ are adjacent in $G_{1} m G_{2}$ if and only if $u_{1}=w_{2}$ or $\left.v_{1}=v_{2}\right)$. Hence the theorem.
3) Theorem

The number of edges in the identical product of any two graphs $G$ and $H$ with $n_{1}$ and $n_{2}$ vertices respectively is $\frac{n_{1} n_{2}\left(n_{4}+n_{2}-2\right)}{2}$ Proof: Let u be any vertex in graph G . Then there are $n_{2}$ vertices in $G \llbracket H$ in the form ( $\mathrm{u}, \mathrm{x}$ ) where x is any vertex in H and these vertices are adjacent to each other. Therefore, there are $\frac{n_{1} n_{2}\left(n_{2}-1\right)}{2}$ edges in this case. Also if $v$ be any vertex in graph $H$, there are $n_{1}$ of the form ( $\mathrm{x}, \mathrm{v}$ ) where y be any vertex in G and these vertices are adjacent to each other. Therefore, there are $\frac{n_{1} n_{2}\left(n_{1}-1\right)}{2}$ edges in this case.
Hence the total number of edges in $G \llbracket H=\frac{n_{1} n_{2}\left(n_{2}-1\right)}{2}++\frac{n_{1} n_{2}\left(n_{1}-1\right)}{2}=\frac{n_{1} n_{2}\left(n_{1}+n_{2}-2\right)}{2}$.
4) Theorem

Identical product of any two graphs is regular
Proof: Let G and H be two graphs with $n_{1}$ and $n_{2}$ vertices respectively. Let ( $\mathrm{u}, \mathrm{v}$ ) be any vertex in $G \mathbb{\square} H$
$d\left(u_{s} v\right)=n_{2}-1+n_{1}-1=n_{2}+n_{1}-2$
Hence identical product is regular.

## III. CONCLUSIONS

In this paper the identical product of two graphs is defined and proved some results relating to this

## REFERENCES

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