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Integer Solution Patterns in Transcendental Equations with Six Unknowns

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Abstract: ***This study presents an analytical investigation of the transcendental equation*** $p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+q^2}=k^5\left(n^2+1\right)$ ***and examines its structural properties using appropriate algebraic techniques.***
Keywords: Transcendental Equation, Integral Solution, Diophantine equation, Linear Transformation.

I. INTRODUCTION

A transcendental equation is an equation that involves transcendental functions whose values must be determined. These equations are generally difficult to solve using elementary algebraic methods. Hence, numerical techniques and analytical approaches are often employed to obtain approximate or exact solutions. From the perspective of number theory, transcendental equations also show interesting connections with integer solutions and special numerical patterns. In certain cases the study of these equations leads to the formulation of problems that are closely related to Diophantine equations, where solutions are restricted to integers. The interaction between transcendental functions and integer-based equations provides deeper insights into the structural properties of numbers and mathematical relations.

Transcendental functions arise frequently in mathematical analysis and appear in various applications across scientific disciplines. A function is termed transcendental when it cannot be expressed as the root of a polynomial equation with finite operations such as addition, subtraction, multiplication, and extraction of roots. Well-known transcendental functions include logarithmic, exponential, trigonometric, hyperbolic, and inverse trigonometric functions. These functions play a crucial role in many areas of mathematics and its applications.

In this study, the fundamental concepts of transcendental equations and their properties are explored. Special attention is given to certain forms of transcendental equations and the methods used to analyze and solve them.

II. METHODOLOGY FOR ANALYSIS

The equation to be solved is

$$p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+s^2}=k^5\left(n^2+1\right) \quad (1)$$

The following linear transformation, $p=(u-v)^5, q=(v-u)^5, r=u^5-10u^3v^2+5uv^4, s=5u^4v-10u^2v^3+v^5$ leads to $p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+s^2}=u^2+v^2$. Hence, $p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+s^2}=u^2+v^2$. reduces to,

$$u^2+v^2=k^5\left(n^2+1\right) \quad (2)$$

Now we find various patterns of solutions of (1) using (2).

A. Pattern 1

Let $k=y^2+z^2$, for $y,z\geq 0$

$$u^2+v^2=\left(y^2+z^2\right)^5\left(n^2+1\right) \quad (3)$$

Using factorization and equating real and imaginary parts we get,

$$u=n\left(y^5-10y^3z^2+5yz^4\right)-\left(5y^4z-10y^2z^3+z^5\right)$$
$$v=\left(y^5-10y^3z^2+5yz^4\right)+n\left(5y^4z-10y^2z^3+z^5\right)$$

Therefore, $u = nf(y,z) - g(y,z)$ and $v = f(y,z) + ng(y,z)$

where, $f(y,z) = y^5 - 10y^3z^2 + 5yz^4$ and $g(y,z) = 5y^4z - 10y^2z^3 + z^5$

Hence, for all $n \geq 0$ the non-zero integral solutions are found to be,

$$p = ((n-1)f(y,z) - (n+1)g(y,z))^5$$

$$q = ((n+1)g(y,z) - (n-1)f(y,z))^5$$

$$r = (nf(y,z) - g(y,z))^5 - 10(nf(y,z) - g(y,z))^3(f(y,z) + ng(y,z))^2 + 5(nf(y,z) - g(y,z))(f(y,z) + ng(y,z))^4$$

$$s = 5(nf(y,z) - g(y,z))^4(f(y,z) + ng(y,z)) - 10(nf(y,z) - g(y,z))^2(f(y,z) + ng(y,z))^2 + (f(y,z) + ng(y,z))^5$$

Numerical examples satisfying the solution are listed below.

TABLE 1

n	p	q	r	s	k	LHS=RHS
1	32768	-32768	0	-32768	2	64
2	33554432	-33554432	-1275068416	1375731712	4	5120
3	-867624000000000	867624000000000	-267743000000000	-10167500000000	9	590490
4	1040650000000000	-1040650000000000	1923350000000000	3240260000000000	10	1700000

B. Pattern 2

Equation (3) can be rewritten as,

$$u^2 + v^2 = 1.(y^2 + z^2)^5(n^2 + 1)$$

Taking $1 = \left(\frac{3^2 + 4^2}{5^2}\right)$, we get $u^2 + v^2 = \left(\frac{3^2 + 4^2}{5^2}\right)(y^2 + z^2)^5(n^2 + 1)$

