



IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 12 Issue: III Month of publication: March 2024 DOI: https://doi.org/10.22214/ijraset.2024.58885

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## **Intrinsic Solution on Quadratic Diophantine Equation** $z^{2} = 170x^{2} + y^{2}$

S. Vidhya<sup>1</sup>, S. Sivaranjani<sup>2</sup>

Assistant Professor<sup>1</sup>, PG Student<sup>2</sup>, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), (Affiliated to Bharathidasan University), Trichy-620018

Abstract: The non-zero unique integer solutions of the Quadratic Diophantine Equation with three unknowns  $z^2 = 170x^2 + y^2$  are examined. There are several absorbing relationships between the answers and a few unique numbers of rank n that arespecial Polygonal numbers, Star number and Gnomonic numbers. Keywords: Three-variable quadratic equation, integral solutions, Polygonal numbers, Star number and Gnomonic number.

#### I. **INTRODUCTION**

There are various distinct kinds of quadratic Diophantine equations. Anyone can look at [1,7] for an in-depth review of a broad spectrum of concerns and a substantial survey of the body of literature. The non-trivial integral solutions to the quadratic Diophantine problem of the form  $lxy + m(x, y) = z^2$  have been examined in [8]. The integral solutions of an distinct Pythagorean triangular problem have been investigated in [9,10]. Two parametric non-trivial integral solutions to the quadratic homogeneity Diophantine problem  $X^2 + PXY + Y^2 = Z^2$ , for which P constitutes a non-zero constant, are given in [11]. The non-trivial integral solutions of the quadratic homogeneity equation,  $l\alpha(x^2 + y^2) + bxy = 4l\alpha^2 z^2$  have been analysed in [12]. This formof quadratic Diophantine equation,  $(x - y)(x - z) + y^2 = 0$  is examined in [13] for its integral solutions at various angles, and their parameterized representations are identified.

we examine an additional fascinating quadratic equation,  $z^2 = 170x^2 + y^2$ , and derive distinct patterns of integral solutions that are non-trivial. Furthermore, a few fascinating relationships between the solutions special Polygonal number and Gnomonic numbers are illustrated.

#### П. **NOTATIONS**

 $T_{m,n} = \left[1 + \frac{(n-1)(m-2)}{2}\right] = \text{Polygonal Number with rank n and sides m.}$ 

 $Star_n = 6n(n-1) + 1 = Star$  number of rank n.

 $Gno_n = (2n-1) =$  Gnomonic number of rank n.

#### III. **METHOD OF ANALYSIS**

The Diophantine equation of quadratic has to be computed for its non-zero integral solution is  $z^2$ 

$$=170x^{2} + y^{2}(1)$$

Assuming,

 $z = z(a,b) = a^2 + 170b^2$ (2)where a and b are non-zero integers.

A. Pattern: 1 Equation (1) can be written as  $z^2 - 170x^2 = v^2(3)$ 



ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538 Volume 12 Issue III Mar 2024- Available at www.ijraset.com

Assuming 
$$y = y(a,b) = a^2 - 170b^2$$
 (4)  
We get,  
 $(a^2 + 170b^2)^2 = (170x^2 + y^2)$   
Using the factorization method, we have  
 $z + \sqrt{170}x = (a + \sqrt{170}b)^2$  (5)  
 $z - \sqrt{170}x = (a - \sqrt{170}b)^2$  (6)  
Comparing the rational and irrational factors,  
 $z = z(a,b) = a^2 + 170b^2$   
 $x = x(a,b) = 2ab$   
The corresponding non-zero distinct solutions are  
 $x = x(a,b) = 2ab$   
 $y = y(a,b) = a^2 - 170b^2$   
 $z = z(a,b) = a^2 + 170b^2$ 

Observations:

1. If a = b and a is even, then a is divisible by 2.

2. If *a* is odd and *b* is even, then *z* is divisible by 3.  
3. 
$$x(b,b) + y(b,b) + z(b,b)T_{10,b} \equiv 0 \pmod{3}$$
  
4.  $x(1,a) - y(1,a) + z(1,a) - 68T_{12,a} - 137GnO_a \equiv 0 \pmod{137}$   
5.  $y(1,1) + z(1,1) + x(1,1)$  is a perfect square.  
6.  $y(2,2) + z(2,2)$  is a Deficient number.  
7.  $11x(b,b) + y(b,b) + z(b,b)$  which represents a Nasty number.  
8. For any values of a and b,  $z - x - y$  is divisible by 2.

B. Pattern: 2

Equation (1) can be written as

 $170x^2 + y^2 = z^2 * 1$ 

Assuming

$$z = z(a,b) = a^{2} + 170b^{2}$$
  
and write  
$$(13 + i12\sqrt{170})(13 - i12\sqrt{170})$$

$$1 = \frac{\left(13 + i12\sqrt{170}\right)\left(13 - i12\sqrt{170}\right)}{24649} (8)$$

Using factorization method, equation (7) can be written as

$$\left(y + i\sqrt{170}x\right)\left(y - i\sqrt{170}x\right) = \left[\frac{13 + i12\sqrt{170}}{157}\right]\left[\frac{13 - i12\sqrt{170}}{157}\right]\left[\left(a + i\sqrt{170}b\right)^2\left(a - i\sqrt{170}b\right)^2\right]$$

(7)

we get

$$(y+i\sqrt{170}x) = \frac{1}{157}(13+i12\sqrt{170})(a-i\sqrt{170}b)^2$$
 (9)



International Journal for Research in Applied Science & Engineering Technology (IJRASET)

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$$(y - i\sqrt{170}x) = \frac{1}{157} (13 + i12\sqrt{170}) (a + i\sqrt{170}b)^2$$
 (10)

Comparing real and imaginary parts, we get

$$x = \frac{1}{157} \Big[ 12a^2 - 2040b^2 + 26ab \Big]$$
$$y = \frac{1}{157} \Big[ 13a^2 - 2210b^2 - 4080ab \Big]$$

Since finding integer solutions is what we specialize in, we have chosen and appropriately such that and are integers.

