iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 **Issue:** III **Month of publication:** March 2023

DOI: https://doi.org/10.22214/ijraset.2023.49368

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue III Mar 2023- Available at www.ijraset.com

Implementation of Fuzzy Logic to Weigh the Vehicle Slow Down Using Boundaries

Dr. K. Sharath Babu
Assistant Professor of Mathematics, Matrusri Engineering College Saidabad(MECS) , Hyderabad

I. INTRODUCTION

It is observed that many real life problems, the cost of transporting a commodity from one place to another cannot be determined exactly. This may be attributed to many reasons such as variations in Crude oil ,fuel prices, Labour charges, power utilization, optimality utilization of man power, expenditure of utilities etc. The inaccurate costs can be conveniently modeled by fuzzy numbers. Thus transportation problems with fuzzy costs are of a great importance and which are reliable too.

An assignment problem with fuzzy costs has been considered by Lin and Wen [2004]. They have modeled the problem into a mixed integer programming problem, using the fuzzy decision of Bellman and Zadeh [1970] and then converted it into a linear fractional programming problem. They have developed a suitable labeling algorithm for the solution of the resultant linear fractional programming problem.

A definition of an optimal solution of a transportation problem with fuzzy cost coefficients and an algorithm for determining this solution was given by Chanas and Kuchta [1996].

Also, there are situations where the commodities supplied by the origins contain certain number of units of impurities and each destination has it's own limitations on the number of units of impurities it can receive. For example, some industries receiving coal may fix some limits on the quantity of sulphur in the coal supplied. A method of solution of a multi-objective time transportation problem with impurity restrictions was developed by Singh and Saxena [2003].

The present chapter is aimed at developing a solution methodology for a transportation problem involving both the above aspects i.e costs are fuzzy and also impurity restrictions are imposed. The demand and supply values are assumed to be crisp numbers

This work (chapter) is divided into five sections. Section III.2 formulates the problem under consideration and also gives the basic notations and assumptions. Section III.3 develops the necessary theory and produces a linear fractional programming problem, which has the same optimal solution as of the problem under consideration. Section III.4 applies the developed method on a numerical example. Some concluding remarks are drawn in section III.5

II. MATHEMATICAL FORMULATION OF THE PROBLEM

The mathematical formulation of the fuzzy transportation problem with additional restrictions is

$$\text{Minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij} \qquad (1)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i,\ (i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ (j = 1,2,\ldots,n)$$

$$\sum_{i=1}^{m} f_i x_{ij} \le p_j,\ (j = 1,2,\ldots,n)$$

$$x_{ij} \ge 0 \text{ and are integer, } i = 1,2,\ldots,m,\ j = 1,2,\ldots,n$$

Here a_i is the amount of commodity available at the i^{th} origin and b_j is the requirement of the commodity at the j^{th} destination. One unit of the commodity at the i^{th} origin contains f_i units of a certain impurity and j^{th} destination cannot receive more than p_j units of the impurity. x_{ij} is the amount of commodity transported from the i^{th} origin to the j^{th} destination.

The costs $C_{ij} = (\alpha_{ij}, \beta_{ij})$ (i =1,2,…,m, j =1,2,…,n) are subnormal fuzzy numbers having strictly increasing linear membership functions defined as follows:

$$\mu_{ij}(C_{ij}) = \begin{cases} q_{ij} & \text{if } C_{ij} = \beta_{ij} \text{ and } x_{ij} > 0 \\ q_{ij}(C_{ij}-\alpha_{ij})/(\beta_{ij}-\alpha_{ij}) & \text{if } \alpha_{ij} \le C_{ij} \le \beta_{ij} \text{ and } x_{ij} > 0 \\ 0 & \text{Otherwise} \end{cases} \qquad (2)$$

The condition $x_{ij} > 0$ is initiated and added to (2) because there is no real expense if $x_{ij} = 0$ in any feasible solution x of (1). We use the notation $< \alpha_{ij}, \beta_{ij} >$ to denote the special type of fuzzy number C_{ij} used in this. Matrix $[C_{ij}]$ is written as $[C_{ij}] = [< \alpha_{ij}, \beta_{ij} >]_{m\times n}$

Matrix $[q_{ij}]$ is defined by $[q_{ij}] = [q_{ij}]_{m\times n}$

Let C_T denote the total cost and a, b be the lower and upper bounds of the total cost respectively. We define the membership function of C_T as linear decreasing function in (3) and use the notation < a, b > to denote fuzzy number C_T. Numbers a and b are constants and subjectively chosen by the decision maker. We may take a as the minimum cost of the transportation problem with α_{ij}'s as costs and b as the maximum cost of the transportation problem with β_{ij}'s as costs, the demand and supply values in both cases being same as those of problem (3.1).

