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Integer Solution Patterns in Transcendental Equations with Six Unknowns

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Abstract: ***This study presents an analytical investigation of the transcendental equation*** $p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+q^2}=k^5\left(n^2+1\right)$ ***and examines its structural properties using appropriate algebraic techniques.***

Keywords: Transcendental Equation, Integral Solution, Diophantine equation, Linear Transformation.

I. INTRODUCTION

A transcendental equation is an equation that involves transcendental functions whose values must be determined. These equations are generally difficult to solve using elementary algebraic methods. Hence, numerical techniques and analytical approaches are often employed to obtain approximate or exact solutions. From the perspective of number theory, transcendental equations also show interesting connections with integer solutions and special numerical patterns. In certain cases the study of these equations leads to the formulation of problems that are closely related to Diophantine equations, where solutions are restricted to integers. The interaction between transcendental functions and integer-based equations provides deeper insights into the structural properties of numbers and mathematical relations.

Transcendental functions arise frequently in mathematical analysis and appear in various applications across scientific disciplines. A function is termed transcendental when it cannot be expressed as the root of a polynomial equation with finite operations such as addition, subtraction, multiplication, and extraction of roots. Well-known transcendental functions include logarithmic, exponential, trigonometric, hyperbolic, and inverse trigonometric functions. These functions play a crucial role in many areas of mathematics and its applications.

In this study, the fundamental concepts of transcendental equations and their properties are explored. Special attention is given to certain forms of transcendental equations and the methods used to analyze and solve them.

II. METHODOLOGY FOR ANALYSIS

The equation to be solved is

$$p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+s^2}=k^5\left(n^2+1\right) \quad (1)$$

The following linear transformation, $p=(u-v)^5, q=(v-u)^5, r=u^5-10u^3v^2+5uv^4, s=5u^4v-10u^2v^3+v^5$ leads to $p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+s^2}=u^2+v^2$. Hence, $p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+s^2}=u^2+v^2$. reduces to,

$$u^2+v^2=k^5\left(n^2+1\right) \quad (2)$$

Now we find various patterns of solutions of (1) using (2).

A. Pattern 1

Let $k=y^2+z^2$, for $y,z\geq 0$

$$u^2+v^2=\left(y^2+z^2\right)^5\left(n^2+1\right) \quad (3)$$

Using factorization and equating real and imaginary parts we get,

$$u=n\left(y^5-10y^3z^2+5yz^4\right)-\left(5y^4z-10y^2z^3+z^5\right)$$

$$v=\left(y^5-10y^3z^2+5yz^4\right)+n\left(5y^4z-10y^2z^3+z^5\right)$$

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Therefore, $u = nf(y,z) - g(y,z)$ and $v = f(y,z) + ng(y,z)$

where, $f(y,z) = y^5 - 10y^3z^2 + 5yz^4$ and $g(y,z) = 5y^4z - 10y^2z^3 + z^5$

Hence, for all $n \geq 0$ the non-zero integral solutions are found to be,

$p = ((n-1)f(y,z) - (n+1)g(y,z))^5$

$q = ((n+1)g(y,z) - (n-1)f(y,z))^5$

$r = (nf(y,z) - g(y,z))^5 - 10(nf(y,z) - g(y,z))^3(f(y,z) + ng(y,z))^2 + 5(nf(y,z) - g(y,z))(f(y,z) + ng(y,z))^4$

$s = 5(nf(y,z) - g(y,z))^4(f(y,z) + ng(y,z)) - 10(nf(y,z) - g(y,z))^2(f(y,z) + ng(y,z))^2 + (f(y,z) + ng(y,z))^5$

Numerical examples satisfying the solution are listed below.

TABLE 1

n	p	q	r	s	k	LHS=RHS
1	32768	-32768	0	-32768	2	64
2	33554432	-33554432	-1275068416	1375731712	4	5120
3	-867624000000000	867624000000000	-267743000000000	-10167500000000	9	590490
4	1040650000000000	-1040650000000000	1923350000000000	3240260000000000	10	1700000

B. Pattern 2

Equation (3) can be rewritten as,

$u^2 + v^2 = 1.(y^2 + z^2)^5(n^2 + 1)$

Taking $1 = \left(\frac{3^2 + 4^2}{5^2}\right)$, we get $u^2 + v^2 = \left(\frac{3^2 + 4^2}{5^2}\right)(y^2 + z^2)^5(n^2 + 1)$

