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Integration of the Loaded Cordeveg-De Fries Equation in a Class of Fast Decreasing Functions

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Annotation: This article is devoted to the integration of the loaded source Korteweg-de Vries equation in the class of rapidly decreasing functions. In this work, the Cauchy problem imposed on the Korteweg-de Vries equation was solved using the inverse problem method of the Sturm-Liouville operator scattering theory. Their Yost solutions are defined and integral Levin images are obtained for them. The givens of the scattering theory were described and some of their necessary properties were given, the Gelfand-Levitan-Marchenko integral equation, which is the main integral equation of the inverse problems of the scattering theory, was derived.
Keywords: Operator, differential, continuous, loaded, kernel, asymptotics, potential, scattering theory, inverse problem, Vronci determinant, analytical function, return coefficient, normalizing constants, integral equation, eigenvalues.

I. INTRODUCTION

In 1967, K. Gardner, J. Green, M. Kruskal, R. Miura, one of the main equations of modern mathematical physics

$$u_t - 6uu_x + u_{xxx} = 0, \qquad u(x,t)\big|_{t=0} = u_0(x), \quad x \in R$$

They managed to find the solution of the Cauchy problem to the Korteweg-de Vries equation using the inverse problem of the scattering theory for the Sturm-Liouville operator. This inverse problem was first introduced by L.D. studied by Faddeev. P. Laks showed that the method of inverse problems has a universal character and introduced the concept of a high-order KdF equation. In modern scientific literature, there is a growing interest in nonlinear evolutionary equations with adapted sources. Such evolutionary equations occupy an important place in plasma physics, hydrodynamics, and solid state physics. In particular, the Korteweg-de Vries equation with an integral source in the class of rapidly decreasing functions was studied by Leon, Latifilar. Such equations are used to study the interaction of long and short capillary-gravitational waves. In modern literature, if the value of the solution or its derivative at a point participates in the coefficients of the equation or on the right side of the equation, such equations are called loaded equations. The study of such equations is considered important both from the point of view of building the general theory of differential equations and from the point of view of their application.

II. MAIN PART

Consider the Sturm-Liouville equation

$$L(0)y := -y'' + u_0(x)y = k^2 y, \quad x \in R \tag{1}$$

where potential $u_0(x)$ satisfies condition (3). In this subsection, information necessary for further presentation concerning the direct and inverse scattering problems for equation (1) will be given. Denote by $f(x,k)$ and $g(x,k)$ the Jost solutions of equation (1) with asymptotic

$$\begin{aligned} f(x,k) &= e^{ikx} + \overset{=}{o}(1), \ x \to \infty, \ (\operatorname{Im} k = 0); \\ g(x,k) &= e^{-ikx} + \overset{=}{o}(1), \ x \to -\infty, \ (\operatorname{Im} k = 0). \end{aligned} \tag{2}$$

Under condition (3), such solutions exist and are uniquely determined by asymptotic (2).

Solutions $f(x,k)$, $g(x,k)$ satisfy representations

$$f(x,k)=e^{ikx}+\int_x^{\infty}K^+(x,z)e^{ikz}dz$$
$$g(x,k)=e^{-ikx}+\int_{-\infty}^{x}K^-(x,z)e^{-ikz}dz, \tag{8}$$

kernels $K^+(x,z),\ K^-(x,z)$ of which are real functions associated with the potential $u_0(x)$ relations:

$$u_0(x)=\mp 2\frac{d}{dx}K^{\pm}(x,x). \tag{9}$$

For real k pairs of functions $\{f(x,k),\ f(x,-k)\}$ and $\{g(x,k), g(x,-k)\}$ are pairs of linearly independent solutions of equation (1), therefore

$$f(x,k)=b(k)g(x,k)+a(k)g(x,-k),$$
$$g(x,k)=-b(-k)f(x,k)+a(k)f(x,-k). \tag{10}$$

Functions $r^+(k)=-\dfrac{b(-k)}{a(k)},\ r^-(k)=\dfrac{b(k)}{a(k)}$ are called reflection coefficients (right and left, respectively). The coefficients $a(k), b(k)$ and $r^+(k)$ have the following properties (see [4] p. 121):