$$\mu_T(C_T) = \mu_T\left(\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij}\right) = \begin{cases} 1, \text{ if } C_T \le a \\ \left(b-\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij}\right)/(b-a), \text{ if } a \le c_T \le b \\ 0, \text{ if } C_T \ge b . \end{cases} \qquad (3)$$

III. FORMULATION AND SOLUTION OF THE PROBLEM

Applying the Bellman-Zadeh's fuzzy decision [1970], we maximize the minimum of the membership functions corresponding to that solution i.e

$$\text{Max - Min } (\mu_{ij}(i =1,2,\dots,m,\ j =1,2,\dots,n), \mu_T(C_T)) \qquad (4)$$

Where x_{ij} is an element of a feasible solution x of problem (1). Then we can represent the problem (1) as follows

$$\underset{x_{ij} > 0}{\text{Max-Min}} (\mu_{ij}, \mu_T(C_T)) \qquad (5)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \ (i =1,2,\dots,m) \ ; \qquad \sum_{i=1}^{m} x_{ij} = b_j, \ (j =1,2,\dots,n)$$

$$\sum_{i=1}^{m} f_i x_{ij} \le p_j, \ (j =1,2,\dots,n) \ ; \qquad x_{ij} \ge 0 \text{ and are integer}$$

$$\text{for } i =1,2,\dots,m,\ j =1,2,\dots,n$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue III Mar 2023- Available at www.ijraset.com

Applying the membership functions of the unit costs and the total transportation cost as defined in (2) and (3) respectively, we can further represent (5) as the following equivalent model.

$$\text{Maximize } \lambda \qquad (6)$$

Subject to

$$\lambda x_{ij} \leq (q_{ij}(C_{ij}^{\lambda} - \alpha_{ij})x_{ij})/ (\beta_{ij} - \alpha_{ij}) \text{ for } i = 1,2,\ldots,m,\ j = 1,2,\ldots,n$$

$$\lambda \leq \left(b - \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{\lambda} x_{ij}\right)/(b-a), \qquad \sum_{j=1}^{n} x_{ij} = a_i,\ (i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ (j = 1,2,\ldots,n)$$

$$\sum_{i=1}^{m} f_i x_{ij} \leq p_j,\ (j = 1,2,\ldots,n)$$

$$C_{ij}^{\lambda} x_{ij} \leq \beta_{ij} x_{ij} \text{ for } i = 1,2,\ldots,m,\ j = 1,2,\ldots,n$$

$$x_{ij} \geq 0 \text{ and integer for } i = 1,2,\ldots,m,\ j = 1,2,\ldots,n$$

Where C_{ij}^{λ} denotes the λ-cut of C_{ij}. In (6), since x_{ij}, C_{ij}^{λ}, and λ are all decision variables, it can be treated as a mixed integer nonlinear programming model

We define E as the set of all pairs (i, j) where x_{ij} is an element of the feasible solution x of (1) and confine our discussion based on E. Then, we can write (6) as follows:

$$\text{Maximize } \lambda \qquad (7)$$

Subject to

$$\lambda \leq (q_{ij}(C_{ij}^{\lambda} - \alpha_{ij})) / (\beta_{ij} - \alpha_{ij}) \text{ for } (i, j) \in E$$

$$\lambda \leq \left(b - \sum_{(i,j) in E} C_{ij}^{\lambda} x_{ij}\right)/(b-a),$$

$$C_{ij}^{\lambda} \leq \beta_{ij} \text{ for } (i, j) \in E$$

Let $d_{ij} = \beta_{ij} - C_{ij}^{\lambda} \geq 0$. Then (3.7) can be expressed as follows:

$$\text{Maximize } \lambda \qquad (8a)$$

Subject to

$$\lambda \leq q_{ij}(\beta_{ij} - \alpha_{ij} - d_{ij}) / (\beta_{ij} - \alpha_{ij}) \text{ for } (i, j) \in E \qquad (8b)$$

$$\lambda \leq \left(b - \sum_{(i,j) in E} (\beta_{ij} - d_{ij}) x_{ij}\right)/(b-a), \qquad (8c)$$

$$d_{ij}, \lambda \geq 0 \text{ for } (i, j) \in E \qquad (8d)$$

Now, the necessary theory for the development of a linear fractional programming problem having the same optimal solution of problem (8a – 8d) is given by the following two theorems.

