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Integral Solutions of the Ternary Cubic Equation $3(x^2+y^2)-4(xy)+2(x+y+1)=522z^3$

G. Janaki[1], J. Rahini[2]
[1]Associate Professor, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 18.
[2]PG Student, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 18.

Abstract: ***The non-homogenous cubic equation with three unknowns represented by the Diophantine equation*** $3(x^2+y^2)-4(xy)+2(x+y+1)=522z^3$ ***is analyzed for its patterns of stringfy integral solutions. A few interesting properties among the solutions and some special polygonal numbers are presented.***
Keywords: Cubic equation, integral solutions, polygonal number, square number, special number.

I. INTRODUCTION

Number theory is a vast and fascinating field of mathematics. Concerned with the properties of numbers in general and integers in particular as well as the wider classes of problems that arises from their study. The study of number theory is very important because of all other branches depends on this branch for their final results.Solving equations in integers is the central problem of Number theory. Number theory may be subdivided into several fields, according to the method use and the type of questions investigated. The term "arithmetic" is also used to refer to Number theory. The word Diophantine refers to the Hellenistic Mathematician of 3rd century, Diophantus of Alexandria who made of such equations and was one of the first Mathematician to introduce symbolism into algebra**.**In this communication, the non-homogeneous cubic equation with three unknowns represented by the equation $3(x^2+y^2)-4(xy)+2(x+y+1)=522z^3$ is considered and in particular a few interesting relations among the solutions are presented.

$T_{m,n}$ = Polygonal number

O_n = Octrahedral number

CS_n = Centered square number

CC_n = Centered cube number

Gno_n = Gnomonic number

SO_n = Stella Octangula number

CH_n = Centered Hexagonal number

TT_n = Truncated tetrahedral number

II. METHOD OF ANALYSIS

The ternary cubic equation to be considered for its quasi integral solution is

$$3(x^2+y^2)-4(xy)+2(x+y+1)=522z^3 \quad (1)$$

After using the transformations,

$$x=r+s,\ y=r-s \quad (2)$$

in (1) leads to $(r+1)^2+5s^2=261z^3$ (3)

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We demonstrate five different patterns of quasi distinct integer solutions of (1) as below:

A. Pattern: 1

Assume

$z = z(a,b) = a^2 + 5b^2$ (4)

Write $261 = (16 + i\sqrt{5})(16 - i\sqrt{5})$ (5)

Applying the factorization method and substituting (4) and (5) in (3),

$((r+1) + i\sqrt{5}s)((r+1) - i\sqrt{5}s) = (16 + i\sqrt{5})(16 - 6i\sqrt{5})((a + i\sqrt{5}b)^3(a - i\sqrt{5}b)^3)$ (6)

Equating real and imaginary parts,the values of r and s are

$$r = r(a,b) = 16a^3 - 240ab^2 - 15a^2b + 25b^3 - 1$$
$$s = s(a,b) = a^3 - 15ab^2 + 48a^2b - 80b^3$$

Substituting the values of u & v in equation (2), the non zero distinct integral solutions of (1) are executed as

$$x = x(a,b) = 17a^3 - 255ab^2 - 33a^2b - 55b^3 - 1$$
$$y = y(a,b) = 15a^3 - 225ab^2 - 63a^2b + 105b^3 - 1$$
$$z = z(a,b) = a^2 + 5b^2$$

Properties

1. $6z(a,a)$ is a Perfect square
2. $42z(1,1)$ is a palindrom number
3. $-x(a,1 + y(a,1) - z(a,1) + SO_a + 6TT_n + T_{25,a} + CH_a + T_{6,a} + 11Gno_a + 2T_{3,a} \equiv 0 (mod\ 177)$
4. $y(1,1) - x(1,1) - 28z(1,1)$ is a Nasty number
5. $22z(1,1) - y(1,1)$ is a happy couple number