1) With real $k\neq 0$

$$a(-k)=\overline{a(k)},\ \ b(-k)=\overline{b(k)},\ \ |a(k)|^2=1+|b(k)|^2; \tag{11}$$

$$a(k)=\frac{1}{2ik}W\{g(x,k),\ f(x,k)\},\ \ b(k)=\frac{1}{2ik}W\{f(x,k), g(x,-k)\},$$
$$a(k)=1+\underline{\underline{O}}\left(\frac{1}{k}\right),\ \ b(k)=\underline{\underline{O}}\left(\frac{1}{k}\right),\ \ k\to\pm\infty; \tag{12}$$

where

$$W\{f(x,k), g(x,k)\}\equiv f(x,k)g'(x,k)-f'(x,k)g(x,k);$$

2) The function $a(k)$ continues analytically into the half-plane $\operatorname{Im}k>0$ and there have a finite number of zero's $k_n=i\chi_n,\ (\chi_n>0),\ n=1,2,3,\ldots,N$, these zero's are simple, and $\lambda_n=-\chi_n^2$ is an eigenvalue of the operator $L(0)$. In addition, there is a relation

$$g(x,i\chi_j)=B_j f(x,i\chi_j),\ \ j=1,2,\ldots,N; \tag{13}$$

3) For real $k\neq 0$ function $r^+(k)$ is continuous

$$\overline{r^+(k)}=r^+(-k),\ \ |r^+(k)|<1,\ \ r^+(k)=o\left(\frac{1}{k}\right),\ \ |k|\to\infty,$$

$$k^2\left[1-|r^+(k)|^2\right]^{-1}=O(1),\ \ |k|\to 0;$$

4) Function $k(a(k)-1)$, where $a(k)$ is defined by the formula

$$a(k)=\prod_{j=1}^{N}\frac{k-i\chi_j}{k+i\chi_j}\exp\left\{-\frac{1}{2i\pi}\int_{-\infty}^{\infty}\frac{\ln\left(1-\left|r^{+}(\xi)\right|^{2}\right)}{\xi-k}d\xi\right\},\ \operatorname{Im}k>0,$$

is continuous and bounded at $\operatorname{Im}k\geq 0$ and

$$\left(a(k)\right)^{-1}=O(1),\ \left|k\right|\to 0,\ \ \operatorname{Im}k\geq 0,$$

$$\lim_{k\to 0}ka(k)\left(r^{+}(k)+1\right)=0,\ \operatorname{Im}k=0;$$

5) Functions

$$R^{\pm}(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}r^{\pm}(k)e^{\pm ikx}dk$$

for each $a>-\infty$ satisfy the condition

$$\left(1+\left|x\right|\right)\left|R^{\pm}(\pm x)\right|\in L^{1}\left(a,\infty\right).$$

The set $\left\{r^{+}(k),\chi_1,\chi_2,\ldots\chi_N,B_1,B_2,\ldots,B_N\right\}$ is called the scatter data for the $L(0)$ operator. The direct scattering problem is to determine the scattering data from the potential $u_0(x)$, and the inverse problem is to restore the potential scattering data $u_0(x)$ of equation (1).

The kernel $K^{+}(x,y)$ in representation (8) is the solution of the Gelfand-Levitan- Marchenco integral equation

$$K^{+}(x,y)+F^{+}(x+y)+\int_{x}^{\infty}K^{+}(x,z)F^{+}(z+y)dz=0,\ \ (y>x) \tag{14}$$

where

$$F^{+}(x)=\sum_{j=1}^{N}\alpha_j^{+}e^{-\chi_j x}+\frac{1}{2\pi}\int_{-\infty}^{\infty}r^{+}(k)e^{ikx}dk, \tag{15}$$

$$\alpha_j^{+}=\frac{B_j}{i\left.\frac{da(z)}{dz}\right|_{z=i\chi_j}},$$

and $a(z)$ is the analytic continuation of the function $a(k)$ to the upper half-plane $\operatorname{Im}k>0$.