Using factorization and equating real and imaginary parts we get,

$u = [n(3y^5 - 30y^3z^2 + 15yz^4 - 20y^4z + 40y^2z^3 - 4z^5) - (15y^4z - 30y^2z^3 + 3z^5 + 4y^5 - 40y^3z^2 + 20yz^4)]/5$

$v = [n(15y^4z - 30y^2z^3 + 3z^5 + 4y^5 - 40y^3z^2 + 20yz^4) - (3y^5 - 30y^3z^2 + 15yz^4 - 20y^4z + 40y^2z^3 - 4z^5)]/5$

In order to get integer solutions, take $y = 5A$ and $z = 5B$ we get,

$u = n(1875A^5 - 18750A^3B^2 + 9375AB^4 - 12500A^4B + 25000A^2B^3 - 2500B^5)$

$-(9374A^4B - 18750A^2B^3 + 1875B^5 + 2500A^5 - 25000A^3B^2 + 12500AB^4)$

$v = n(9375A^4B - 18750A^2B^3 + 1875B^5 + 2500A^5 - 25000A^3B^2 + 12500AB^4)$

$+(1875A^5 - 18750A^3B^2 + 9375AB^4 - 12500A^4B + 25000A^2B^3 - 2500B^5)$

Therefore, $u = nf(A,B) - g(A,B)$ and $v = ng(A,B) + f(A,B)$

where, $f(A,B)=(1875A^5-18750A^3B^2+9375AB^4-12500A^4B+25000A^2B^3-2500B^5)$

$g(A,B)=(9375A^4B-18750A^2B^3+1875B^5+2500A^5-25000A^3B^2+12500AB^4)$

Hence, for all $n\geq 0$, the non-zero integral solutions are found to be,

$p=((n-1)f(A,B)-(n+1)g(A,B))^5$

$q=((n+1)g(A,B)-(n-1)f(A,B))^5$

$r=(nf(A,B)-g(A,B))^5-10(nf(A,B)-g(A,B))^3(f(A,B)+ng(A,B))^2+5(nf(A,B)-g(A,B))(f(A,B)+ng(A,B))$

$s=5(nf(A,B)-g(A,B))^4(f(A,B)+ng(A,B))-10(nf(A,B)-g(A,B))^2(f(A,B)+ng(A,B))^2+(f(A,B)+ng(A,B))^5$

Numeric examples satisfying the solution are listed below.

TABLE 2

n	p	q	r	s	k	LHS=RHS
1	52521900000000000000000	-52521900000000000000000	-9737500000000000000000	74062500000000000000000	50	625000000
2	-18895700000000000000000000	188957000000000000000000000	4376380000000000000000000	74062500000000000000000	100	50000000000
3	50088800000000000000000000	-50088800000000000000000000	-4146350000000000000000000	30459100000000000000000000	125	305176000000
4	-45916500000000000000000000000	45916500000000000000000000000	4878530000000000000000000	10871300000000000000000000	100	1700000000000

C. Pattern 3

Taking (3) as in pattern (2) and replacing 1 as $\left(\frac{6^2+8^2}{10^2}\right)$, we get $u^2+v^2=\left(\frac{6^2+8^2}{10^2}\right)(y^2+z^2)^5(n^2+1)$

Using factorization and equating real and imaginary parts we get,

$u=[n(6y^5-60y^3z^2+30yz^4-40y^4z+80y^2z^3-8z^5)-(30y^4z-60y^2z^3+6z^5+8y^5-80y^3z^2+40yz^4)]/10$

$v=[n(30y^4z-60y^2z^3+6z^5+8y^5-80y^3z^2+40yz^4)+(6y^5-60y^3z^2+30yz^4-40y^4z+80y^2z^3-8z^5)]/10$

In order to get integer solutions, take $y=10A$ and $z=10B$ we get,

$u=n(60000A^5-600000A^3B^2+300000AB^4-400000A^4B+800000A^2B^3-80000B^5)$

$-(300000A^4B-600000A^2B^3+60000B^5+80000A^5-800000A^3B^2+800000AB^4)$

$v=n(300000A^4B-600000A^2B^3+60000B^5+80000A^5-800000A^3B^2+400000AB^4)$

$+(60000A^5-600000A^3B^2+3000000AB^4-4000000A^4B+800000A^2B^3-80000B^5)$

Therefore, $u=nf(A,B)-g(A,B)$ and $v=ng(A,B)+f(A,B)$

where, $f(A,B)=(60000A^5-600000A^3B^2+300000AB^4-400000A^4B+800000A^2B^3-80000B^5)$

