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Integer Solutions on Ternary Quadratic Diophantine Equation $3(x^2 + y^2) - 5xy = 36z^2$

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Abstract: The non-zero distinct integer solution to the Quadratic Diophantine equation with three unknowns are examined. We obtain the various patterns of integer points that satisfy the ternary Quadratic equation. A few interesting relationship between the solution and the unique pattern.

Keywords: Integral solutions, Ternary Quadratic equation, Polygonal number

I. INTRODUCTION

Number theory is a type of mathematics concerned with the properties and relationship of number, specifically integers. Number theory has a long history and has made significant contributions to many fields of mathematics. A Diophantine equation with three variables is called a Ternary Diophantine equation. Find an integer solution for x, y, z such that the given equation is satisfied in order to solve Ternary Quadratic Diophantine Equation. In mathematics a Diophantine equation is an equation, typically a polynomial equation in two or more unknown with integer coefficients for which only integer solution are of interest. The Diophantine equations provide an infinite fields for study [1-3]. In particular, when solving Quadratic equation involving three unknowns, reference [4-10]. This correspondence belongs to an addition exciting equation, which is $3(x^2 + y^2) - 5xy = 36z^2$, which is a homogeneous quadratic equation with three unknown. The Objective is to determine the infinite number of non-zero integral points of this equation. A few amazing relationship between the solution are additionally shown.

II. NOTATION

 $T_{m,n} = n \left[1 + \frac{(n-1)(m-2)}{2} \right] = \text{Polygonl number with rank n and side m.}$ $O_n = \frac{(2n^3 + n)}{3} = \text{Octahendral number of rank n.}$ $4DF_n = \frac{n^2(n^2 - 1)}{12} = \text{Four dimensional figurate whose generating polygonal is a square.}$

 $SO_n = n(2n^2 - 1) =$ Stella octangula number of rank n.

III. METHOD OF ANALYSIS

(2)

To find the non-zero integral solution of the ternary quadratic Diophantine equation

$$3(x^2 + y^2) - 5xy = 36z^2 \tag{1}$$

The replacement of linear transformations

x = p + q and y = p - q

As (1) result in

$$p^2 + 11q^2 = 36z^2 \tag{3}$$



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A. Pattern 1

$$z = z(a,b) = a^{2} + 11b^{2}$$
(4)

When the integer a and b are non-zero

consider,

$$36 = (5 + i\sqrt{11})(5 - i\sqrt{11}) \tag{5}$$

By applying factorization techniques and the equation (4) and (5) in (3)

$$(p+i\sqrt{11}q)(p-i\sqrt{11}q) = (5+i\sqrt{11})(5-i\sqrt{11})(a+i\sqrt{11}b)^2(a-i\sqrt{11}b)^2$$
(6)

By comparing the real and imaginary components of the equation and equating the like term, we obtain

$$p = 5a^{2} - 55b^{2} - 22b$$
$$q = a^{2} - 11b^{2} + 10ab$$

Equation (2) can be solved as an integer by substituting the above values for p and q

$$x = x(a,b) = 6a2 - 66b2 - 12ab$$

$$y = y(a,b) = 4a2 - 44b2 - 32ab$$

$$z = z(a,b) = a2 + 11b2$$

Observation

1. $y(a,a) - 4x(2a,2a) = 216a^2$ Nasty number. 2. x(1,1) + y(1,1) = -144 Perfect square. 3. $x(a,a) + y(a,a) + 12T_{26,a} + 66Gno_a \equiv 0 \pmod{66}$ 4. $y(a,a) + 8T_{20,a} + 32Gno_a \equiv 0 \pmod{32}$ 5. $2x(a,a) + 12T_{24,a} + 60Gno_a \equiv 0 \pmod{60}$ 6. $(2a, 1) + x(a, 1) + 4T_{24,a} + 16Gna_{24,a} = 0 \pmod{64}$

6. $x(2a,1) + y(a,1) - 4T_{16,a} + 16Gno_a \equiv 0 \pmod{126}$

B. Pattern 2

Equation (3) is written as,

$$p^2 - 36z^2 = -11q^2 \tag{7}$$

compute equation (7)

$$(p+6z)(p-6z) = q \times -11q$$
 (8)

Equation (8) is expressed as a ratio as

$$\frac{(p+6z)}{-11q} = \frac{q}{(p-6z)} = \frac{a}{b}$$

The two equations that follow are identical to this

$$bp + 11aq + 6bz = 0 \tag{9}$$

$$ap - bq - 6az = 0 \tag{10}$$

When we apply cross-multiplication, we obtain

$$p = 66a2 - 6b2$$
$$q = 11a2 + b2$$
$$z = -6ab - 6ab$$

The equivalent integer answer (1) can be obtained by substituting the two variables of p and q which appear previous them

$$x = x(a,a) = 77a^{2} - 5a^{2}$$

y = y(a,b) = 55a^{2} - 7b^{2}
z = z(a,b) = -6ab - 6ab



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Observation

- 1. y(1,1) + z(1,1) = 36 Perfect square
- 2. 2x(a, a) + 2y(a, a) + 2z(a, a) = 216 Cubical integer
- 3. 2x(a, a) y(a, a) = 96 Nasty number