Lemma 1. Let the functions $y(x,\lambda)$ and $z(x,\mu)$ respectively be solutions of

$$Ly(x,\lambda)=\lambda y(x,\lambda)\ ,\ Lz(x,\mu)=\mu z(x,\lambda).$$

In that case

$$\frac{d}{dx}W\left\{y(x,\lambda),z(x,\mu)\right\}=\left(\lambda-\mu\right)y(x,\lambda)z(x,\mu)$$

equality will be fair.

The proof of this lemma starts from simple calculations.
The following theorem holds true (see [4], p. 231)

Theorem 1. Specifying the scattering data uniquely determines the potential $u_0(x)$.

Consider the following equation

$$u_t + u(x_0,t)\left(u_{xxx} - 6uu_x\right) = G(x,t), \tag{16}$$

where

$$G(x,t) = -\gamma(t)u(x_1,t)u_x - 4\sum_{m=1}^{N}\xi_m \frac{\partial}{\partial x}\left(\left|\varphi_m\right|^2\right). \tag{17}$$

For equation (16), we will look for the Lax pair [5] in the form

$$-\phi_{xx} + \left(u(x,t) - k^2\right)\phi = 0, \tag{18}$$

$$\phi_t = u(x_0,t)\left(-u_x + 4ik^3\right)\phi + u(x_0,t)\left(2u + 4k^2\right)\phi_x + F(x,t). \tag{19}$$

Using the identity $\phi_{xxt} = \phi_{txx}$, and taking into account equalities (16)-(19), we obtain

$$-F_{xx} + \left(u(x,t) - k^2\right)F = -G(x,t)\phi. \tag{20}$$

Putting $\phi(x,t) = g(x,k,t)$, we are looking for a solution to equation (20) in the form

$$F(x,t) = B(x)g(x,k,t) + C(x)g(x,-k,t).$$

Then to determine $B(x)$ and $C(x)$ we obtain the following system of equations

$$B'(x)g(x,k,t) + C'(x)g(x,-k,t) = 0,$$

$$B'(x)g'(x,k,t) + C'(x)g'(x,-k,t) = G(x,t)g(x,k,t)$$

Solution

$$B(x) = -\frac{1}{2ik}\int_{-\infty}^{x} g(s,k,t)g(s,-k,t)G(s,t)ds$$

$$C(x) = \frac{1}{2ik}\int_{-\infty}^{x} g^2(s,k,t)G(s,t)ds.$$

Therefore, using expression (17), equation (19) can be rewritten as follows

$$\frac{\partial g(x,k,t)}{\partial t} = u(x_0,t)\left(-u_x + 4ik^3\right)g(x,k,t) + u(x_0,t)\left(2u + k^2\right)\frac{\partial g(x,k,t)}{\partial x} +$$

$$+\frac{\gamma(t)u(x_1,t)g(x,k,t)}{2ik}\int_{-\infty}^{x} g(s,k,t)\overline{g(s,k,t)}u_s(s,t)ds -$$

$$-\frac{\gamma(t)u(x_1,t)\overline{g(x,k,t)}}{2ik}\int_{-\infty}^{x} g^2(s,k,t)u_s(s,t)ds +$$

$$+\frac{4g(x,k,t)}{2ik}\int_{-\infty}^{x} g(s,k,t)\overline{g(s,k,t)}\sum_{m=1}^{N}\xi_m \frac{\partial}{\partial x}\left(\left|\varphi_m(s,t)\right|^2\right)ds -$$

$$-\frac{4\overline{g(x,k,t)}}{2ik}\int_{-\infty}^{x} g^2(s,k,t)\sum_{m=1}^{N}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(s,t)\right|^2\right)ds. \tag{21}$$

