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Integral Solutions of the Bi-Quadratic Equation

$$2(x^2-y^2)(7x^2+y^2)=520\,(z^2-w^2)\,t^2+24s^2$$

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Abstract:** The biquadratic with six unknowns of the form* $2(x^2-y^2)(7x^2+y^2)=520\,(z^2-w^2)\,t^2+24s^2$ ***has been studied for its non-trivial distinct integral solutions. A few interesting relations among the solutions and polygonal numbers are presented.

I. INTRODUCTION

Carl Friedrich Gauss, a well-known mathematician who lived from 1777 to 1855, is credited with saying that “number theory is the queen of mathematics and mathematics is the queen of all sciences.” The study of integer and rational number features that go beyond standard manipulations in common arithmetic is known as number theory, or higher arithmetic. Number theorists were created in practically every major civilization throughout history. In the ancient and medieval eras, these individuals were typically geometers, but they may also have been philosophers, calendar calculators, astronomers, astrologers, priests,or magicians. Due to their variety, the quadratic offers an infinite topic for study. One may specifically refer to [4] through [20]. This problem of communication with the non-homogeneous quadratic equation denoted by,

$$2(x^2-y^2)(7x^2+y^2)=520\,(z^2-w^2)\,t^2+24s^2$$

is examined for its distinct non-zero integral solution. Some intriguing connections between the solutions are presented.

II. NOTATIONS

SPECIAL NUMBER	NOTATIONS	DEFINITIONS
Triangular number	$T_{3,n}$	$\frac{n(n+1)}{2}$
Hexadecagonal number	$T_{16,n}$	$n(7n-6)$
Nonadecagonal number	$T_{19,n}$	$\frac{n(17n-15)}{2}$
Icositetragonal number	$T_{24,n}$	$n(11n-10)$
Octahedral number	O_n	$\frac{1}{3}(2n^3+n)$
Centered square number	CS_n	$n^2+(n-1)^2$
Gnomonic number	Gno_n	$(2n-1)$
Stella octangular number	SO_n	$n(2n^2-1)$
Star number	$Star_n$	$6n(n-1)+1$

Rhombic dodecagonal number	RD_n	$(2n-1)(2n^2-2n+1)$
Pentadecagonal number	$T_{15,n}$	$\frac{n(13n-11)}{2}$

III. METHOD OF ANALYSIS

The Quadratic equation under consideration is

$$2(x^2-y^2)(7x^2+y^2)=520\,(z^2-w^2)\;t^2+24s^2 \qquad (1)$$

On substituting the linear transformation,

$$x=u+v;\,y=u-v;$$

$$z=2u+v;w=2u-v;s=2uv \qquad (2)$$

in (1) , leads to

$$u^2+v^2=65t^2 \qquad (3)$$

Equation (3) is solved through different methods and thus we obtain four different patterns of integral solutions to (1)

A. *Pattern I*

Assume

$$t=t(a,b)=a^2+b^2 \qquad (4)$$

Where a and b are non-zero integers.

Write $65=(1+8i)(1-8i)$ (5) Applying the factorization method and substituting (4) and (5) in (3),

$$(u+iv)(u-iv)=(1+8i)(1-8i)(a+ib)^2\,(a-ib)^2 \qquad (6)$$

Equating real and imaginary parts, the values of u and v are

$$u=a^2-b^2-16ab$$

$$v=8a^2-8b^2+2ab$$

Substituting the values of u and v in equation (2), the non zero distinct integral solutions of (1) are executed as,

$$x=x(a,b)=9a^2-9b^2-14ab$$

$$y=y(a,b)=-7a^2+7b^2-18ab$$

$$z=z(a,b)=10a^2-10b^2-30ab$$

$$w=w(a,b)=-6a^2+6b^2-34ab \qquad (7)$$

$$s=s(a,b)=16a^4-96a^2b^2-252a^3b+252ab^3+16b^4$$

$$t=t(a,b)=a^2+b^2$$

Thus (7) gives the integral solution of (1)