1) Theorem 1

Let λ_x be the optimal value of the objective function of problem (3.8a – 3.8d). Suppose that

$$b < \left(\sum_{(i,j) in E} (\beta_{ij} - d_{ij}) x_{ij} - a \min\{q_{ij}/(i,j) \in E\}\right) / (1 - \min\{q_{ij}/(i,j) \in E\})$$

Then $\lambda_x = q_{ij}(\beta_{ij} - \alpha_{ij} - d_{ij}) / (\beta_{ij} - \alpha_{ij})$ for $(i, j) \in E$

$$= \left(b - \sum_{(i,j) in E} (\beta_{ij} - d_{ij}) x_{ij}\right)/(b-a)$$

Proof:

The problem (3.8a – 3.8d) can be written into a linear programming model in variables λ, d_{ij} as

$$\text{Maximize } \lambda \qquad (9a)$$

Subject to

$d_{ij} + ((\beta_{ij} - \alpha_{ij})/ q_{ij})\lambda \leq (\beta_{ij} - \alpha_{ij})$ for $(i, j) \in E$ (9b)

$- \sum_{(i,j) in E} d_{ij}x_{ij} + (b-a)\lambda \leq b- \sum_{(i,j) in E} \beta_{ij}x_{ij}$ (9c)

$\lambda, d_{ij} \geq 0$ for $(i, j) \in E$ (9d)

The dual of the above problem is:

Minimize $\sum_{(i,j) in E} (\beta_{ij} - \alpha_{ij})w_k + (b- \sum_{(i,j) in E} \beta_{ij}x_{ij})w_{m+n}$ (10a) Subject to

$w_k - x_{ij}w_{m+n} \geq 0$ for $(i, j) \in E$ (10b)

$\sum_{(i,j) in E} ((\beta_{ij} - \alpha_{ij})/q_{ij})w_k + (b-a)w_{m+n} \geq 1$ (10c)

$w_k \geq 0$ for $k = 1,2,\ldots, m+n$ (10d)

Let $s_1, s_2, \ldots s_{m+n}$ be the slack variables of (9b) and (9c) respectively. Similarly, let $u_1, u_2, \ldots, u_{m+n}$ be the surplus variables of (10b) and (10c) respectively.

Since $b < (\sum_{(i,j) in E} (\beta_{ij} - d_{ij})x_{ij} - a \min\{q_{ij}/(i, j) \in E\}) / (1- \min\{q_{ij}/(i, j) \in E\})$,

we have $\min\{q_{ij}/(i, j) \in E\} > (b- \sum_{(i,j) in E} (\beta_{ij} - d_{ij})x_{ij})/(b-a)$

By comparing (c) and the above inequality,
we have $\lambda < \min\{q_{ij}/(i, j) \in E\}$
so that $\lambda < q_{ij}$ for all $(i, j) \in E$
so that $C_{ij}^{\lambda} < \beta_{ij}$ for all $(i, j) \in E$ (Since the membership functions are increasing)
i.e $\beta_{ij} - C_{ij}^{\lambda} > 0$ for all $(i, j) \in E$
i.e $d_{ij} > 0$ for all $(i, j) \in E$.
Applying the complementary slackness theorem, we obtain using (10b)
$u_1 = u_2 = \ldots = u_{m+n-1} = 0$
Hence $w_i - x_{ij}w_{m+n} = 0$ for $i = 1,2,\ldots,m+n-1$
Now, if $w_{m+n} = 0$ then $w_1 = w_2 = \ldots = w_{m+n-1} = 0$.
This is a contradiction to (10c)
Hence $w_{m+n} > 0$
Assuming that the solution is non-degenerate
$w_i = w_{m+n}x_{ij} > 0$ for $i = 1,2,\ldots,m+n-1$
i.e $w_1 > 0, w_2 > 0, \ldots, w_{m+n} > 0$
Now by complementary slackness theorem, we obtain from (9b) and (9c)
$s_1 = s_2 = \ldots = s_{m+n} = 0$
Therefore $\lambda_x = q_{ij}(\beta_{ij} - \alpha_{ij} - d_{ij}) / (\beta_{ij} - \alpha_{ij})$ for $(i, j) \in E$

$= (b- \sum_{(i,j) in E} (\beta_{ij} - d_{ij})x_{ij}) / (b-a)$

2) Theorem 2

Let λ_x be the optimal value of the objective function of problem (3.8a – 3.8d) and

$b < (\sum_{(i,j) in E} (\beta_{ij} - d_{ij})x_{ij} - a \min\{q_{ij}/(i,j) \in E\}) / (1- \min\{q_{ij}/(i,j) \in E\})$.