$g(A,B) = (300000A^4B - 600000A^2B^3 + 60000B^5 + 80000A^5 - 800000A^3B^2 + 400000AB^4)$

Hence, for all $n \geq 0$, the non-zero integral solutions are found to be,

$p = ((n-1)f(A,B) - (n+1)g(A,B))^5$

$q = ((n+1)g(A,B) - (n-1)f(A,B))^5$

$r = (nf(A,B) - g(A,B))^5 - 10(nf(A,B) - g(A,B))^3(f(A,B) + ng(A,B))^2 + 5(nf(A,B) - g(A,B))(f(A,B) + ng(A,B))^4$

$s = 5(nf(A,B) - g(A,B))^4(f(A,B) + ng(A,B)) - 10(nf(A,B) - g(A,B))^2(f(A,B) + ng(A,B))^2 + (f(A,B) + ng(A,B))^5$

Numeric examples satisfying the solution are listed below.

TABLE 3

n	p	q	r	s	k	LHS=RHS
1	1706340000000000000000000000	-1706340000000000000000000000	-3267360000000000000000000000	7581200000000000000000000000	200	646002000000
2	-3986730000000000000000000000	3986730000000000000000000000	1167060000000000000000000000	1467788000000000000000000000	400	511941000000000
3	-5277320000000000000000000000	5277320000000000000000000000	9135790000000000000000000000	-1423330000000000000000000000	500	309947000000000
4	-4366610000000000000000000000	4366610000000000000000000000	9211150000000000000000000000	-1756050000000000000000000000	900	100383000000000000

D. Pattern 4

Taking (3) as in pattern (2) and replacing 1 as $\left(\frac{5^2 + 12^2}{13^2}\right)$

$u^2 + v^2 = \left(\frac{5^2 + 12^2}{13^2}\right)(y^2 + z^2)^5(n^2 + 1)$

Using factorization and equating real and imaginary parts we get,

$u = [n(5y^5 - 50y^3z^2 + 25yz^4 - 60y^4z + 120y^2z^3 - 12z^5) - (25y^4z - 50y^2z^3 + 5z^5 + 12y^5 - 120y^3z^2 + 60yz^4)]/13$

$v = [n(25y^4z - 50y^2z^3 + 5z^5 + 12y^5 - 120y^3z^2 + 60yz^4) + (5y^5 - 50y^3z^2 + 25yz^4 - 60y^4z + 120y^2z^3 - 12z^5)]/13$

In order to get integer solutions, take $y = 13A$ and $z = 13B$ we get,

$u = n(142805A^5 - 1428050A^3B^2 + 714025AB^4 - 1713660A^4B + 3427320A^2B^3 - 342732B^5)$

$-(714025A^4B - 1428050A^2B^3 + 142805B^5 + 342732A^5 - 3427320A^3B^2 + 1713660AB^4)$

$v = n(714025A^4B - 1428050A^2B^3 + 142805B^5 + 342732A^5 - 3427320A^3B^2 + 1713660AB^4)$

$+(142805A^5-1428050A^3B^2+714025AB^4-1713660A^4B+3427320A^2B^3-342732B^5)$

Therefore, $u=nf(A,B)-g(A,B)$ and $v=ng(A,B)+f(A,B)$

where, $f(A,B)=(142805A^5-1428050A^3B^2+714025AB^4-1713660A^4B+3427320A^2B^3-342732B^5)$

$g(A,B)=(714025A^4B-1428050A^2B^3+142805B^5+342732A^5-3427320A^3B^2+1713660AB^4)$

Hence, for all $n\geq 0$, the non-zero integral solutions are found to be,

$p=((n-1)f(A,B)-(n+1)g(A,B))^5$

$q=((n+1)g(A,B)-(n-1)f(A,B))^5$

$r=(nf(A,B)-g(A,B))^5-10(nf(A,B)-g(A,B))^3(f(A,B)+ng(A,B))^2+5(nf(A,B)-g(A,B))(f(A,B)+ng(A,B))^4$

$s=5(nf(A,B)-g(A,B))^4(f(A,B)+ng(A,B))-10(nf(A,B)-g(A,B))^2(f(A,B)+ng(A,B))^2+(f(A,B)+ng(A,B))^5$

Numeric examples satisfying the solution are listed below.