Observations

1. $2s(1,1)+2t(1,1)-x(1,1)$ is a perfect square.
2. $5x(1,-1)+2z(1,-1)+2w(1,-1)+t(2,1)$ is a happy couple number
3. $z(A^2,4)-480DF_A+10T_{24,A}+50\,Gno_A\equiv 0\pmod{210}$
4. $8x(2A,A)-3z(A,A)+6t(2A,A)+w(A,A)-12T_{15,A}-33Gno_A\equiv 0\pmod{33}$

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5. $2s(A,1)-5y(2A+1,A)+t(A,A)-1536DF_A+756O_A-43T_{16,A}-622DGno_A\equiv 0(\text{mod}\,689)$
6. $4t(B+1,B)+s(B,2)-768DF_B+756O_B+4T_{8,B}-1134Gno_B\equiv 0(\text{mod}\,1394)$
7. $2w(3A,1)+3z(A+2,A)-162SO_4-5Star_A+66Gno_A\equiv 0(\text{mod}\,443)$

B. Pattern II

Assume

$t=t(a,b)=a^2+b^2$ (8) Where a and b are non-zero integers.

Write $65=(8+i)(8-i)$ (9) Applying the factorization method and substituting (8) and (9) in (3),

$(u+iv)(u-iv)=(8+i)(8-i)(a+ib)^2(a-ib)^2$ (10) Equating real and imaginary parts, the values of u and v are

$u=8a^2-8b^2-2ab$

$v=a^2-b^2+16ab$

Substituting the values of u and v in equation (2), the non zero distinct integral solutions of (1) are executed as,

$$\left.\begin{aligned} x &= x(a,b)=9a^2-9b^2+14ab\\ y &= y(a,b)=7a^2-7b^2-18ab\\ z &= z(a,b)=17a^2-17b^2-12ab\\ w &= w(a,b)=15a^2-15b^2-20ab\\ s &= s(a,b)=16a^4-96a^2b^2+252a^3b-252ab^3+16b^4\\ t &= t(a,b)=a^2+b^2 \end{aligned}\right\} \quad (11)$$

Thus (11) gives the distinct integral solution of (1)

Observations

1. $4x(1,1)-4y(1,1)+z(1,1)+2t(1,1)$ is a perfect square.
2. $4x(1,1)-2y(1,1)+12z(1,1)-4w(1,1)-4s(1,1)-30t(1,1)$ is a cubical number.
3. $s(1,1)+z(1,1)+t(5,0)$ is a palindrom number.
4. $y(2A+1,2)-4T_{16,A}+10Gno_A\equiv 0(\text{mod}\,67)$
5. $w(A+1,A)+z(2A-1,A)-15Star_A+20Gno_A\equiv 0(\text{mod}\,323)$
6. $10w(2A,A)+t(2A^2,3)+z(2A,1)-192DF_A-68T_{19,A}-Gno_A\equiv 0(\text{mod}\,248)$
7. $y(2A-1,A^2)+10t(A^2,A)+z(A,0)-144DF_A+54O_A-152T_{3,A}+43Gno_A\equiv 0(\text{mod}\,36)$

C. Pattern III

Assume

$t=t(a,b)=a^2+b^2$ (12)

Where a and b are non-zero integers.

Write $65=(4+7i)(4-7i)$ (13) Applying the factorization method and substituting (12) and (13) in (3),

$$(u+iv)(u-iv)=(4+7i)(4-7i)(a+ib)^2(a-ib)^2 \quad (14)$$

Equating real and imaginary parts, the values of u and v are

$$u=4a^2-4b^2-14ab$$

$$v=7a^2-7b^2+8ab$$

Substituting the values of u and v in equation (2), the non zero distinct integral solutions of (1) are executed as,

$$x=x(a,b)=11a^2-11b^2-6ab$$

$$y=y(a,b)=-3a^2+3b^2-22ab$$

$$z=z(a,b)=15a^2-15b^2-20ab$$

$$w=w(a,b)=a^2-b^2-36ab \quad (15)$$

$$s=s(a,b)=56a^4-336a^2b^2-132a^3b+132ab^3+56b^4$$

$$t=t(a,b)=a^2+b^2$$

Thus, (15) gives the distinct integral solution of (1)