Passing in equality (21) to the limit $x\to\infty$, due to (11), (12) and the asymptotic of the Jost solution, we derive

$$\frac{da(k,t)}{dt}=\frac{\gamma(t)u(x_1,t)a(k,t)}{2ik}\int_{-\infty}^{\infty} g(s,k,t)\overline{g(s,k,t)}u_s(s,t)ds+$$

$$+\frac{\gamma(t)u(x_1,t)b(k,t)}{2ik}\int_{-\infty}^{\infty} g^2(s,k,t)u_s(s,t)ds-$$

$$-\frac{4a(k,t)}{2ik}\int_{-\infty}^{\infty} g(s,k,t)\overline{g(s,k,t)}\sum_{m=1}^{N}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(s,t)\right|^2\right)ds-$$

$$-\frac{4b(k,t)}{2ik}\int_{-\infty}^{\infty} g^2(s,k,t)\sum_{m=1}^{N}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(s,t)\right|^2\right)ds, \tag{22}$$

$$\frac{db(-k,t)}{dt}=8ik^3\beta(t)u(x_0,t)b(-k,t)+$$

$$+\frac{\gamma(t)u(x_1,t)b(-k,t)}{2ik}\int_{-\infty}^{\infty} g(s,k,t)\overline{g(s,k,t)}u_s(s,t)ds+$$

$$+\frac{\gamma(t)u(x_1,t)a(-k,t)}{2ik}\int_{-\infty}^{\infty} g^2(s,k,t)u_s(s,t)ds-$$

$$-\frac{4b(-k,t)}{2ik}\int_{-\infty}^{\infty} g(s,k,t)\overline{g(s,k,t)}\sum_{m=1}^{N}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(s,t)\right|^2\right)ds-$$

$$-\frac{4a(-k,t)}{2ik}\int_{-\infty}^{\infty} g^2(s,k,t)\sum_{m=1}^{N}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(s,t)\right|^2\right)ds. \tag{23}$$

Now consider the following lemma:

Lemma 2. The following equalities hold

$$\int_{-\infty}^{\infty}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(x,t)\right|^2\right)g^2(x,k,t)dx=0 \tag{24}$$

$$\int_{-\infty}^{\infty}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(x,t)\right|^2\right)g(x,k,t)\overline{g(x,k,t)}dx=0 \tag{25}$$

Proof: We will prove equation (24), for this we first divide equation (24) into two parts, that is, as follows

$$\int_{-\infty}^{\infty}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(x,t)\right|^2\right)g^2(x,k,t)dx=\frac{\xi_m}{2}\int_{-\infty}^{\infty}\frac{\partial}{\partial x}\left(\left|\varphi_m(x,t)\right|^2\right)g^2(x,k,t)dx+$$

$$+\frac{\xi_m}{2}\int_{-\infty}^{\infty}\frac{\partial}{\partial x}\left(\left|\varphi_m(x,t)\right|^2\right)g^2(x,k,t)dx=\frac{\xi_m}{2}g^2(x,k,t)\left|\varphi_m(x,t)\right|^2\Big|_{-\infty}^{\infty}-$$

$$-\frac{\xi_m}{2}\int_{-\infty}^{\infty}\left|\varphi_m(x,t)\right|^2 2g(x,k,t)g'(x,k,t)dx+\frac{\xi_m}{2}\int_{-\infty}^{\infty}\frac{\partial}{\partial x}\left|\varphi_m(x,t)\right|^2 g^2(x,k,t)dx=$$

$$=\frac{\xi_m}{2}\int_{-\infty}^{\infty}\left(\frac{\partial}{\partial x}\left(\left|\varphi_m(x,t)\right|^2\right)g^2(x,k,t)-\left|\varphi_m(x,t)\right|^2 2g(x,k,t)\overline{g(x,k,t)}\right)dx=$$

$$=\frac{\xi_m}{2}\int_{-\infty}^{\infty}\left[\left(\varphi_m'\overline{\varphi}_m+\varphi_m\overline{\varphi}'_m\right)g^2-2gg'\varphi_m\overline{\varphi}_m\right]dx=$$

$$=\frac{\xi_m}{2}\int_{-\infty}^{\infty}\left(g\overline{\varphi}_m W\left\{g,\varphi_m\right\}+\varphi_m g W\left\{g,\overline{\varphi}_m\right\}\right)dx.$$

