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Integer Solutions on Ternary Quadratic Diophantine Equation $3(x^2 + y^2) - 5xy = 36z^2$

G. Janaki[1], P. Sangeetha[2], R. Rasini[3]
[1]Associate Professor, [2]Assistant Professor, [3]PG Student, PG and Research Department of Mathematics Cauvery College for Women (Autonomous) Trichy-18, India (Affiliated Bharathidasan University)

Abstract: The non-zero distinct integer solution to the Quadratic Diophantine equation with three unknowns are examined. We obtain the various patterns of integer points that satisfy the ternary Quadratic equation. A few interesting relationship between the solution and the unique pattern.
Keywords: Integral solutions, Ternary Quadratic equation, Polygonal number

I. INTRODUCTION

Number theory is a type of mathematics concerned with the properties and relationship of number, specifically integers. Number theory has a long history and has made significant contributions to many fields of mathematics. A Diophantine equation with three variables is called a Ternary Diophantine equation. Find an integer solution for x, y, z such that the given equation is satisfied in order to solve Ternary Quadratic Diophantine Equation. In mathematics a Diophantine equation is an equation, typically a polynomial equation in two or more unknown with integer coefficients for which only integer solution are of interest. The Diophantine equations provide an infinite fields for study [1-3]. In particular, when solving Quadratic equation involving three unknowns, reference [4-10].This correspondence belongs to an addition exciting equation, which is $3(x^2 + y^2) - 5xy = 36z^2$, which is a homogeneous quadratic equation with three unknown. The Objective is to determine the infinite number of non-zero integral points of this equation. A few amazing relationship between the solution are additionally shown.

II. NOTATION

$T_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right] =$ Polygonl number with rank n and side m.

$O_n = \frac{(2n^3 + n)}{3} =$ Octahendral number of rank n.

$4DF_n = \frac{n^2(n^2-1)}{12} =$ Four dimensional figurate whose generating polygonal is a square.

$SO_n = n(2n^2 - 1) =$ Stella octangula number of rank n.

III. METHOD OF ANALYSIS

To find the non-zero integral solution of the ternary quadratic Diophantine equation

$$3(x^2 + y^2) - 5xy = 36z^2 \qquad (1)$$

The replacement of linear transformations

$$x = p + q \quad \text{and} \quad y = p - q \qquad (2)$$

As (1) result in

$$p^2 + 11q^2 = 36z^2 \qquad (3)$$

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A. Pattern 1

consider, $z = z(a,b) = a^2 + 11b^2$ (4)

When the integer a and b are non-zero

$$36 = (5 + i\sqrt{11})(5 - i\sqrt{11}) \quad (5)$$

By applying factorization techniques and the equation (4) and (5) in (3)

$$(p + i\sqrt{11}q)(p - i\sqrt{11}q) = (5 + i\sqrt{11})(5 - i\sqrt{11})(a + i\sqrt{11}b)^2(a - i\sqrt{11}b)^2 \quad (6)$$

By comparing the real and imaginary components of the equation and equating the like term, we obtain

$$p = 5a^2 - 55b^2 - 22b$$

$$q = a^2 - 11b^2 + 10ab$$

Equation (2) can be solved as an integer by substituting the above values for p and q

$$x = x(a,b) = 6a^2 - 66b^2 - 12ab$$

$$y = y(a,b) = 4a^2 - 44b^2 - 32ab$$

$$z = z(a,b) = a^2 + 11b^2$$

Observation

1. $y(a,a) - 4x(2a,2a) = 216a^2$ Nasty number.
2. $x(1,1) + y(1,1) = -144$ Perfect square.
3. $x(a,a) + y(a,a) + 12T_{26,a} + 66Gno_a \equiv 0(\bmod 66)$
4. $y(a,a) + 8T_{20,a} + 32Gno_a \equiv 0(\bmod 32)$
5. $2x(a,a) + 12T_{24,a} + 60Gno_a \equiv 0(\bmod 60)$
6. $x(2a,1) + y(a,1) - 4T_{16,a} + 16Gno_a \equiv 0(\bmod 126)$