Consider a = 157A and b = 157B

$$x = 1884A^{2} - 320280B^{2} + 4082AB$$
$$y = 2041A^{2} - 346970B^{2} - 640560AB$$
$$z = 24649A^{2} + 4190330B^{2}$$

Observations

1. If A=B, then x-y-z is divisible by 2. 2.  $x(A, A) - y(A, A) - 134235T_{12,A} - 268470GnO_A \equiv 0 \pmod{268470}$ 3.  $z(B, B) - y(B, B) - x(B, B) - 393913T_{30,A} \equiv 0 \pmod{5120869}$ 4.  $z(A,1) + x(A,1) - 2041T_{28,A} - 14287GnO_A \equiv 0 \pmod{3884337}$ 5.  $y(B,B) + z(B,B) - 293590T_{24,A} \equiv 0 \pmod{2935900}$ 

C. Pattern: 3

Equation (1) can be written as  $z^{2} - y^{2} = 170x^{2}$ and we get  $(z + y)(z - y) = 170x \cdot x$  (11)

a) Case 1 Equation (1) can be written as  $\frac{z+y}{170x} = \frac{x}{z-y} = \frac{P}{Q}$ (12) From equation (12), we get two equations -170Px + Qy + Qz = 0 Qx + Py - Pz = 0Applying cross ratio method, we get the integer solutions are x = x(P,Q) = -2PQ $y = y(P,Q) = Q^2 - 170P^2$ 

 $z = z(P,Q) = -Q^2 - 170P^2$ 

Observations 1. For all values of P and Q, x + y - z is divisible by 2.



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2. 
$$2y(1, P) - x(1, P) - 2z(1, P) - 2T_{6,P} - 2GnO_P \equiv 0 \pmod{2}$$
  
3.  $x(P,1) - 4y(P,1) - 4z(P,1) - 170T_{18,P} - 594GnO_P \equiv 0 \pmod{594}$   
4. Each of the following expressions  
(i)  $z(Q,Q) + 2x(Q,Q) - y(Q,Q)$   
(ii)  $T_{30,P} - 3x(Q,1) + T_{18,P}$ 

represents a nasty number.

b) Case 2 Equation (11) can be written as  $\frac{z+y}{x} = \frac{170x}{z-y} = \frac{P}{Q}$ (13)

From equation (13), we have two equations

-Px + Qy + Qz = 0170Qx + Py - Pz = 0

Applying cross ratio method, we get the integer solutions are

$$x = x(P,Q) = -2QP$$
  

$$y = y(P,Q) = 170Q^{2} - P^{2}$$
  

$$z = z(P,Q) = -P^{2} - 170Q^{2}$$

Observations

1. 
$$y(1,Q) - x(1,Q) - 9z(1,Q) - 680T_{7,Q} - 511GnO_Q \equiv 0 \pmod{519}$$
  
2.  $10x(1,Q) + 3y(1,Q) - 85Star_Q - 245GnO_Q \equiv 0 \pmod{257}$   
3.  $y(Q,Q) - 6x(Q,Q) - z(Q,Q) - 44T_{18,Q} - 154GnO_Q \equiv 0 \pmod{154}$   
4.  $3T_{16,P} - 3x(P,1)$  represents a Nasty number.  
5.  $y(1,1) - x(1,1)$  represents a Palindromic number.  
6.  $z(1,1) + y(1,1) - 5x(1,1)$  is a Dudeney number.

D. Pattern: 4 Equation (1) can be written as  $z^2 - 170x^2 = y^2 * 1$ and write  $1 = (\sqrt{170} + 13)(\sqrt{170} - 13)$ Assuming,  $y = y(a,b) = a^2 - 170b^2$  (15) We have  $z^2 - 170x^2 = (a^2 - 170b^2) [(\sqrt{170} + 13)(\sqrt{170} - 13)]$ Using factorization method, we get  $z + \sqrt{170}x = (a + \sqrt{170}b)^2 [(\sqrt{170} + 13)](16)$ 

(14)



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$$z - \sqrt{170}x = (a + \sqrt{170}b)^2 [(\sqrt{170} + 13)]$$
 (17)  
From equation (16), we get

Thus, the corresponding non-zero distinct integer solutions are

$$x = x(a,b) = a^{2} + 170b^{2} + 26ab$$
  

$$y = y(a,b) = a^{2} - 170b^{2}$$
  

$$z = z(a,b) = 13a^{2} + 2210b^{2} + 340ab$$

Observations

1. 
$$z(b,b) - x(b,b) - 169T_{30,b} \equiv 0 \pmod{2197}$$
  
2.  $z(a,1) - y(a,1) + x(a,1) - T_{28,a} - 189GnO_a \equiv 0 \pmod{2739}$   
3.  $z(b,b) - y(b,b) - x(b,b) - 507T_{12,b} - 1014GnO_b \equiv 0 \pmod{1014}$   
4.  $x(1,1) + y(1,1)$  represents a perfect number.  
5.  $x(1,1) - y(1,1)$  represents even composite number.

## IV. CONCLUSION

The Diophantine quadratic equation using special polygonal numbers has been presented. Futhermore, one can look for integer solutions of the quadratic diophantine equations with other special numbers.

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