Where we generate the following expression using Lemma 1

$$\frac{\xi_m}{2}\int_{-\infty}^{\infty}\left(g\overline{\varphi}_m W\left\{g,\varphi_m\right\}+\varphi_m g W\left\{g,\overline{\varphi}_m\right\}\right)dx=$$

$$=\frac{\xi_m}{2}\int_{-\infty}^{\infty}\left(\frac{1}{\lambda-\lambda_m}\left(\frac{d}{dx}W\left\{g,\overline{\varphi}_m\right\}\right)\cdot W\left\{g,\varphi_m\right\}+\frac{1}{\lambda-\lambda_m}\left(\frac{d}{dx}W\left\{g,\varphi_m\right\}\right)W\left\{g,\overline{\varphi}_m\right\}\right)dx=$$

$$=\frac{\xi_m}{2(\lambda-\lambda_m)}\int_{-\infty}^{\infty}\frac{d}{dx}\left\{W\left\{g,\overline{\varphi}_m\right\}W\left\{g,\varphi_m\right\}\right\}dx=\frac{\xi_m}{2(\lambda-\lambda_m)}W\left\{g,\overline{\varphi}_m\right\}W\left\{g,\varphi_m\right\}\Big|_{x=-\infty}^{x=+\infty}=0;$$

Equation (25) is proved similarly.

Multiplying (23) by $a(k,t)$ and subtracting from it equality (22) multiplied by $b(-k,t)$, according to (15), we obtain

$$\frac{dr^+(k,t)}{dt}=8ik^3u(x_0,t)r^+(k,t)-\frac{\gamma(t)u(x_1,t)}{2ika^2(k,t)}\int_{-\infty}^{\infty}g^2(s,k,t)u_s(s,t)ds.$$

Let's calculate the integral on the right side of the previous equality. To do this, we use formula (11), and we have

$$\int_{-\infty}^{\infty}g^2(s,k,t)u_s(s,t)ds=g^2(s,k,t)u(s,t)\Big|_{s=-\infty}^{s=+\infty}-$$

$$-2\int_{-\infty}^{\infty}\left(g''(s,k,t)+k^2g(s,k,t)\right)g'(s,k,t)ds=$$

$$=-2\int_{-\infty}^{\infty}g'(s,k,t)g''(s,k,t)ds-2k^2\int_{-\infty}^{\infty}g(s,k,t)g'(s,k,t)ds=$$

$$=-\int_{-\infty}^{\infty}\left(g'^2(s,k,t)\right)'ds-k^2\int_{-\infty}^{\infty}\left(g^2(s,k,t)\right)'ds=$$

$$=-g'^2(s,k,t)\Big|_{-\infty}^{\infty}-k^2g^2(s,k,t)\Big|_{-\infty}^{\infty}=k^2a^2(k,t)e^{-2ikx}+2k^2a(k,t)b(-k,t)+$$

$$+k^2b^2(-k,t)e^{2ikx}-k^2e^{-2ikx}-k^2a^2(k,t)e^{-2ikx}+2k^2a(k,t)b(-k,t)-$$
$$-k^2b^2(-k,t)e^{2ikx}+k^2e^{-2ikx}=4k^2a(k,t)b(-k,t).$$

According to this and equality (22), we have

$$\frac{da(k,t)}{dt}=0.$$

Therefore, we deduce that

$$\frac{d\lambda_j(t)}{dt}=0, \tag{26}$$

$$\frac{dr^+(k,t)}{dt}=\left(8ik^3u(x_0,t)-2ik\gamma(t)u(x_1,t)\right)r^+(k,t). \tag{27}$$