Using factorization and equating real and imaginary parts we get,

$$u = [n(3y^5 - 30y^3z^2 + 15yz^4 - 20y^4z + 40y^2z^3 - 4z^5) - (15y^4z - 30y^2z^3 + 3z^5 + 4y^5 - 40y^3z^2 + 20yz^4)]/5$$

$$v = [n(15y^4z - 30y^2z^3 + 3z^5 + 4y^5 - 40y^3z^2 + 20yz^4) - (3y^5 - 30y^3z^2 + 15yz^4 - 20y^4z + 40y^2z^3 - 4z^5)]/5$$

In order to get integer solutions, take $y = 5A$ and $z = 5B$ we get,

$$u = n(1875A^5 - 18750A^3B^2 + 9375AB^4 - 12500A^4B + 25000A^2B^3 - 2500B^5)$$

$$-(9374A^4B - 18750A^2B^3 + 1875B^5 + 2500A^5 - 25000A^3B^2 + 12500AB^4)$$

$$v = n(9375A^4B - 18750A^2B^3 + 1875B^5 + 2500A^5 - 25000A^3B^2 + 12500AB^4)$$

$$+(1875A^5 - 18750A^3B^2 + 9375AB^4 - 12500A^4B + 25000A^2B^3 - 2500B^5)$$

Therefore, $u = nf(A,B) - g(A,B)$ and $v = ng(A,B) + f(A,B)$

where, $f(A,B) = (1875A^5 - 18750A^3B^2 + 9375AB^4 - 12500A^4B + 25000A^2B^3 - 2500B^5)$

$g(A,B) = (9375A^4B - 18750A^2B^3 + 1875B^5 + 2500A^5 - 25000A^3B^2 + 12500AB^4)$

Hence, for all $n \geq 0$, the non-zero integral solutions are found to be,

$$p = ((n-1)f(A,B) - (n+1)g(A,B))^5$$

$$q = ((n+1)g(A,B) - (n-1)f(A,B))^5$$

$$r = (nf(A,B) - g(A,B))^5 - 10(nf(A,B) - g(A,B))^3(f(A,B) + ng(A,B))^2 + 5(nf(A,B) - g(A,B))(f(A,B) + ng(A,B))$$

$$s = 5(nf(A,B) - g(A,B))^4(f(A,B) + ng(A,B)) - 10(nf(A,B) - g(A,B))^2(f(A,B) + ng(A,B))^2 + (f(A,B) + ng(A,B))^5$$

Numeric examples satisfying the solution are listed below.

TABLE 2

n	p	q	r	s	k	LHS=RHS
1	52521900000000000000000	-52521900000000000000000	-97375000000000000000	74062500000000000000	50	625000000
2	-18895700000000000000000000	18895700000000000000000000	437638000000000000000000000	74062500000000000000	100	50000000000
3	500888000000000000000000000	-500888000000000000000000000	-41463500000000000000000000000	30459100000000000000000000000	125	305176000000
4	-459165000000000000000000000000	459165000000000000000000000000	48785300000000000000000000000	10871300000000000000000000000	100	170000000000

C. Pattern 3

Taking (3) as in pattern (2) and replacing 1 as $\left(\frac{6^2+8^2}{10^2}\right)$, we get $u^2 + v^2 = \left(\frac{6^2+8^2}{10^2}\right)(y^2+z^2)^5(n^2+1)$

Using factorization and equating real and imaginary parts we get,

$$u = [n(6y^5 - 60y^3z^2 + 30yz^4 - 40y^4z + 80y^2z^3 - 8z^5) - (30y^4z - 60y^2z^3 + 6z^5 + 8y^5 - 80y^3z^2 + 40yz^4)]/10$$

$$v = [n(30y^4z - 60y^2z^3 + 6z^5 + 8y^5 - 80y^3z^2 + 40yz^4) + (6y^5 - 60y^3z^2 + 30yz^4 - 40y^4z + 80y^2z^3 - 8z^5)]/10$$

In order to get integer solutions, take $y = 10A$ and $z = 10B$ we get,

$$u = n(60000A^5 - 600000A^3B^2 + 300000AB^4 - 400000A^4B + 800000A^2B^3 - 80000B^5)$$
$$-(300000A^4B - 600000A^2B^3 + 60000B^5 + 80000A^5 - 800000A^3B^2 + 800000AB^4)$$

$$v = n(300000A^4B - 600000A^2B^3 + 60000B^5 + 80000A^5 - 800000A^3B^2 + 400000AB^4)$$
$$+(60000A^5 - 600000A^3B^2 + 3000000AB^4 - 4000000A^4B + 800000A^2B^3 - 80000B^5)$$