TABLE 4

n	p	q	r	s	k	LHS=RHS
1	884223000000000000000000000000	-884223000000000000000000000000	907154000000000000000000000000	-212686000000000000000000000000	338	8822940000000
2	790907000000000000000000000000	-790907000000000000000000000000	119427000000000000000000000000000000000	216651000000000000000000000000000000000000	845	2276940000000000
3	-220184000000000000000000000000	220184000000000000000000000000	-220184000000000000000000000000	0	169	1378580000000
4	-738816000000000000000000000000000000	738816000000000000000000000000000000	355193000000000000000000000000000000	100290000000000000000000000000000000000	676	1624670000000000

III. CONCLUSION

In this post, we have explained the essential solutions to the transcendental equation $p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+s^2}=k^5\left(n^2+1\right)$ under complex patterns utilizing various numerical examples. Additionally, one could look for detailed answers to these similar types of equations.

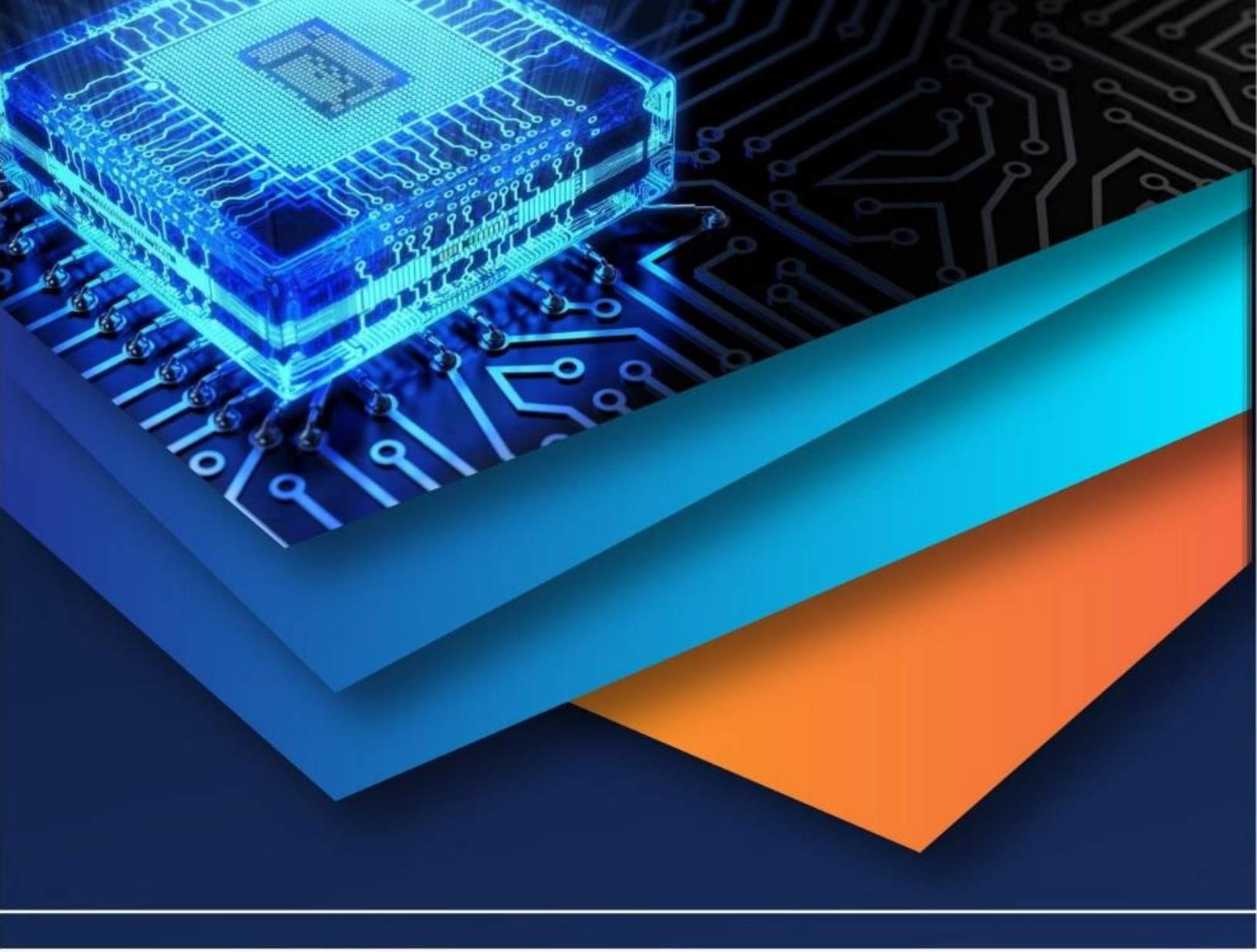
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