Now let's move on to finding the evolution of normalization numbers $B_n,\ n=1,2,\ldots,N$ corresponding to the eigenvalues $\lambda_n,\ n=1,2,3,\ldots,N$. To do this, we rewrite equality (21) in the following form

$$\frac{\partial g(x,k,t)}{\partial t}=u(x_0,t)\left(-u_x+4ik^3\right)g+u(x_0,t)\left(2u(x,t)+4k^2\right)\frac{\partial g(x,k,t)}{\partial x}+$$
$$+\frac{\gamma(t)u(x_1,t)g(x,k,t)}{2ik}\Big[g(x,k,t)\overline{g(x,k,t)}u(x,t)-$$
$$-\int_{-\infty}^{x}u(s,t)\left(g'(s,k,t)\overline{g(s,k,t)}+g(s,k,t)\overline{g}'(x,k,t)\right)ds\Big]-$$
$$-\frac{\gamma(t)u(x_1,t)\overline{g(x,k,t)}}{2ik}\left[g^2(x,k,t)u(x,t)-\int_{-\infty}^{x}2g'(s,k,t)g(s,k,t)u(s,t)ds\right]+$$
$$+\frac{4g(x,k,t)}{2ik}\int_{-\infty}^{x}g(s,k,t)\overline{g(s,k,t)}\sum_{m=1}^{N}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(x,t)\right|^2\right)ds-$$
$$-\frac{4\overline{g(x,k,t)}}{2ik}\int_{-\infty}^{x}g^2(s,k,t)\sum_{m=1}^{N}\xi_m\frac{\partial}{\partial x}\left(\left|\varphi_m(x,t)\right|^2\right)ds=$$
$$=u(x_0,t)\left(-u_x+4ik^3\right)g(x,k,t)+u(x_0,t)\left(2u+4k^2\right)\frac{\partial g(x,k,t)}{\partial x}-$$
$$-\gamma(t)u(x_1,t)g'(x,k,t)-ik\gamma(t)u(x_1,t)g(x,k,t)-2g(x,k,t)\xi_m\int_{-\infty}^{x}\left|\varphi_m(x,t)\right|^2ds.$$

Using equality (13), setting $k=k_n$, taking into account the asymptotics of the Jost solution at $x\to+\infty$ and equating the coefficients at $e^{-\chi_n x}$, we find an analogue of the Gardner-Green-Kruskal-Miura equations

$$\frac{dB_n(t)}{dt}=\left(8\chi_n^3u(x_0,t)+2\chi_n\gamma(t)u(x_1,t)-2\xi_nA_n(t)\right)B_n(t),\ n=1,2,3,\ldots,N \tag{28}$$

Thus, the following theorem has been proved.

Theorem 2. If function $u(x,t),\ \varphi_m(x,t),\ m=\overline{1,N},\ x\in R,\ t>0$ is a solution to problem (1)-(5), then scattering data $\left\{r^+(k,t),\lambda_n(t),B_n(t),\ n=\overline{1,N}\right\}$ of operator $L(t)$ with potential $u(x,t)$ satisfy differential equations (26), (27) and (28).

Remark 1. Consider the kernel of the Gelfand-Levitan-Marchenko integral equation

$$F^+(x,t)=\sum_{j=1}^{N}\alpha_j^+(t)e^{-\chi_j x}+\frac{1}{2\pi}\int_{-\infty}^{\infty}r^+(k,t)e^{ikx}dk$$

with scattering data from Theorem 2. Then the data $\left\{r^+(k,t),\chi_1(t),\dots,\chi_N(t),B_1(t),\dots,B_n(t)\right\}$ satisfies conditions A-E. Therefore, according to Theorem 1, the potential $u(x,t)$ in the operator $L(t)$ is uniquely determined.

Remark 2. The obtained relations (26)-(28) completely determine the evolution of the scattering data for the operator $L(t)$ and thus make it possible to apply the method of the inverse scattering problem to solve problem (1)-(5). Let function $u_0(x)\left(1+|x|\right)\in L^1(R)$ be given. Then the solution of problem (1)-(5) is found using the following algorithm. We solve the direct scattering problem with the initial function $u_0(x)$ and obtain scattering data $\left\{r^+(k),\chi_n,B_n,n=\overline{1,N}\right\}$ for the operator $L(0)$.

