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Integral Solutions of the Ternion Quadratic Equation

$$a^2 + g^2 = 401s^2$$

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***Abstract:** In order to find its non zero unique integral solutions for the quadratic diophantine equation with three unknowns given by is analysed. The equation under consideration exhibits multiple patterns of solutions. The solutions are presented with a few fascinating aspects.*
***Keywords:** Quadratic equation with three unknowns, integral solutions, polygonal numbers.*

I. INTRODUCTION

The quadratic diophantine equation with three unknowns offers a numerous researching opportunities because of their range[1-3]. For quadratic equations containing three unknowns, one should specially refer [4-19]. This communication deals with yet another fascinating ternary quadratic equation $a^2 + g^2 = 401s^3$ with three unknown factors that can be used to determine any one of an infinite numbers of non-zero integral solutions.

A. Notations

$T_{4,n} = n^2$ (Tetragonal number)

$T_{10,n} = n(4n-3)$ (Decagonal number)

$T_{12,n} = n(5n-4)$ (Dodecagonal number)

$T_{16,n} = n(7n-6)$ (Hexadecagonal number)

$T_{20,n} = n(9n-8)$ (Icosagonal number)

$T_{22,n} = n(10n-9)$ (Icosidigonal number)

$T_{24,n} = n(11n-10)$ (Icositetragonal number)

$T_{28,n} = n(13n-12)$ (Icosioctagonal number)

$CS_n = n^2 + (n-1)^2$ (Centered square number)

$Gno_n = 2n-1$ (Gnomonic number)

$CH_n = 3n^2 - 3n + 1$ (Centered hexagonal number)

II. METHOD OF ANALYSIS

The ternion quadratic equation to be solved for its non-zero integral solution is

$$a^2 + g^2 = 401s^2 \tag{1}$$

various patterns of solutions of (1) are listed below

A. Pattern 1:

Assume,

$$s = s(l,m) = l^2 + m^2 \quad (2)$$

Where l and m are non-zero integers and write

$$401 = (20+i)(20-i) \quad (3)$$

Using (2) and (3) in (1), we get

$$a^2 + g^2 = (20+i)(20-i)(l^2+m^2)^2$$

Using factorization method, we have

$$(a+ig)(a-ig) = (20+i)(20-i)(l+im)^2(l-im)^2$$

If we analyse the positive and negative aspects, we get

$$a+ig = (20+i)(l+im)^2 \quad (4)$$

$$a-ig = (20-i)(l-im)^2 \quad (5)$$

Equating the actual and fictitious elements either in (4) or (5), we get

$$a(l,m) = 20l^2 - 2lm - 20m^2$$

$$g(l,m) = l^2 + 40lm - m^2$$

$$s(l,m) = l^2 + m^2$$

PROPERTIES

1) $g(1,1) + 2a(1,1)$ is a perfect square
2) $g(1,1) + 2a(1,1) - 3s(1,1)$ is a nasty number
3) $60s(l,1) - 40a(l,1) - 4T_{12,l} - 7Gno_l \equiv 0 \pmod{27}$
4) $a(1,l) + g(1,l) + 31s(1,l) - 2T_{12,l} - 25Gno_l \equiv 0 \pmod{2}$
5) $g(l,1) - a(l,1) - 19s(l,1) - 21Gno_l \equiv 0 \pmod{42}$
6) $a(l,l) + 30s(l,l) - 20T_{4,l} = 0$
7) $s(m,m) - 2T_{4,m} = 0$
8) $20a(1,1) + g(1,1) + 7s(1,1)$ is a perfect number
9) $a(1,1) + 10g(1,1) + 60s(1,1)$ is a Disarium number
10) $a(1,1) + g(1,1) + 3s(1,1)$ is a palindrome number