B. Pattern:2

Instead of (5), Write $261 = (4 + i7\sqrt{5})(4 - i7\sqrt{5})$ (7)

Equating real and imaginary parts, we get

$$r = r(a,b) = 4a^3 - 60ab^2 - 105a^2b + 175b^3 - 1$$
$$s = s(a,b) = 7a^3 - 105ab^2 + 12a^2b - 20b^3$$

substituting (7) and (4) in (3) and the factorization method, as shown in pattern1, the corresponding integral solutions of (1) are represented by

$$x = x(a,b) = 11a^3 - 165ab^2 - 93a^2b + 155b^3 - 1$$
$$y = y(a,b) = -3a^3 + 45ab^2 - 117a^2b + 195b^3 - 1$$
$$z = z(a,b) = a^2 + 5b^2$$

Properties

1. $6y(1,1)$ is a Ruth-Aaron number
2. $y(a,1) - x(a,1) + 14az(a,1) + 12CC_a - 128Gno_a \equiv 0 (mod\ 180)$
3. $y(a,a) + z(a,a)$ is a cubical integer
4. $y(1,1) - x(1,1)$ is a palindrom number
5. $17z(1,1)$ is a duck number

C. Pattern: 3

Instead of(5), Write $261 = (9 + i6\sqrt{5})(9 - i6\sqrt{5})$ (8)

Equating real and imaginary parts, we get

$$r = r(a,b) = 9a^3 - 135ab^2 - 90a^2b + 150b^3 - 1$$
$$s = s(a,b) = 6a^3 - 90ab^2 + 27a^2b - 45b^3$$

substituting (8) and (4) are in (3) and the factorization method, as shown in pattern 1, the corresponding integral solutions of (1) are represented by

$$x = x(a,b) = 15a^3 - 255ab^2 - 63a^2b - 105b^3 - 1$$
$$y = y(a,b) = 3a^3 - 45ab^2 - 117a^2b + 195b^3 - 1$$

$$z = z(a,b) = a^2 + 5b^2$$

Properties

1. $24z(1,1)$ is a perfect square
2. $y(a,1) - x(a,1) + 54z(a,1) + 6SO_a - 87Gno_a \equiv 0(mod\ 447)$
3. $z(a,a) - x(a,a)$ is a palindrom number
4. $6y(1,1)$ is a duck number
5. $2y(1,1) - z(1,1)$ is a cubical integer

D. Pattern: 4

instead of (5), Write $261 = \frac{(96+i6\sqrt{5})(96-i6\sqrt{5})}{36}$ (9)

Equating real and imaginary parts, we get

$$r = u(a,b) = \frac{1}{6}(96a^3 - 1440ab^2 - 90a^2b + 150b^3) - 1$$
$$s = v(a,b) = \frac{1}{6}(6a^3 - 90ab^2 + 288a^2b - 480b^3)$$
$$z = z(a,b) = a^2 + 5b^2$$

Since finding integral solutions is the topic we are interested in, we have choose a and b in such a way that r, s, and z are integers.

Let us take $a = 6A$ and $b = 6B$, we have

$$r = u(A,B) = 3456A^3 - 3240AB^2 - 51840A^2B + 5400B^3 - 1$$
$$s = v(A,B) = 216A^3 - 3240AB^2 + 10368A^2B - 17280B^3$$
$$z = z(A,B) = 36A^2 + 180B^2$$

In terms of (2), the integral solutions of (1) are given by

$$x = x(A,B) = 3672A^3 - 6480AB^2 - 41472A^2B + 11880B^3 - 1$$
$$y = y(A,B) = 3240A^3 - 62208A^2B + 22680B^3 - 1$$
$$z = z(A,B) = 36A^2 + 180B^2$$

Properties

1. $6z(1,1)$ is a Perfect square
2. $z(1,1)$ is a cubical integer
3. $5z\ (1,1)$ is a duck number
4. $12z\ (1,1)$ is a palindrom number
5. $x(a,1) + y(a,1) - 192az(a,1) + 103680C_{2,a} + 72360Gno_a \equiv 0(mod\ 42118)$