Also let $\gamma_{ij} = (\beta_{ij} - \alpha_{ij})/q_{ij}$ for $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$.

Then $\lambda_x = (b- \sum_{(i,j) in E} \alpha_{ij}x_{ij}) / (b-a + \sum_{(i,j) in E} \gamma_{ij}x_{ij})$

Proof:

By theorem 1, we have

$\lambda_x = ((\beta_{ij} - \alpha_{ij} - d_{ij})x_{ij}) / (\gamma_{ij}x_{ij})$ for $(i,j) \in E$

$= (b- \sum_{(i,j) in E} (\beta_{ij} - d_{ij})x_{ij}) / (b-a)$

Hence, by Componendo and Dividendo principle we have

$$\lambda_x = (b- \sum_{(i,j) in E} (\beta_{ij} - d_{ij})x_{ij} + \sum_{(i,j) in E} (\beta_{ij} - \alpha_{ij} - d_{ij})x_{ij}) / (b-a + \sum_{(i,j) in E} \gamma_{ij}x_{ij})$$

$$= (b- \sum_{(i,j) in E} \alpha_{ij}x_{ij}) / (b-a + \sum_{(i,j) in E} \gamma_{ij}x_{ij}) \qquad (11)$$

A. The Fractional Programming Model

By theorem 2, problem (6) can be restated as

$$\text{Maximize} \quad (b- \sum_{i=1}^{m}\sum_{j=1}^{n} \alpha_{ij}x_{ij}) / (b-a + \sum_{i=1}^{m}\sum_{j=1}^{n} \gamma_{ij}x_{ij}) \qquad (12)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \ (i = 1,2,\ldots,m) \ : \qquad \sum_{i=1}^{m} x_{ij} = b_j, \ (j = 1,2,\ldots,n)$$

$$\sum_{i=1}^{m} f_i x_{ij} \le p_j, \ (j = 1,2,\ldots,n) \ ; \qquad x_{ij} \ge 0 \text{ and integer for } i = 1,2,\ldots,m, \ j = 1,2,\ldots,n$$

Problem (12) is a linear fractional programming problem and its optimal solution may be obtained by the algorithm by Kantiswarup [1965]. The optimal value of the objective function of problem (12) is λ_x. ,d_{ij} for $(i, j) \in E$ can be obtained from

$\lambda_x = (\beta_{ij} - \alpha_{ij} - d_{ij})/\gamma_{ij}$ for $(i, j) \in E$ (13)

Then the fuzzy costs corresponding to the maximal value of λ are given by

$C_{ij}^{\lambda} = \beta_{ij} - d_{ij}$ for $(i, j) \in E$ (14)

IV. ILLUSTRATED PROBLEMS

Let us consider the following example:

$$\text{Minimize} \sum_{i=1}^{3}\sum_{j=1}^{3} C_{ij}x_{ij} \qquad (15)$$

Subject to

$x_{11} + x_{12} + x_{13} = 4$; $x_{21} + x_{22} + x_{23} = 5$; $x_{31} + x_{32} + x_{33} = 6$; $x_{11} + x_{21} + x_{31} = 5$; $x_{12} + x_{22} + x_{32} = 5$;

$x_{13} + x_{23} + x_{33} = 5$

$2x_{11} + x_{21} + 0.x_{31} \le 4$

$2x_{12} + x_{22} + 0.x_{32} \le 1$

$2x_{13} + x_{23} + 0.x_{33} \le 9$

$x_{ij} \ge 0$ and are integers for $i, j = 1,2,3$

Where $[C_{ij}] = \begin{pmatrix} <4,13> & <3,12> & <2,6> \\ <4,13> & <6,14> & <7,15> \\ <7,10> & <4,8> & <6,12> \end{pmatrix}$

and $[q_{ij}] = \begin{pmatrix} 0.9 & 0.6 & 0.8 \\ 0.9 & 0.8 & 0.8 \\ 0.6 & 0.8 & 0.6 \end{pmatrix}$