B. *Pattern 2*

We write 401 in the form as

$$401 = (1+20i)(1-20i) \quad (6)$$

Where l and m are non-zero integers and write

Using (6) and (2) in (1), we get

$$a^2 + g^2 = (1+20i)(1-20i)(l^2+m^2)^2$$

Using factorization method, we have

$$(a+ig)(a-ig) = (1+20i)(1-20i)(l+im)^2(l-im)^2$$

If we analyse the positive and negative aspects, we get

$$a+ig = (1+20i)(l+im)^2 \quad (7)$$

$$a-ig = (1-20i)(l-im)^2 \quad (8)$$

Equating the actual and fictitious elements either in (7) or (8), we get

$$a(l,m) = l^2 - 40lm - m^2$$

$$g(l,m)=20l^2+2lm-20m^2$$
$$s(l,m)=l^2+m^2$$

PROPERTIES

1) $a(1,1)+38s(1,1)$ is a perfect square
2) $a(1,1)+g(1,1)+20s(1,1)$ is a perrin number
3) $a(l,1)-g(l,1)+39s(l,1)-10CS_l+11Gno_l \equiv 0 \pmod{37}$
4) $3g(1,l)-a(1,l)+59s(1,l)-23Gno_l \equiv 0 \pmod{141}$
5) $s(m,1)-a(m,1)-20Gno_m \equiv 0 \pmod{22}$
6) $a(l,l)+10g(l,l)+10s(l,l)=0$
7) $a(l,l)+30s(l,l)-20T_{4,l}=0$
8) $g(1,1)+s(1,1)$ is a deficient number
9) $a(1,1)+31s(1,1)$ is a palindrome number
10) $g(1,1)+2s(1,1)$ is a nasty number

C. *Pattern 3*

Observe that (1) is written as

$$a^2+g^2=400s^2+s^2 \tag{9}$$

$$\frac{a-20s}{s+g}=\frac{s-g}{a+20s}=\frac{\alpha}{\beta}\ ,\quad \beta\neq 0$$

which is equivalent to the system of double equation

$$a\beta-g\alpha-(20\beta+\alpha)s=0$$
$$-a\alpha-g\beta+(\beta-20\alpha)s=0 \tag{10}$$

By applying the cross multiplication method to solve (10), the relevant non-zero unique integral solution to (1) are obtained as

$$a(\alpha,\beta)=20\alpha^2-2\alpha\beta-20\beta^2$$
$$g(\alpha,\beta)=\alpha^2+40\alpha\beta-\beta^2$$
$$s(\alpha,\beta)=-\alpha^2-\beta^2$$

PROPERTIES

1) $2a(1,1)+g(1,1)+2s(1,1)$ is a happy number
2) $g(1,1)+7s(1,1)$ is a silverback number
3) $25g(1,\alpha)-a(1,\alpha)-5s(1,\alpha)-501Gno_\alpha \equiv 0 \pmod{511}$
4) $2g(\alpha,1)x(\alpha,1)-30s(\alpha,1)-2Star_\alpha-47Gno_\alpha \equiv 0 \pmod{140}$
5) $2g(1,\beta)-a(1,\beta)-42s(1,\beta)-20CH_\beta 71Gno_\beta \equiv 0 \pmod{146}$
6) $g(\alpha,\alpha)+2s(\alpha,\alpha)-36T_{4,\alpha}=0$
7) $2a(\beta,\beta)-4s(\beta,\beta)-6T_{4,\beta}=0$
8) $20a(1,1)-31s(1,1)$ is a perrin number
9) $g(1,1)-a(1,1)-s(1,1)$ is a palindrome number
10) $10g(1,1)-50a(1,1)-25s(1,1)$ is a perfect digital invariant number