- Using theorem 2, we find the scattering data for $t>0$ $\left\{r^+(k,t),\chi_n(t),B_n(t),n=\overline{1,N}\right\}$ Using the method based on the Gelfand-Levitan-Marchenco integral equation, we solve the inverse scattering problem, i.e. find $u(x,t)$ from the scattering data for $t>0$ obtained in the previous step. After that, it is easy to find the solution $\varphi_m(x,t)$ of equation $L(t)\varphi_m(x,t):=-\varphi_m''(x,t)+u(x,t)\varphi_m(x,t)=\lambda_m\varphi_m(x,t),\ m=1,2,\dots,N$

Let us give an example illustrating the stated algorithm.
Consider the following problem:

$$\begin{cases} u_t+u_{xxx}-6uu_x+\gamma(t)u(\ln 2,t)u_x=4\xi_1\dfrac{\partial}{\partial x}|\varphi_1|^2 \\ -\varphi_1''(x,t)+u(x,t)\varphi_1(x,t)=\lambda(t)\varphi_1(x,t)\end{cases} \tag{29}$$

$$u(x,0)=-\frac{2}{\operatorname{ch}^2 x},\ x\in R, \tag{30}$$

there are

$$\int_{-\infty}^{\infty}|\varphi_1(x,t)|^2dx=A_1(t)=\frac{1}{2}e^{-10t},$$

$$\gamma(t)=\frac{42e^{10t}+9e^{20t}+16}{64e^{20t}}\quad \xi_n=1.$$

It is not difficult to find the scattering data of the operator $L(0)$:

$$\lambda(0)=-1,\ r^+(k,0)=0,\ B_1(0)=1.$$

By virtue of Theorem 2, we have

$$\lambda(t)=\lambda(0)=-1,\ r^{+}(k,t)=0,\ B_1(t)=e^{\mu(t)},\ A_1(t)=\frac{1}{2}e^{-\mu(t)},$$

where

$$\mu(t)=8\int_0^t d\tau+2\int_0^t \gamma(\tau)u(\ln 2,\tau)d\tau-2\int_0^t A_1(t)dt.$$

Substituting these data in formula (15), we find the kernel

$$F^{+}(x)=\sum_{j=1}^{N}\alpha_j^{+}e^{-\chi_j x}+\frac{1}{2\pi}\int_{-\infty}^{\infty} r^{+}(k)e^{ikx}dk$$

$$F_{+}(x,t)=2e^{-x+\mu(t)}$$

Next, solving the integral equation Gelfand-Levitan-Marchenco

$$K_{+}(x,y,t)+2e^{\mu(t)}\cdot e^{-(x+y)}+2e^{\mu(t)}\cdot e^{-y}\int_x^{\infty}K_{+}(x,s,t)e^{-s}ds=0,$$

$$K_{+}(x,y,t)=-\frac{2e^{\mu(t)}e^{-(x+y)}}{1+e^{\mu(t)}e^{-2x}}$$

Where do we find the solution of the Cauchy problem:

$$u(x,t)=-2\frac{d}{dx}K_{+}(x,x,t)=-\frac{2}{\mathrm{ch}^2\left(x-5t\right)},\ \varphi_1(x,t)=\frac{e^{-x}+3e^{-3x+10t}}{1+e^{-2x+10t}}$$

III. CONCLUSION

This article is devoted to the integration of the Korteweg-de Vries equation with an adapted source load in the class of rapidly decreasing functions. This article provides the necessary information on the exact and inverse problems of the scattering theory for the Sturm-Liouville operator, which are necessary for the following statements. First, the Yost solutions of the Sturum-Liouville operator on the entire axis are defined and integral images are obtained for them, the givens of the scattering theory are described and some of their necessary properties are given, the Gelfand-Levitan equation, which is the main integral equation of the inverse problems of the scattering theory, The Marchenco integral equation was derived. The problem of finding the solution of the Cauchy problem in the class of rapidly decreasing functions, which is applied to the Korteweg-de Vries equation with an adapted source load, is studied. In this case, the method of inverse problems of the scattering theory was used to determine the solution of the Cauchy problem imposed on the Korteweg-de Vries equation with an adapted source load in the class of rapidly decreasing functions. Equations for $t>0$ calculating the evolution of the Sturm-Liouville operator given by the scattering theory have been derived. The algorithm of applying the method of inverse problems of scattering theory is given. An example is shown in order to show the correctness of the obtained results.