Then we have

$$[\alpha_{ij}] = \begin{pmatrix} 4 & 3 & 2 \\ 4 & 6 & 7 \\ 7 & 4 & 6 \end{pmatrix} \quad [\beta_{ij}] = \begin{pmatrix} 13 & 12 & 6 \\ 13 & 14 & 15 \\ 10 & 8 & 12 \end{pmatrix} \quad [\gamma_{ij}] = \begin{pmatrix} 10 & 15 & 5 \\ 10 & 10 & 10 \\ 5 & 5 & 10 \end{pmatrix}$$

a is taken as the minimum cost of the transportation problem with costs as α_{ij}'s (a = 54) and b is taken as the maximum cost of the transportation problem with costs as β_{ij}'s (b =192). Now, using (12), the following fractional programming problem is developed corresponding to problem (15)

Maximize $(192 - 4x_{11} - 3x_{12} - 2x_{13} - 4x_{21} - 6x_{22} - 7x_{23} - 7x_{31} - 4x_{32} - 6x_{33})$ / (16)

$(138 + 10x_{11} + 15x_{12} + 5x_{13} + 10x_{21} + 10x_{22}\ \ 10x_{23} + 5x_{31} + 5x_{32} + 10x_{33})$

Subject to

$x_{11} + x_{12} + x_{13} = 4$; $x_{21} + x_{22} + x_{23} = 5$; $x_{31} + x_{32} + x_{33} = 6$; $x_{11} + x_{21} + x_{31} = 5$; $x_{12} + x_{22} + x_{32} = 5$; $x_{13} + x_{23} + x_{33} = 5$; $2x_{11} + x_{21} + 0.x_{31} \leq 4$;
$2x_{12} + x_{22} + 0.x_{32} \leq 1$; $2x_{13} + x_{23} + 0.x_{33} \leq 9$; $x_{ij} \geq 0$ integers for i, j = 1,2,3

The optimal solution of problem (16) is obtained by using the method by Kantiswarup [1965] as follows
$x_{13} = 4$, $x_{21} = 4$, $x_{23} = 1$, $x_{31} = 1$, $x_{32} = 5$ with max $\lambda_x = 0.563$
for (i, j) $\in$ E, we have
$\lambda_x = (\beta_{ij} - \alpha_{ij} - d_{ij})/\gamma_{ij}$ so that $d_{ij} = \beta_{ij} - \alpha_{ij} - \lambda_x\gamma_{ij}$
$\therefore$ We have
$d_{13} = 4 - (0.563)(5) = 1.185$; $d_{21} = 9 - (0.563)(10) = 3.37$; $d_{23} = 8 - (0.563)(10) = 2.37$
$d_{31} = 3 - (0.563)(5) = 0.185$; $d_{32} = 4 - (0.563)(5) = 1.185$;

The fuzzy costs corresponding to $\lambda = 0.563$ are

$C_{ij}^{\lambda} = \beta_{ij} - d_{ij}$ for (i,j) $\in$ E
$\therefore$ We have

$C_{13}^{0.563} = 6-1.185 = 4.815$; $C_{21}^{0.563} = 13 - 3.37 = 9.63$; $C_{23}^{0.563} = 15 - 2.37 = 12.63$
$C_{31}^{0.563} = 10 - 0.185 = 9.815$; $C_{32}^{0.563} = 8 - 1.185 = 6.815$
$\therefore$ Total transportation cost = $\sum_{(i,j) in E} C_{ij}^{0.563} x_{ij} = 114.3$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue III Mar 2023- Available at www.ijraset.com