Therefore, $u = nf(A,B) - g(A,B)$ and $v = ng(A,B) + f(A,B)$

where, $f(A,B) = (60000A^5 - 600000A^3B^2 + 300000AB^4 - 400000A^4B + 800000A^2B^3 - 80000B^5)$

$$g(A,B)=(300000A^4B-600000A^2B^3+60000B^5+80000A^5-800000A^3B^2+400000AB^4)$$

Hence, for all $n\geq 0$, the non-zero integral solutions are found to be,

$$p=((n-1)f(A,B)-(n+1)g(A,B))^5$$

$$q=((n+1)g(A,B)-(n-1)f(A,B))^5$$

$$r=(nf(A,B)-g(A,B))^5-10(nf(A,B)-g(A,B))^3(f(A,B)+ng(A,B))^2+5(nf(A,B)-g(A,B))(f(A,B)+ng(A,B))^4$$

$$s=5(nf(A,B)-g(A,B))^4(f(A,B)+ng(A,B))-10(nf(A,B)-g(A,B))^2(f(A,B)+ng(A,B))^2+(f(A,B)+ng(A,B))^5$$

Numeric examples satisfying the solution are listed below.

TABLE 3						
n	p	q	r	s	k	LHS=RHS
1	1706340000000000000 000000000000	- 1706340000000000000 000000000000	- 3267360000000000000 00000000000	758120000000000000 000000000000	200	64600200 0000
2	- 3986730000000000000 0000000000000000	3986730000000000000 0000000000000000	1167060000000000000 0000000000000000	146778800000000000 0000000000000000 00	400	51194100 000000
3	- 5277320000000000000 0000000000000000	5277320000000000000 0000000000000000	9135790000000000000 00000000000000000	- 142333000000000000 00000000000000000 000	500	30994700 0000000
4	- 4366610000000000000 0000000000000000000 0	4366610000000000000 0000000000000000000 0	9211150000000000000 0000000000000000000 00	- 175605000000000000 0000000000000000 0000000	900	10038300 00000000 0

D. Pattern 4

Taking (3) as in pattern (2) and replacing 1 as $\left(\frac{5^2+12^2}{13^2}\right)$

$$u^2+v^2=\left(\frac{5^2+12^2}{13^2}\right)(y^2+z^2)^5(n^2+1)$$

Using factorization and equating real and imaginary parts we get,

$$u=[n(5y^5-50y^3z^2+25yz^4-60y^4z+120y^2z^3-12z^5)-(25y^4z-50y^2z^3+5z^5+12y^5-120y^3z^2+60yz^4)]/13$$

$$v=[n(25y^4z-50y^2z^3+5z^5+12y^5-120y^3z^2+60yz^4)+(5y^5-50y^3z^2+25yz^4-60y^4z+120y^2z^3-12z^5)]/13$$

In order to get integer solutions, take $y=13A$ and $z=13B$ we get,

$$u=n(142805A^5-1428050A^3B^2+714025AB^4-1713660A^4B+3427320A^2B^3-342732B^5)$$

$$-(714025A^4B-1428050A^2B^3+142805B^5+342732A^5-3427320A^3B^2+1713660AB^4)$$

$$v=n(714025A^4B-1428050A^2B^3+142805B^5+342732A^5-3427320A^3B^2+1713660AB^4)$$

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$+(142805A^5-1428050A^3B^2+714025AB^4-1713660A^4B+3427320A^2B^3-342732B^5)$

Therefore, $u=nf(A,B)-g(A,B)$ and $v=ng(A,B)+f(A,B)$

where, $f(A,B)=(142805A^5-1428050A^3B^2+714025AB^4-1713660A^4B+3427320A^2B^3-342732B^5)$

$g(A,B)=(714025A^4B-1428050A^2B^3+142805B^5+342732A^5-3427320A^3B^2+1713660AB^4)$

Hence, for all $n\geq 0$, the non-zero integral solutions are found to be,

$$p=((n-1)f(A,B)-(n+1)g(A,B))^5$$

$$q=((n+1)g(A,B)-(n-1)f(A,B))^5$$

$$r=(nf(A,B)-g(A,B))^5-10(nf(A,B)-g(A,B))^3(f(A,B)+ng(A,B))^2+5(nf(A,B)-g(A,B))(f(A,B)+ng(A,B))^4$$

$$s=5(nf(A,B)-g(A,B))^4(f(A,B)+ng(A,B))-10(nf(A,B)-g(A,B))^2(f(A,B)+ng(A,B))^2+(f(A,B)+ng(A,B))^5$$

Numeric examples satisfying the solution are listed below.