B. Pattern 2

Equation (3) is written as,

$$p^2 - 36z^2 = -11q^2 \quad (7)$$

compute equation (7)

$$(p + 6z)(p - 6z) = q \times -11q \quad (8)$$

Equation (8) is expressed as a ratio as

$$\frac{(p + 6z)}{-11q} = \frac{q}{(p - 6z)} = \frac{a}{b}$$

The two equations that follow are identical to this

$$bp + 11aq + 6bz = 0 \quad (9)$$

$$ap - bq - 6az = 0 \quad (10)$$

When we apply cross-multiplication, we obtain

$$p = 66a^2 - 6b^2$$

$$q = 11a^2 + b^2$$

$$z = -6ab - 6ab$$

The equivalent integer answer (1) can be obtained by substituting the two variables of p and q which appear previous them

$$x = x(a,a) = 77a^2 - 5a^2$$

$$y = y(a,b) = 55a^2 - 7b^2$$

$$z = z(a,b) = -6ab - 6ab$$

Observation

1. $y(1,1)+z(1,1)=36$ Perfect square
2. $2x(a,a)+2y(a,a)+2z(a,a)=216$ Cubical integer
3. $2x(a,a)-y(a,a)=96$ Nasty number
4. $x(a,a)-y(a,a)+z(1,1)-2T_{26,a}-11Gno_a\equiv 0(\mathrm{mod}\,1)$
5. $x(a,a)-z(a,a)-12T_{16,a}-36Gno_a\equiv 0(\mathrm{mod}\,36)$
6. $x(a^2,a)+y(a^2,a)-3360DF_a-20T_{14,a}-50Gno_a\equiv 0(\mathrm{mod}\,50)$

C. Pattern 3

Equation (3) as, $p^2=36z^2-11q^2$

Consider, $p=p(a,b)=36a^2+11b^2$ (11)

$$(6a+\sqrt{11}b)^2(6a-\sqrt{11}b)^2=(6z+\sqrt{11}q)(6z-\sqrt{11}q) \quad (12)$$

When we compare rational and irrational we get,

$$z=\frac{36a^2+11b^2}{6}$$

$$q=12ab$$

Substituting the above mentioned p and q value into equation (2)

$$x=x(a,b)=36a^2-11b^2+12ab$$

$$y=y(a,b)=36a^2-11b^2-12ab$$

$$z=z(a,b)=\frac{36a^2+11b^2}{6}$$

When we substitute a=6A and b=6B in x, y and z we get

$$x=x(A,B)=1296A^2-396B^2+432AB$$

$$y=y(A,B)=1296A^2-396B^2-432AB$$

$$z=z(A,B)=216A^2+66B^2$$

Observation

1. $x(a,a)-z(a,a)-105T_{22,a}-472Gno_a-a\equiv 0(\mathrm{mod}\,472)$
2. $x(a,1)+z(a,1)-108T_{30,a}-918Gno_a\equiv 0(\mathrm{mod}\,588)$
3. $x(a,a)+y(a,a)-66a^2$ Nasty number
4. $x(a,a)+z(1,1)-222T_{14,a}-555Gno_a\equiv 0(\mathrm{mod}\,837)$
5. $z(a,a)-94T_{8,a}-94Gno_a\equiv 0(\mathrm{mod}\,94)$
6. $y(a^2,a)-z(a^2,a)-5180DF_a+216SO_a-412T_{5,a}+5Gno_a\equiv 0(\mathrm{mod}\,5)$

D. Pattern 4

Equation (3) can be expressed as follows

$$p^2+11q^2=36z^2\times 1 \quad (13)$$

‘1’ Should be written as

$$1=\frac{(5+i\sqrt{11})(5-i\sqrt{11})}{36} \quad (14)$$

$$36=(5+i\sqrt{11})(5-i\sqrt{11})$$

we can write using equation(14),(15) and the value of z

$$(p+i\sqrt{11}q)(p-i\sqrt{11}q)=(5+i\sqrt{11})(5-i\sqrt{11})(a^2+\sqrt{11}b^2)^2\frac{(5+i\sqrt{11})(5-i\sqrt{11})}{36}$$

Equating positive factor

$$(p+i\sqrt{11}q)=\frac{1}{6}(5+i\sqrt{11})(5+i\sqrt{11})(a+i\sqrt{11}b)^2$$

$$(p+i\sqrt{11}q)=\frac{(14a^2-154b^2-220ab)+i\sqrt{11}(10a^2-110b^2+28ab)}{6}$$

When we solve the preceding equation for the real and imaginary part, we obtain

$$p=\frac{(14a^2-154b^2-220ab)}{6}$$

$$q=\frac{(10a^2-110b^2+28ab)}{6}$$

Substituting the following values of p and q into equation (2)

$$x=\frac{(24a^2-264b^2-192ab)}{6}$$

$$y=\frac{(4a^2-44b^2-248ab)}{6}$$

$$z=a^2+11b^2$$

Now if replace a=6A and b=6B in x, y and z we get

$$x=x(A,B)=144A^2-1584B^2-1152AB$$

$$y=y(A,B)=24A^2-264B^2-1488AB$$

$$z=z(A,B)=36A^2-396B^2$$

Observation

1. $x(a,1)+y(a,1)-28T_{14,a}+1250Gno_a\equiv(\mathrm{mod}\,3098)$
2. $y(a,a)+z(a,a)+288T_{11,a}+504Gno_a\equiv 0(\mathrm{mod}\,504)$
3. $y(a^2,a)-x(a^2,a)+5760DF_a+168SO_a-100T_{26,a}-466Gno_a\equiv 0(\mathrm{mod}\,466)$
4. $z(a,a)-54T_{14,a}-189Gno_a\equiv 0(\mathrm{mod}\,189)$
5. $y(a,a)-x(a,a)=864a^2$ Nasty number
6. $x(a^2,1)-y(a,a)-6912DF_a+576SO_a-64T_{11,a}+176Gno_a\equiv 0(\mathrm{mod}\,176)$