Observations

1. $\frac{1}{2}x(1,-1)-y(1,1)+z(-1,1)+w(1,-1)$ is a perfect square.
2. $z(1,-1)-2s(1,1)+4t(1,1)$ is a cubical number.
3. $x(1,1)+10t(5,4)$ is a Duck number.
4. $z(2A+1,A^2)+720DF_A+60O_A-5T_{12,A}-50Gno_A\equiv 0(\mathrm{mod}\ 65)$
5. $x(2A-1,A^2)+2y(A+1,2)+528DF_A+3RD_A+3T_{12,A}+60Gno_A\equiv 0(\mathrm{mod}\ 122)$
6. $x(2A-1,A)+2y(A+1,2)+528DF_A+3RD_A-3T_{12,A}+60Gno_A\equiv 0(\mathrm{mod}122)$
7. $s(1,A)-2688DF_A-66SO_A+40T_{16,A}+153Gno_A\equiv 0(\mathrm{mod}\ 97)$

D. Pattern 4

(3) can be written as

$$u^2=65t^2-v^2 \quad (16)$$

Assuming $u=65a^2-b^2$ and

$$1=\left(\frac{(\sqrt{65}+1)(\sqrt{65}-1)}{64}\right) \quad (17)$$

Substituting (16) in (17), using the method of factorization

$$\frac{(\sqrt{65}+1)(\sqrt{65}-1)(\sqrt{65}a-b)^2(\sqrt{65}a+b)^2}{64}=(\sqrt{65}t+v)(\sqrt{65}t-v)$$

Now define

$$(\sqrt{65}t+v)=\frac{(\sqrt{65}+1)(\sqrt{65}a+b)^2}{8}$$

$$(\sqrt{65}t-v)=\frac{(\sqrt{65}-1)(\sqrt{65}a-b)^2}{8} \quad (18)$$

Equating the like in (18) we get

$$t = \frac{1}{8}\left(65a^2 + b^2 + 2ab\right)$$

$$v = \frac{1}{8}\left(65a^2 + b^2 + 130ab\right)$$

Since our interest is on finding integer solution we have choose a and b Suitably so that t and v integers,

$$t = 65A^2 + B^2 + 2AB$$

$$v = 65A^2 + B^2 + 130AB$$

$$u = 520A^2 - 8B^2$$

Thus, using the values of u and v performing a few calculations the
Values of x, y and z are obtained as follows:

$$x = x(A,B) = 585A^2 - 7B^2 + 130AB$$

$$y = y(A,B) = 455A^2 - 9B^2 - 130AB$$

$$z = z(A,B) = 1105A^2 - 15B^2 + 130AB \qquad (19)$$

$$w = w(A,B) = 975A^2 - 17B^2 - 130AB$$

$$s = s(A,B) = 67600A^2 + 135200A^3B - 2080AB^3 - 16B^4$$

$$t = t(A,B) = 65A^2 + B^2 + 2AB$$

Thus (19) represents the non-trivial integral solution of (1)

Observations

1. $w(1,1) - y(1,1)$ is a cubical number.
2. $x(1,1) - t(1,1) - y(1,1)$ is a perfect square.
3. $s(1,1) + t(1,1)$ is a Duck number.
4. $x(A,1) + t(A,A) + y(1,A) - 116T_{13,A} - 391Gno_A \equiv 0(\mathrm{mod}\,1489)$
5. $w(A,2) + t(2A,1) - y(0,A) - 622CS_A - 183Gno_A \equiv 0(\mathrm{mod}\,506)$
6. $x(1,A) - t(2,2A) + T_{24,A} - 56Gno_A \equiv 0(\mathrm{mod}\,381)$
7. $y(2A,1) - 728T_{7,A} - 416Gno_A \equiv 0(\mathrm{mod}\,407)$

IV. CONCLUSION

In this study, an attempt has been made to complete the set of non-trivial distinct integral solutions for the non-homogeneous biquadratic equation. Finally, in order to find more solution to the biquadratic equation under consideration, one can look for further biquadratic equations that involve many variables.

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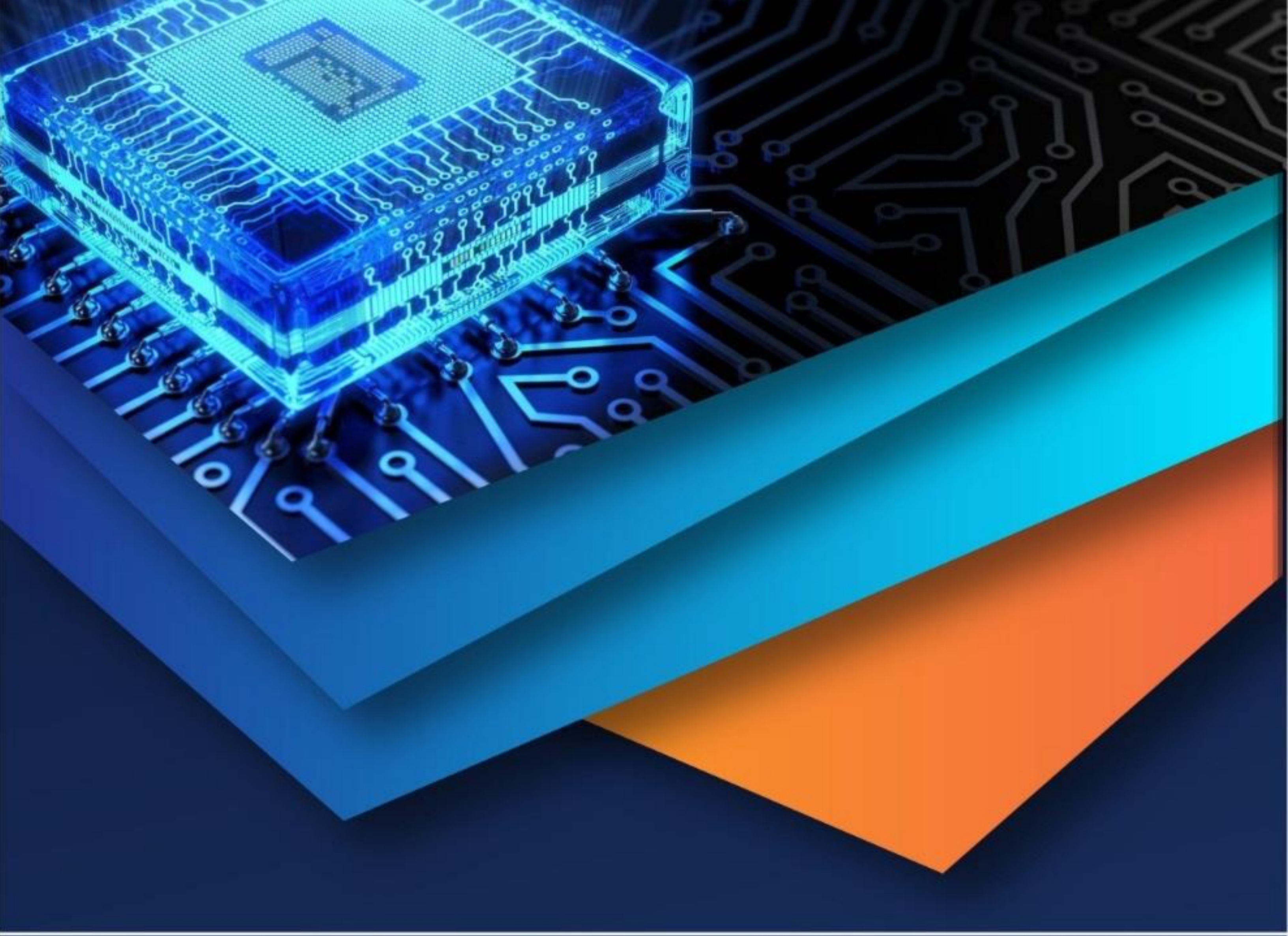

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