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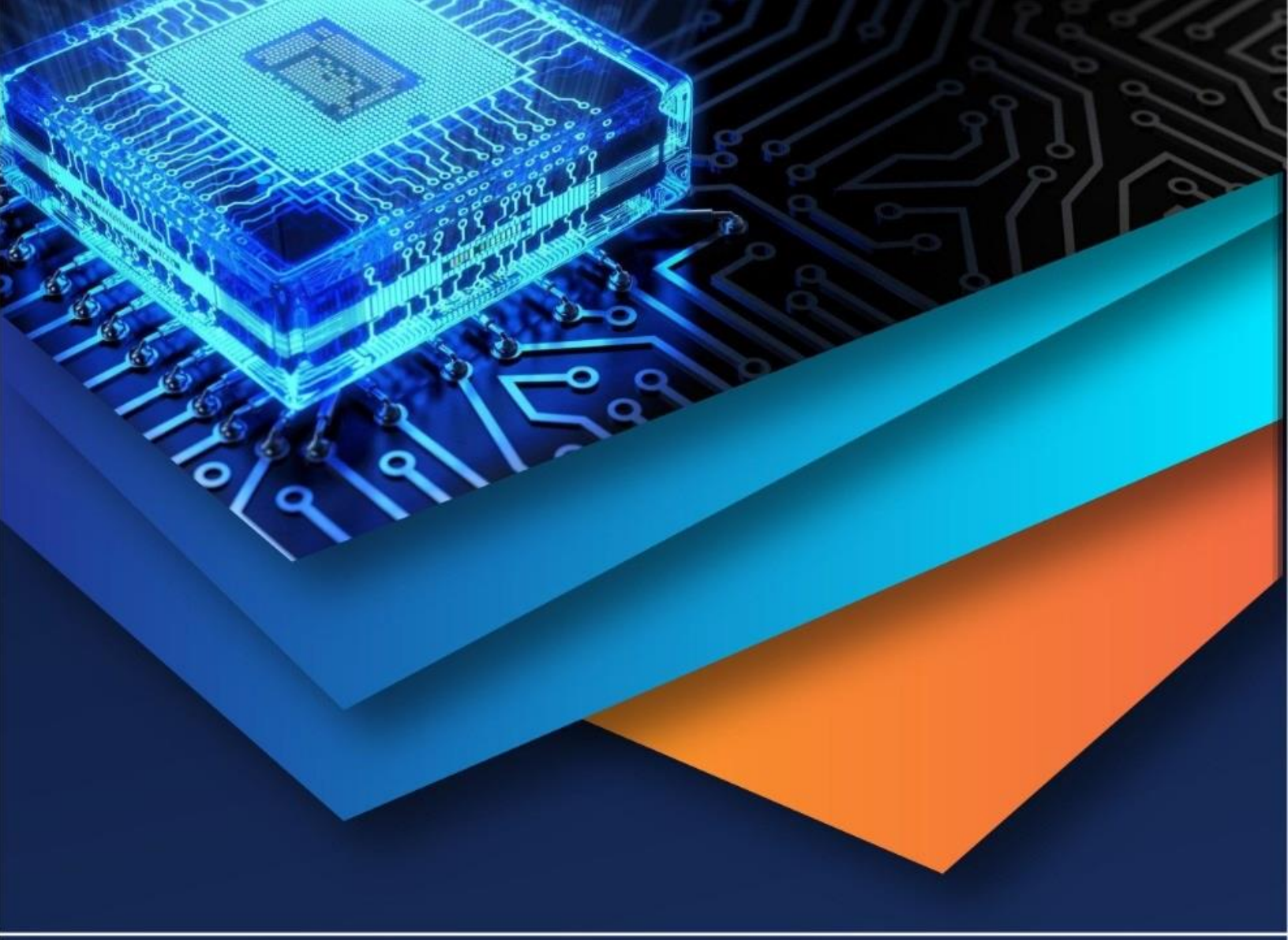

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