E. Pattern: 5

instead of (5), Write $261 = \frac{(48+i3\sqrt{5})(48-i3\sqrt{5})}{9}$ (10)

Equating real and imaginary parts, we get

$$r = r(a,b) = \frac{1}{3}(48a^3 - 720ab^2 - 45a^2b + 75b^3) - 1$$
$$s = s(a,b) = \frac{1}{3}(3a^3 - 45ab^2 + 144a^2b - 240b^3)$$
$$z = z(a,b) = a^2 + 5b^2$$

Since finding integral solutions is the topic we are interested in, we have choose a and b in such a way that r, s, and z are integers.

Let us take $a = 3A$ and $b = 3B$, we have

$$u = u(A,B) = 432A^3 - 6480AB^2 - 405A^2B + 675B^3 - 1$$
$$v = v(A,B) = 27A^3 - 405AB^2 + 1296A^2B - 2160B^3$$
$$z = z(A,B) = 9A^2 + 45B^2$$

In terms of (2), the integral solutions of (1) are given by

$$x = x(A,B) = 459A^3 - 6885AB^2 - 891A^2B + 1485B^3 - 1$$
$$y = y(A,B) = 405A^3 - 6075AB^2 - 1701A^2B + 2835B^3 - 1$$
$$z = z(A,B) = 36A^2 + 180B^2$$

Properties

1. $\frac{z(1,1)}{6}$ $is\ a\ perfect\ square$

2. $y(a,1) - x(a,1) + 6az(a,1) + 654CH_a + 441Gno_a \equiv 0(mod\ 4533)$

3. $y(a,1) - x(a,1) + a6z(a,1) + 327$ star a $+ 441Gno_a \equiv 0(mod\ 4206)$

4. $z(1,1)+10$ is a cubical integer

5. $z(1,1)+9$ is a woodall number

III. CONCLUSION

In this paper, we have presented five different patterns of non-zero distinct integer solutions of the non-homogeneous cone given by

$$3(x^2 + y^2) - 4(xy) + 2(x + y + 1) = 522z^3$$

To conclude, one may search for other patterns of non-zero integer distinct solutions and their corresponding properties for other choices of cubic Diophantine equations.

REFERENCES

[1] Dickson L.E., "History of the theory of numbers", Chelsia Publishing Co., Sol II, New York, 1952

[2] Carmichael R.D., "The Theory of Numbers and Diophantine Analysis", Dover Publications, New York, 1959