D. Pattern 4

Equation (1) is written as

$$a^2 + g^2 = 400s^2 * 1 \tag{11}$$

Write 1 as

$$1 = \frac{(20+21i)(20-21i)}{29^2} \tag{12}$$

and write 400 as

$$400 = (20+i)(20-i) \tag{13}$$

Using (2), (12) and (13) in (11), we get

$$a^2 + g^2 = (20+i)(20-i)(l^2+m^2)\frac{(20+21i)(20-21i)}{29^2}$$

Employing the method of factorization the above equation is written as

$$(a+ig)(a-ig) = (20+i)(20-i)(l+im)^2(l-im)^2\frac{(20+21i)(20-21i)}{29^2}$$

If we analyse the positive and negative aspects, we get

$$a+ig = \frac{1}{29}(20+i)(20+21i)(l+im)^2 \tag{14}$$

$$a-ig = \frac{1}{29}(20-i)(20-21i)(l-im)^2 \tag{15}$$

Equating the actual and fictitious elements either in (14) or (15), we get

$$a(l,m) = \frac{1}{29}(379l^2 - 880lm - 379m^2)$$

$$g(l,m) = \frac{1}{29}(440l^2 + 758lm - 440m^2)$$

Since our interest is on finding integer solution

Let us take $l = 29L$ and $m = 29M$

$$a(L,M) = 379L^2 - 880LM - 379M^2$$

$$g(L,M) = 440L^2 + 758LM - 440M^2$$

$$s(L,M) = 29L^2 + 29M^2$$

PROPERTIES

1) $a(1,1) + 16s(1,1)$ is a abudant number
2) $-14s(1,1) - a(1,1)$ is a perrin number
3) $2g(L,1) - a(L,1) - 19s(L,1) - 526CS_L - 1724Gno_L \equiv 0 \pmod{1248}$
4) $g(1,L) - a(1,L) + 3s(1,L) - 2T_{28,L} - 831Gno_L \equiv 0 \pmod{955}$
5) $36s(L,1) - a(L,1) - 133T_{12,L} - 706Gno_L \equiv 0 \pmod{2129}$
6) $g(M,1) - a(M,1) + s(M,1) - 10T_{20,M} - 859Gno_M \equiv 0 \pmod{827}$
7) $g(1,M) - a(1,M) + 2s(M,1) - 832Gno_M \equiv 0 \pmod{967}$
8) $g(L,L) - 13s(L,L) - 4T_{4,L} = 0$
9) $a(M,M) + g(M,M) + 3s(M,M) - 52T_{4,M} = 0$
10) $g(1,1) - 13s(1,1)$ is a perfect square

E. Pattern 5

Equation (1) is written as

$$401s^2 - g^2 = a^2 * 1 \tag{16}$$

Assume,

$$a(l,m) = 401l^2 - m^2 \tag{17}$$

Write 1 as

$$1 = (\sqrt{401} + 20)(\sqrt{401} - 20) \tag{18}$$

Using (17) and (18) in (16) and applying the method of factorization, we get

$$(\sqrt{401}s + g) = (\sqrt{401}l + m)^2 (\sqrt{401} - 20)$$

Equating then rational and irrational factors, we get

$$a(l,m) = 401l^2 - m^2$$

$$g(l,m) = 8020l^2 - 802lm + 20m^2$$

$$s(l,m) = 401l^2 - 40lm + m^2$$

PROPERTIES

1) $2g(1,l) + 7a(1,l) - 25s(1,l) - 2T_{10,l} + 299Gno_l \equiv 0(\mathrm{mod}\,8523)$

2) $g(1,l) - 2a(1,l) - 2T_{24,l} + 391Gnol \equiv 0(\mathrm{mod}\,6827)$

3) $20a(m,1) - g(m,1) - 401Gno_m \equiv 0(\mathrm{mod}\,380)$

4) $a(1,m) + g(1,m) + s(1,m) - 2T_{22,m} + 412Gno_m \equiv 0(\mathrm{mod}\,8410)$

5) $g(1,m) + s(1,m) - 3T_{16,m} + 412Gno_m \equiv 0(\mathrm{mod}\,8009)$

6) $10a(l,1) - 10s(l,1) - 200Gno_l \equiv 0(\mathrm{mod}\,180)$

7) $a(l,l) - s(l,l) - 38T_{4,l} = 0$

8) $a(m,m) - g(m,m) + 10s(m,m) - 382T_{4,m} = 0$

9) $g(m,1) - a(m,1) - 19s(m,1) + 21Gno_m = 0$

10) $a(1,1)$ represents a perfect square

III. CONCLUSION

For the ternion quadratic equation $a^2 + g^2 = 401s^2$ we have given numerous non-zero unique integral solutions patterns. To conclude, one can look for further options for solutions and their respective attributes among the various choices.

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