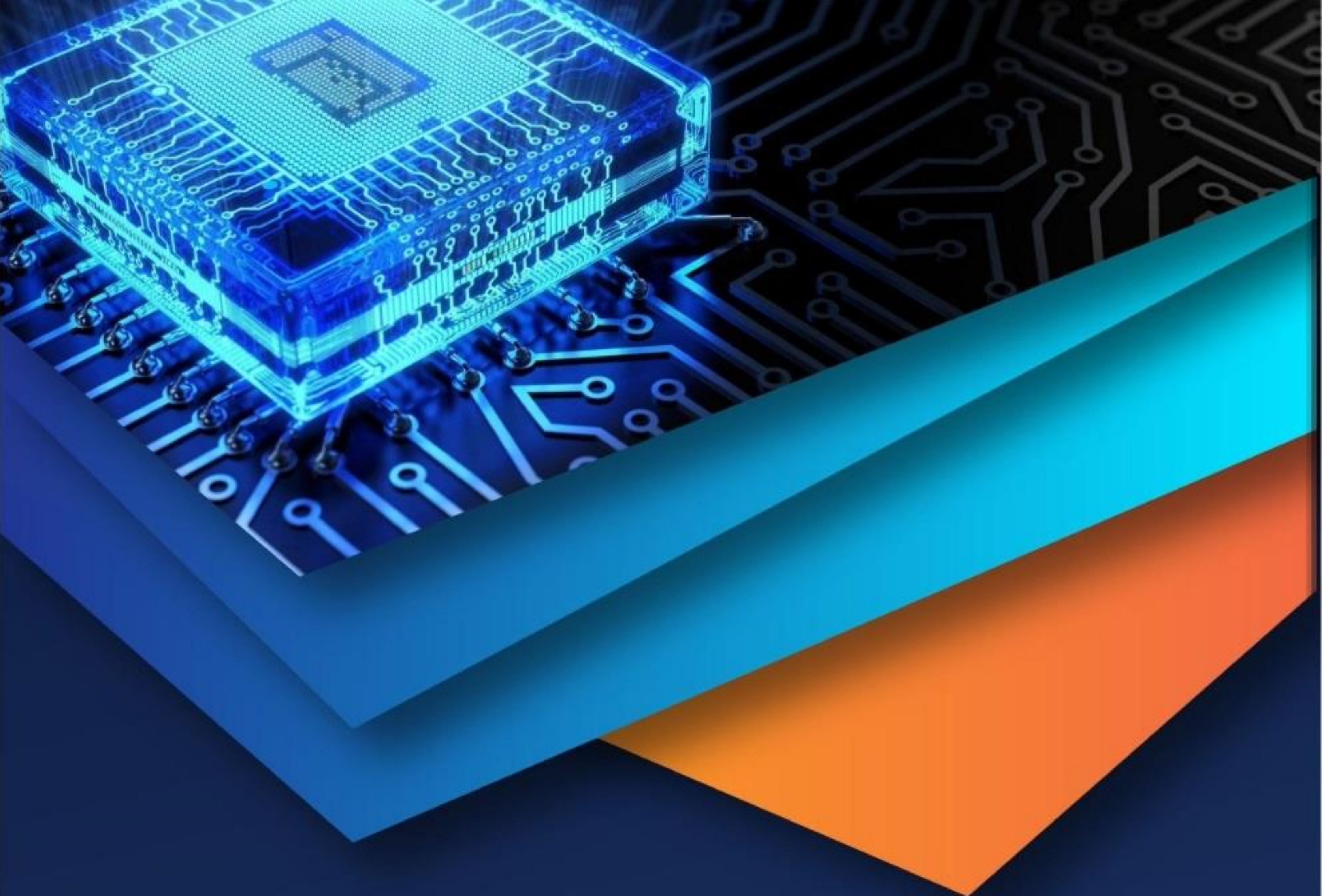
V. DISCUSSION AND CONCLUSIONS

The developed procedure for obtaining an optimal solution of a transportation problem with fuzzy costs and involving impurity restrictions is computationally efficient. The optimal values of x_{ij}'s can be obtained by the method by Kantiswarup [1965]. A computer program can be developed with ease in any programming language for this purpose. The optimal values of d_{ij}'s and hence the unit costs can be obtained using expressions given in the method.

The fuzzy transportation problem considered in this work may not possess an optimal solution, in some cases. This may be attributed to the presence of the impurity restrictions. In such cases, a feasible solution and therefore an optimal solution may be obtained by relaxing the impurity restrictions of the destinations to some extent.

REFERENCES

[1] Lin and Wen ; A GA Based fuzzy controller with slideing mode.
[2] Bellman, R.E., Zadeh, L.A.: Decision making in a fuzzy environment, Management Science17 (1970) B141-B164.
[3] Chanas and kuchta, A new approach to fuzzy goal programming, Proceedings of the 2nd International Conference in Fuzzy Logic and Technology, Leicester, United Kingdom, September 5-7, 2001
[4] Singh & Saxena , Analysis of intelligence techniques on sensor less speed control of Doubly fed — Induction machine.
[5] Fuzzy Set Theory Fuzzy Logic and Their Applications Paperback – January 1, 2013 by A.K. Bhargava .
[6] Fuzzy Set Theory—and Its Applications, Zimmermann, Hans-Jürgen
[7] Klir, G.J., Yuan, B.: Fuzzy Sets and Fuzzy Logic-Theory and Applications, Prentice-Hall Inc. (1995).
[8] Operations research , S.D.Sharma &V.K. Kapoor, Khanna Publications.



10.22214/IJRASET

INDEX COPERNICUS
45.98

IMPACT FACTOR:
7.129

IMPACT FACTOR:
7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY