iJRASET
International Journal For Research in Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 12 **Issue:** III **Month of publication:** March 2024

DOI: https://doi.org/10.22214/ijraset.2024.58706

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 12 Issue III Mar 2024- Available at www.ijraset.com

Integral Solutions of the Bi-Quadratic Equation

$$2\left(x^2-y^2\right)\left(7x^2+y^2\right)=520\left(z^2-w^2\right)t^2+24s^2$$

Dr. R. Radha[1], Ms. K. Saranya[2]
[1]*Associate Professor,* [2]*PG Scholar, PG and Research Department Mathematics, Cauvery College for Women (Autonomous), Tiruchirapalli-620018, Tamilnadu,*

Abstract: ***The biquadratic with six unknowns of the form*** $2\left(x^2-y^2\right)\left(7x^2+y^2\right)=520\left(z^2-w^2\right)t^2+24s^2$ ***has been studied for its non-trivial distinct integral solutions. A few interesting relations among the solutions and polygonal numbers are presented.***

I. INTRODUCTION

Carl Friedrich Gauss, a well-known mathematician who lived from 1777 to 1855, is credited with saying that "number theory is the queen of mathematics and mathematics is the queen of all sciences." The study of integer and rational number features that go beyond standard manipulations in common arithmetic is known as number theory, or higher arithmetic. Number theorists were created in practically every major civilization throughout history. In the ancient and medieval eras, these individuals were typically geometers, but they may also have been philosophers, calendar calculators, astronomers, astrologers, priests,or magicians. Due to their variety, the quadratic offers an infinite topic for study. One may specifically refer to [4] through [20]. This problem of communication with the non-homogeneous quadratic equation denoted by,

$$2\left(x^2-y^2\right)\left(7x^2+y^2\right)=520\left(z^2-w^2\right)t^2+24s^2$$

is examined for its distinct non-zero integral solution. Some intriguing connections between the solutions are presented.

II. NOTATIONS

SPECIAL NUMBER	NOTATIONS	DEFINITIONS
Triangular number	$T_{3,n}$	$\frac{n(n+1)}{2}$
Hexadecagonal number	$T_{16,n}$	$n(7n-6)$
Nonadecagonal number	$T_{19,n}$	$\frac{n(17n-15)}{2}$
Icositetragonal number	$T_{24,n}$	$n(11n-10)$
Octahedral number	O_n	$\frac{1}{3}(2n^3+n)$
Centered square number	CS_n	$n^2+(n-1)^2$
Gnomonic number	Gno_n	$(2n-1)$
Stella octangular number	SO_n	$n(2n^2-1)$
Star number	$Star_n$	$6n(n-1)+1$

©IJRASET: All Rights are Reserved

Rhombic dodecagonal number	RD_n	$(2n-1)(2n^2-2n+1)$
Pentadecagonal number	$T_{15,n}$	$\frac{n(13n-11)}{2}$

III. METHOD OF ANALYSIS

The Quadratic equation under consideration is

$$2(x^2-y^2)(7x^2+y^2)=520\,(z^2-w^2)\,t^2+24s^2 \quad (1)$$

On substituting the linear transformation,

$$x=u+v;\ y=u-v;$$

$$z=2u+v;\ w=2u-v;\ s=2uv \quad (2)$$

in (1) , leads to

$$u^2+v^2=65t^2 \quad (3)$$

Equation (3) is solved through different methods and thus we obtain four different patterns of integral solutions to (1)

A. Pattern I

Assume

$$t=t(a,b)=a^2+b^2 \quad (4)$$

Where a and b are non-zero integers.

Write $65=(1+8i)(1-8i)$ (5) Applying the factorization method and substituting (4) and (5) in (3),

$$(u+iv)(u-iv)=(1+8i)(1-8i)(a+ib)^2\,(a-ib)^2 \quad (6)$$

Equating real and imaginary parts, the values of u and v are

$$u=a^2-b^2-16ab$$

$$v=8a^2-8b^2+2ab$$

Substituting the values of u and v in equation (2), the non zero distinct integral solutions of (1) are executed as,

$$\begin{aligned}
x &= x(a,b)=9a^2-9b^2-14ab\\
y &= y(a,b)=-7a^2+7b^2-18ab\\
z &= z(a,b)=10a^2-10b^2-30ab\\
w &= w(a,b)=-6a^2+6b^2-34ab \quad (7)\\
s &= s(a,b)=16a^4-96a^2b^2-252a^3b+252ab^3+16b^4\\
t &= t(a,b)=a^2+b^2
\end{aligned}$$

Thus (7) gives the integral solution of (1)

Observations

1. $2s(1,1)+2t(1,1)-x(1,1)$ is a perfect square.
2. $5x(1,-1)+2z(1,-1)+2w(1,-1)+t(2,1)$ is a happy couple number
3. $z(A^2,4)-480DF_A+10T_{24,A}+50Gno_A\equiv 0\,(\mathrm{mod}\,210)$
4. $8x(2A,A)-3z(A,A)+6t(2A,A)+w(A,A)-12T_{15,A}-33Gno_A\equiv 0(\mathrm{mod}\,33)$

5. $2s(A,1)-5y(2A+1,A)+t(A,A)-1536DF_A+756O_A-43T_{16,A}-622DGno_A\equiv 0(\mathrm{mod}\,689)$
6. $4t(B+1,B)+s(B,2)-768DF_B+756O_B+4T_{8,B}-1134Gno_B\equiv 0(\mathrm{mod}1394)$
7. $2w(3A,1)+3z(A+2,A)-162SO_4-5Star_A+66Gno_A\equiv 0(\mathrm{mod}\,443)$

B. Pattern II

Assume

$$t=t(a,b)=a^2+b^2 \quad (8)$$

Where a and b are non-zero integers.

Write $65=(8+i)(8-i)$ (9) Applying the factorization method and substituting (8) and (9) in (3),

$$(u+iv)(u-iv)=(8+i)(8-i)(a+ib)^2(a-ib)^2 \quad (10)$$

Equating real and imaginary parts, the values of u and v are

$$u=8a^2-8b^2-2ab$$

$$v=a^2-b^2+16ab$$

Substituting the values of u and v in equation (2), the non zero distinct integral solutions of (1) are executed as,

$$\begin{aligned}
x &= x(a,b)=9a^2-9b^2+14ab\\
y &= y(a,b)=7a^2-7b^2-18ab\\
z &= z(a,b)=17a^2-17b^2-12ab\\
w &= w(a,b)=15a^2-15b^2-20ab \quad (11)\\
s &= s(a,b)=16a^4-96a^2b^2+252a^3b-252ab^3+16b^4\\
t &= t(a,b)=a^2+b^2
\end{aligned}$$

Thus (11) gives the distinct integral solution of (1)

Observations

1. $4x(1,1)-4y(1,1)+z(1,1)+2t(1,1)$ is a perfect square.
2. $4x(1,1)-2y(1,1)+12z(1,1)-4w(1,1)-4s(1,1)-30t(1,1)$ is a cubical number.
3. $s(1,1)+z(1,1)+t(5,0)$ is a palindrom number.
4. $y(2A+1,2)-4T_{16,A}+10Gno_A\equiv 0(\mathrm{mod}\,67)$
5. $w(A+1,A)+z(2A-1,A)-15Star_A+20Gno_A\equiv 0(\mathrm{mod}\,323)$
6. $10w(2A,A)+t(2A^2,3)+z(2A,1)-192DF_A-68T_{19,A}-Gno_A\equiv 0(\mathrm{mod}\,248)$
7. $y(2A-1,A^2)+10t(A^2,A)+z(A,0)-144DF_A+54O_A-152T_{3,A}+43Gno_A\equiv 0(\mathrm{mod}\,36)$

C. Pattern III

Assume

$$t=t(a,b)=a^2+b^2 \quad (12)$$

Where a and b are non-zero integers.

Write $65=(4+7i)(4-7i)$ (13) Applying the factorization method and substituting (12) and (13) in (3),

$$(u+iv)(u-iv)=(4+7i)(4-7i)(a+ib)^2(a-ib)^2 \quad (14)$$

Equating real and imaginary parts, the values of u and v are

$$u = 4a^2 - 4b^2 - 14ab$$

$$v = 7a^2 - 7b^2 + 8ab$$

Substituting the values of u and v in equation (2), the non zero distinct integral solutions of (1) are executed as,

$$x = x(a,b) = 11a^2 - 11b^2 - 6ab$$

$$y = y(a,b) = -3a^2 + 3b^2 - 22ab$$

$$z = z(a,b) = 15a^2 - 15b^2 - 20ab$$

$$w = w(a,b) = a^2 - b^2 - 36ab \quad (15)$$

Thus, (15) gives

$$s = s(a,b) = 56a^4 - 336a^2b^2 - 132a^3b + 132ab^3 + 56b^4$$

$$t = t(a,b) = a^2 + b^2$$

the distinct integral solution of (1)

Observations

1. $\frac{1}{2}x(1,-1) - y(1,1) + z(-1,1) + w(1,-1)$ is a perfect square.
2. $z(1,-1) - 2s(1,1) + 4t(1,1)$ is a cubical number.
3. $x(1,1) + 10t(5,4)$ is a Duck number.
4. $z(2A+1, A^2) + 720DF_A + 60O_A - 5T_{12,A} - 50Gno_A \equiv 0 (\mathrm{mod}\ 65)$
5. $x(2A-1, A^2) + 2y(A+1,2) + 528DF_A + 3RD_A + 3T_{12,A} + 60Gno_A \equiv 0 (\mathrm{mod}\ 122)$
6. $x(2A-1, A) + 2y(A+1,2) + 528DF_A + 3RD_A - 3T_{12,A} + 60Gno_A \equiv 0 (\mathrm{mod} 122)$
7. $s(1, A) - 2688DF_A - 66SO_A + 40T_{16,A} + 153Gno_A \equiv 0 (\mathrm{mod}\ 97)$

D. Pattern 4

(3) can be written as

$$u^2 = 65t^2 - v^2 \quad (16)$$

Assuming $u = 65a^2 - b^2$ and

$$1 = \left(\frac{(\sqrt{65}+1)(\sqrt{65}-1)}{64}\right) \quad (17)$$

Substituting (16) in (17), using the method of factorization

$$\frac{(\sqrt{65}+1)(\sqrt{65}-1)(\sqrt{65}a-b)^2(\sqrt{65}a+b)^2}{64} = (\sqrt{65}t+v)(\sqrt{65}t-v)$$

Now define

$$(\sqrt{65}t+v) = \frac{(\sqrt{65}+1)(\sqrt{65}a+b)^2}{8}$$

$$(\sqrt{65}t-v) = \frac{(\sqrt{65}-1)(\sqrt{65}a-b)^2}{8} \quad (18)$$

Equating the like in (18) we get

$$t = \frac{1}{8}\left(65a^2 + b^2 + 2ab\right)$$

$$v = \frac{1}{8}\left(65a^2 + b^2 + 130ab\right)$$

Since our interest is on finding integer solution we have choose a and b Suitably so that t and v integers,

$$t = 65A^2 + B^2 + 2AB$$

$$v = 65A^2 + B^2 + 130AB$$

$$u = 520A^2 - 8B^2$$

Thus, using the values of u and v performing a few calculations the
Values of x, y and z are obtained as follows:

$$x = x(A,B) = 585A^2 - 7B^2 + 130AB$$

$$y = y(A,B) = 455A^2 - 9B^2 - 130AB$$

$$z = z(A,B) = 1105A^2 - 15B^2 + 130AB \qquad (19)$$

$$w = w(A,B) = 975A^2 - 17B^2 - 130AB$$

$$s = s(A,B) = 67600A^2 + 135200A^3B - 2080AB^3 - 16B^4$$

$$t = t(A,B) = 65A^2 + B^2 + 2AB$$

Thus (19) represents the non-trivial integral solution of (1)

Observations

1. $w(1,1) - y(1,1)$ is a cubical number.
2. $x(1,1) - t(1,1) - y(1,1)$ is a perfect square.
3. $s(1,1) + t(1,1)$ is a Duck number.
4. $x(A,1) + t(A,A) + y(1,A) - 116T_{13,A} - 391Gno_A \equiv 0 \pmod{1489}$
5. $w(A,2) + t(2A,1) - y(0,A) - 622CS_A - 183Gno_A \equiv 0 \pmod{506}$
6. $x(1,A) - t(2,2A) + T_{24,A} - 56Gno_A \equiv 0 \pmod{381}$
7. $y(2A,1) - 728T_{7,A} - 416Gno_A \equiv 0 \pmod{407}$

IV. CONCLUSION

In this study, an attempt has been made to complete the set of non-trivial distinct integral solutions for the non-homogeneous biquadratic equation. Finally, in order to find more solution to the biquadratic equation under consideration, one can look for further biquadratic equations that involve many variables.

REFERENCES

[1] Dickson, L. E. “History of Theory of Numbers and Diophantine Analysis”, Vol.2, Dover Publications, New York 2005.
[2] Mordell L.J., “Diophantine Equations” , Academic Press, New York, 1970.
[3] R D. Carmicheal, “The Theory of Numbers and Diophantine Analysis”, Dover Publications, New York 1959.
[4] R. Radha, G. Janaki, “On Cubic Equation $x^3 + y^3 = 456zw^2$,” International Journal of Management, IT and Engineering, Vol. 8 , Issue 8(1), 310-313.

©IJRASET: All Rights are Reserved | SJ Impact Factor 7.538 | ISRA Journal Impact Factor 7.894 |

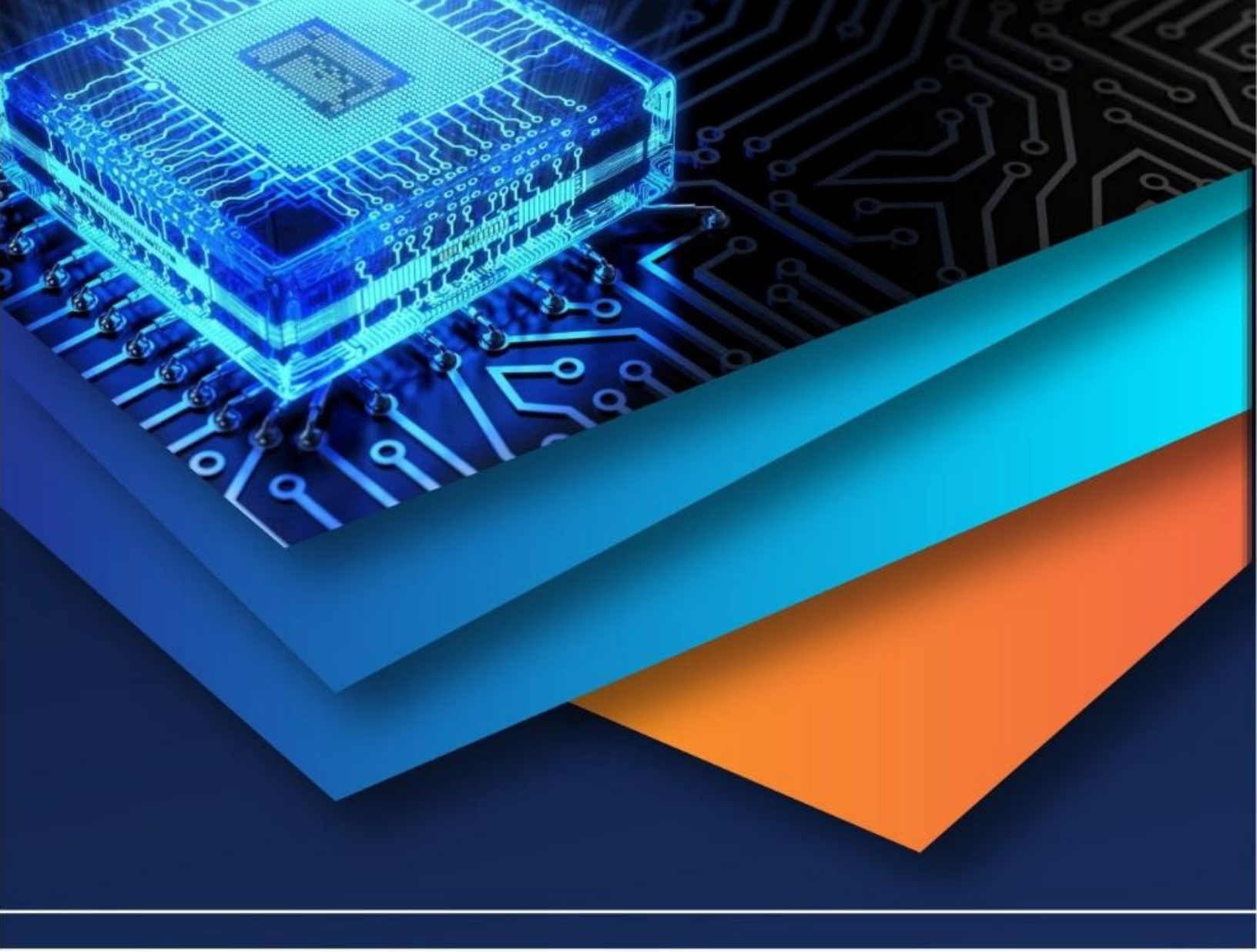
[5] R. Radha, G. Janaki, "Integral solutions of " $(x^2 - y^2)(7x^2 + 7y^2 - 13xy) = 2(z^2 - w^2)t^2$ ", Journal of Scientific Computing, Volume 8, Issue 12, December 2019, Pg.No:99 -104

[6] R. Radha, G. Janaki, "Integral solution of the ternary cubic equation $6(x^2 + y^2) - 11xy + x + y + 1 = 144z^3$", Kala: The Journal of Indian Art History Congress, Volume-27, No . 1(IX) : 2021,Page No:163-168.

[7] R. Radha, G. Janaki, "Integral solutions of $4w^2 - x^2 - y^2 + z^2 = t^2$", International Journal of Science and Research (IJSR), Volume 5 Issue 1 January 2016, Page No. 1718 -1720

[8] R. Radha, G. Janaki, "Observations on $x^2 - 4xy + y^2 + 22x = 0$",International journal of Engineering science and computing, Volume 6 Issue No 5 May 2016, Page No: 6256-6259.

[9] R. Radha, G. Janaki, "Observations on $X^2 + Y^2 = 2Z^2 - 62W^2$", International Research Journal of Engineering and Technology (IRJET) Volume: 04 Issue: 03 | Mar -2017, Page No: 1026-1030.



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 (24*7 Support on Whatsapp)