TABLE 4

n	p	q	r	s	k	LHS=RHS
1	884223000000000000000000000000000	-8842230000000000000000000000000	907154000000000000000000000000	-2126860000000000000000000000000	338	8822940000000
2	790907000000000000000000000000000	-7909070000000000000000000000000	11942700000000000000000000000000000000	216651000000000000000000000000000000000	845	227694000000000
3	-2201840000000000000000000000000	2201840000000000000000000000000	-22018400000000000000000000000	0	169	1378580000000
4	-73881600000000000000000000000000000000	73881600000000000000000000000000000000	355193000000000000000000000000000000	1002900000000000000000000000000000000	676	1624670000000000

III. CONCLUSION

In this post, we have explained the essential solutions to the transcendental equation $p^2-\sqrt{p^4+q^4-(pq)^2}+\sqrt[5]{r^2+s^2}=k^5(n^2+1)$ under complex patterns utilizing various numerical examples. Additionally, one could look for detailed answers to these similar types of equations.

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REFERENCES

[1] AI-Zaid H, Brindza B and Printer A, "On Positive integer solutions of the Equation $xy + yz + zn = n$", Canad.Math.Bull, Vol-39, PP:199, 1996.

[2] Burton D, Elementary Number theory. Tata Hill Publishing company Ltd., New Delhi, 2002.

[3] Dickson L.E. History of Theory of numbers, vol 2, Chelsea publishing company, New York, 1952.

[4] Hua L.K. "Introduction to the Theory of Numbers", Springer-Verlag, Berlin-New York, 1982.

[5] Gopalan M.A, Devibala S, "A remarkable Transcendental Equation" Antrtica.J.Math.3(2),(2006),209-215.

[6] Gopalan M.A ,Kalinga Rani J, "On the transcendental equation $x + g\sqrt{x} + y + h\sqrt{y} = z + g\sqrt{z}$", International Journal of Mathematical Sciences, Vol-9, No-1-2, 177-182,2010.

[7] Gopalan M.A. and Pandichelvi, "Observations on the transcendental equation $z = \sqrt{x} + \sqrt[3]{kx + y^2}$", Diophantus J.Math., 1(2), 59-68,2012.

[8] Gopalan M.A. Manju Somanth and Vanitha N, "On the Special Transcendental Equation", Reflections des ERAJMS. Vol-7, Issue-2.187-192, 2012.

[9] Janaki G, Saranya P, "On the Ternary Quadratic Diophantine Equation $5(x^2 + y^2) - 6xy + 4(x + y) + 4 = 40z^3$", International Journal of Science and Research, Vol 5, issue 3,March 2016, PP:227-229.

[10] Janaki G, Saranya P, "On the Ternary Quadratic Diophantine Equation $5(x^2 + y^2) - 6xy = 4z^2$", Imperial Journal of Interdisciplinary Research, Vol 2, Issue 3, 2016, PP: 396-397.

[11] Janaki G, Saranya P, (2016). Special Pairs of Rectangles and Narcissistic Numbers of Order 3 and 4. IRJET, 3(08), 721-723.

[12] Janaki G,Saranya P, (2016). Special Rectangles and Narcissistic Numbers of Order 3 and 4. International Journal for Research in Applied Science and Engineering Technology, 4(6), 630-633.

[13] Mordel L.J. Diophantine Equations, Academic press, New York, 1969.

[14] Saranya C, Janaki G and Salini R. "Integer Solutions of Some Exponential Diophantine Equations". The Mathematics Education, Vol-58, no-1, March 2024.

[15] Saranya P, Janaki G "On the Quintic Non-Homogeneous Diophantine Equation $x^4 - y^4 = 40(z^2 - w^2)p^3$", International Journal of Engineering Science and Computing, vol-7, issue-2, February 2017, Pg No:4685-4687, Impact Factor:5.611, ISSN No:2349-4182.

[16] Saranya P, Janaki G, "Explication of the transcendental equation $p + \sqrt{p^3 + q^3 - pq} + \sqrt[3]{r^2 + s^2} = (m^2 + 1)t^3$", International Research Journal of Engineering and Technology (IRJET), Vol 4, issue 11, Nov 2017, PP-1042-1044.

[17] Saranya P, Janaki G, "On the Exponnential Diophantine Equation $36^x + 3^y = z^2$", International Research Journal of Engineering and Technology (IRJET), Vol 2, issue 11, Nov 2017, PP-1042-1044.

[18] Saranya P, Janaki G, "Ascertainment of the Exponential Equation $p^{3c} - a(p^{2c} - p^b) = p^{b+c}$", compliance Engineering journal (CEJ), Vol 10, issue 8, July 2019, PP-225-229.

[19] Saranya P, Janaki G, "On some Non-Extendable Gaussian Triples involving Mersenne and Gnomonic number", Aryabhatta Journal of Mathematics and informatics, Vol-12, issue-1, Jan-June 2020, Pg No:81-84, Impact factor:5.856, ISSN No:0975-7139(P).