IV. CONCLUSION

This paper presents an unlimited number of different integer solutions, all non-zero, to the ternary quadratic Diophantine problem $3(x^2+y^2)-5xy=36z^2$ .Because quadratic equations are various, one can search for other quadratic equation with variable higher than properties using special number.

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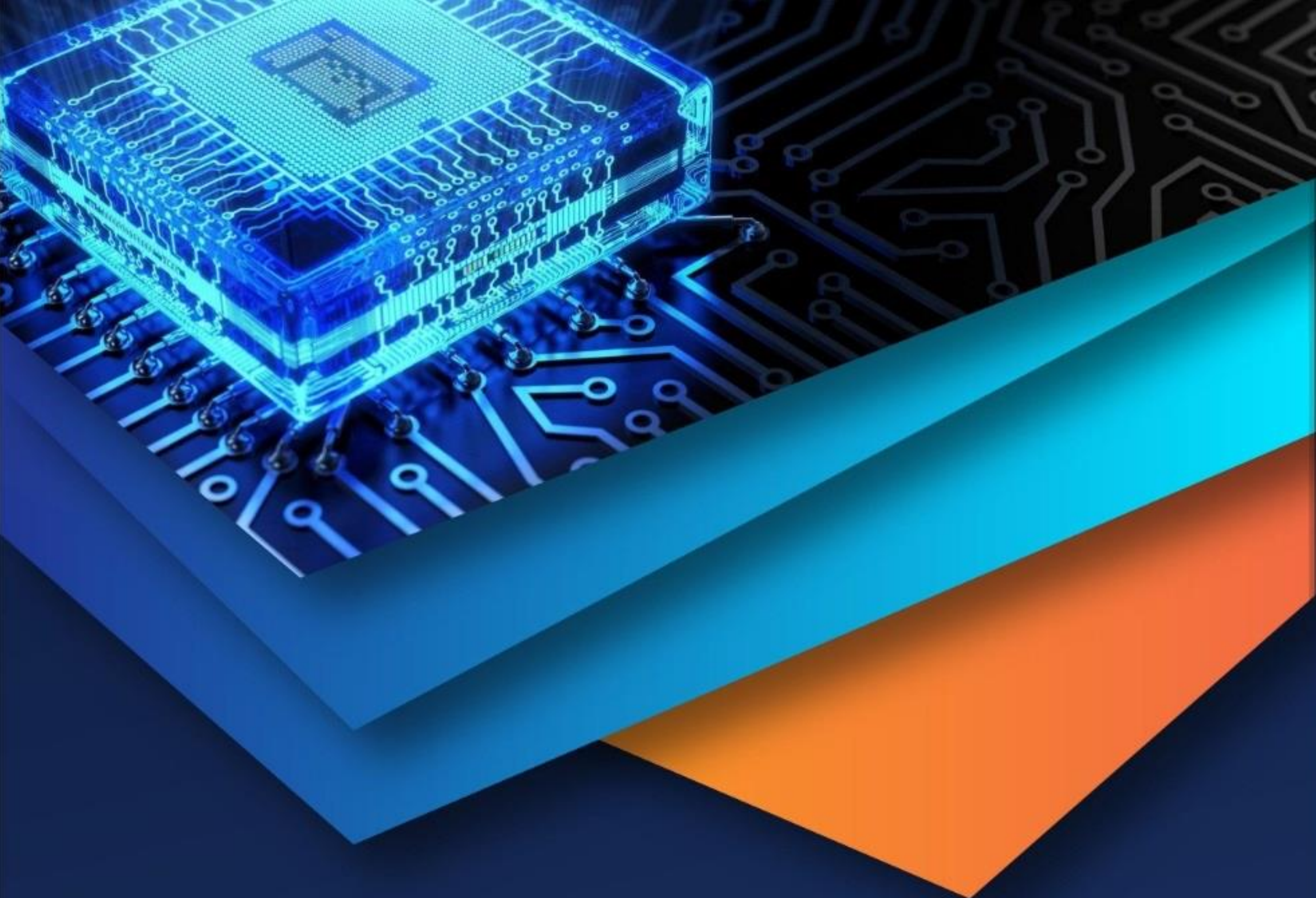

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