[3] Mordell L .J., "Diophantine Equations", Academic Press, London 1969

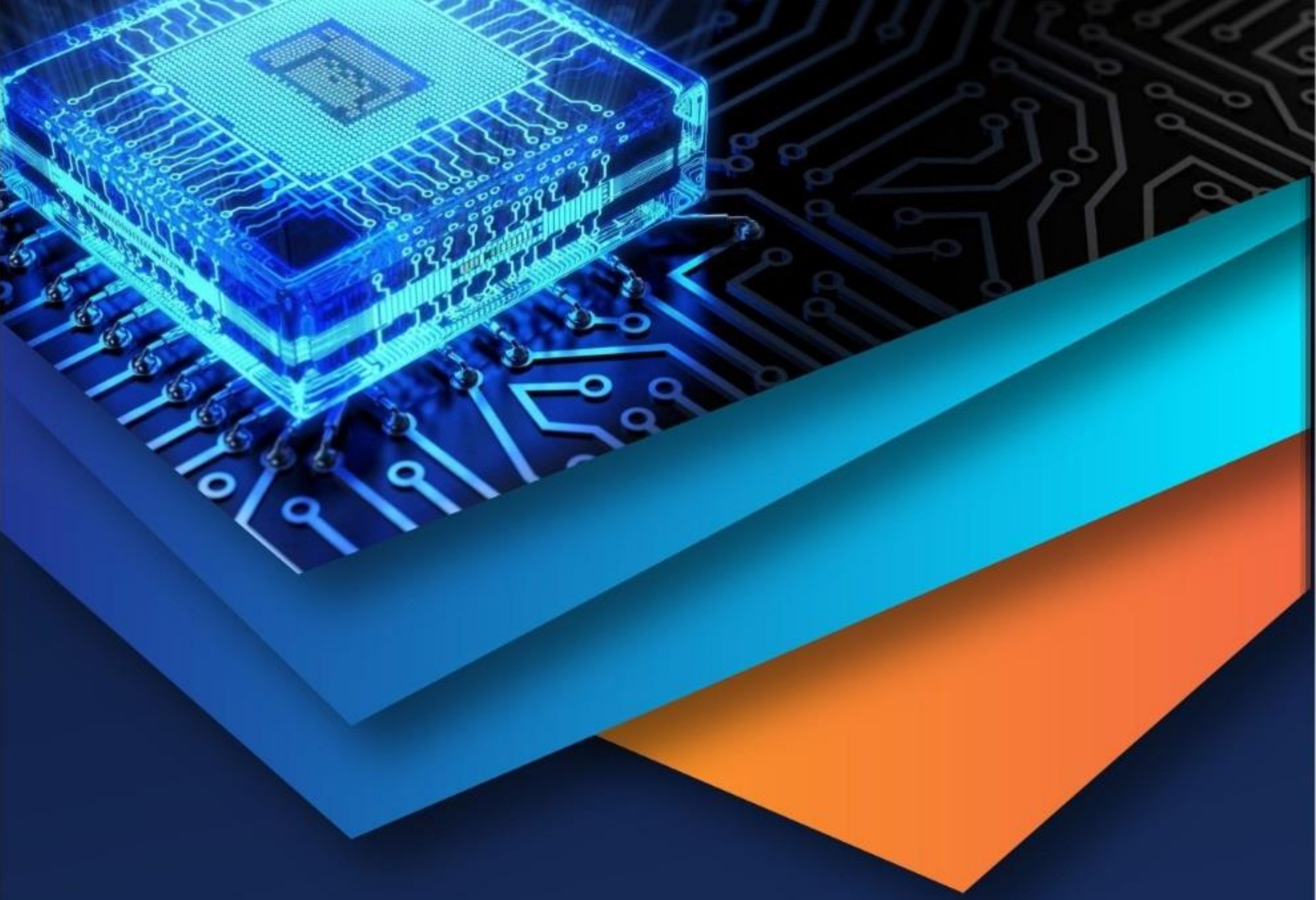
[4] Telang S. G., "Number Theory", Tata Mc Graw-Hill Publishing Company, New Delhi 1996

[5] Janaki G, Saranya C, "Observations on Ternary Quadratic Diophantine Equation $6(x^2 + y^2) - 11xy + 3x + 3y + 9 = 72z^2$", International Journal of Innovative Research and science, Engineering and Technology, volume 5, Issue 2, February 2016

[6] Janaki G, Saranya C, "Integral Solutions of the Ternary Cubic Equation $3(x^2 + y^2) - 4xy + 2(x + y + 1) = 972z^3$", International Research Journal of Engineering and Technology vol-4, Issue 3, Pg. No: 665-669, March 201

[7] Janaki G, Radha R, "On Ternary Quadratic Diophantine Equation $15x^2 + 15y^2 + 24xy = 438z^2$", International Journal for Research in Applied Science and Engineering Technology", vol 6, Issue I, Pg. No: 2656-2660, January 2018

[8] Janaki G, Gowri Shankari A, "Properties of the Ternary Cubic Equation $5x^2 - 3y^2 = z^3$", International Journal for Research in Applied Science and Engineering Technology, vol 10, Issue III,Pg.No: 231-234, August 2022



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