4.
$$x(a,a) - y(a,a) + z(1,1) - 2T_{26,a} - 11Gno_a \equiv 0 \pmod{1}$$

5. $x(a,a) - z(a,a) - 12T_{16,a} - 36Gno_a \equiv 0 \pmod{36}$

6.
$$x(a^2, a) + y(a^2, a) - 3360 DF_a - 20T_{14,a} - 50Gno_a \equiv 0 \pmod{50}$$

C. Pattern 3

Equation (3) as,

Consider,

$$p = p(a,b) = 36a^2 + 11b^2$$
(11)

$$(6a + \sqrt{11b})^2 (6a - \sqrt{11b})^2 = (6z + \sqrt{11q})(6z - \sqrt{11q})$$
(12)

When we compare rational and irrational we get,

$$z = \frac{36a^2 + 11b^2}{6}$$
$$q = 12ab$$

 $p^2 = 36z^2 - 11q^2$

Substituting the above mentioned p and q value into equation (2)

$$x = x(a,b) = 36a^{2} - 11b^{2} + 12ab$$
$$y = y(a,b) = 36a^{2} - 11b^{2} - 12ab$$
$$z = z(a,b) = \frac{36a^{2} + 11b^{2}}{6}$$

When we substitute a=6A and b=6B in x, y and z we get

$$x = x(A,B) = 1296A^{2} - 396B^{2} + 432AB$$

$$y = y(A,B) = 1296A^{2} - 396B^{2} - 432AB$$

$$z = z(A,B) = 216A^{2} + 66B^{2}$$

Observation

1. $x(a, a) - z(a, a) - 105T_{22,a} - 472Gno_a - a \equiv 0 \pmod{472}$ 2. $x(a, 1) + z(a, 1) - 108T_{30,a} - 918Gno_a \equiv 0 \pmod{472}$ 3. $x(a, a) + y(a, a) - 66a^2$ Nasty number 4. $x(a, a) + z(1, 1) - 222T_{14,a} - 555Gno_a \equiv 0 \pmod{837}$ 5. $z(a, a) - 94T_{8,a} - 94Gno_a \equiv 0 \pmod{94}$ 6. $y(a^2, a) - z(a^2, a) - 5180DF_a + 216SO_a - 412T_{5,a} + 5Gno_a \equiv 0 \pmod{5}$

D. Pattern 4

Equation (3) can be expressed as follows

$$p^2 + 11q^2 = 36z^2 \times 1 \tag{13}$$

'1' Should be written as

$$1 = \frac{(5 + i\sqrt{11})(5 - i\sqrt{11})}{36} \tag{14}$$



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$$36 = (5 + i\sqrt{11})(5 - i\sqrt{11})$$

we can write using equation(14),(15) and the value of z

$$(p+i\sqrt{11}q)(p-i\sqrt{11}q) = (5+i\sqrt{11})(5-i\sqrt{11})(a^2+\sqrt{11}b^2)^2 \frac{(5+i\sqrt{11})(5-i\sqrt{11})}{36}$$

Equating positive factor

$$(p+i\sqrt{11}q) = \frac{1}{6}(5+i\sqrt{11})(5+i\sqrt{11})(a+i\sqrt{11}b)^2$$
$$(p+i\sqrt{11}q) = \frac{(14a^2-154b^2-220ab)+i\sqrt{11}(10a^2-110b^2+28ab)}{6}$$

When we solve the preceding equation for the real and imaginary part, we obtain

$$p = \frac{(14a^2 - 154b^2 - 220ab)}{6}$$
$$q = \frac{(10a^2 - 110b^2 + 28ab)}{6}$$

Substituting the following values of p and q into equation (2)

$$x = \frac{(24a^2 - 264b^2 - 192ab)}{6}$$
$$y = \frac{(4a^2 - 44b^2 - 248ab)}{6}$$
$$z = a^2 + 11b^2$$

Now if replace a=6A and b=6B in x, y and z we get

$$x = x(A,B) = 144A^{2} - 1584B^{2} - 1152AB$$
$$y = y(A,B) = 24A^{2} - 264B^{2} - 1488AB$$
$$z = z(A,B) = 36A^{2} - 396B^{2}$$

Observation

1.
$$x(a,1) + y(a,1) - 28T_{14,a} + 1250Gno_a \equiv \pmod{3098}$$

2. $y(a,a) + z(a,a) + 288T_{11,a} + 504Gno_a \equiv 0 \pmod{504}$

3.
$$y(a^2, a) - x(a^2, a) + 5760 DF_a + 168SO_a - 100T_{26,a} - 466Gno_a \equiv 0 \pmod{466}$$

4. $z(a, a) - 54T_{14,a} - 189Gno_a \equiv 0 \pmod{189}$

5.
$$y(a,a) - x(a,a) = 864a^2$$
 Nasty number

6. $x(a^2, 1) - y(a, a) - 6912DF_a + 576SO_a - 64T_{11,a} + 176Gno_a \equiv 0 \pmod{176}$

IV. CONCLUSION

This paper presents an unlimited number of different integer solutions, all non-zero, to the ternary quadratic Diophantine problem $3(x^2 + y^2) - 5xy = 36z^2$. Because quadratic equations are various, one can search for other quadratic equation with variable higher than properties using special number.

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