# Intrinsic Solution on Quadratic Diophantine Equation $z^{2}=170 x^{2}+y^{2}$ 

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#### Abstract

The non-zero unique integer solutions of the Quadratic Diophantine Equation with three unknowns $z^{2}=170 x^{2}+y^{2}$ are examined. There are several absorbing relationships between the answers and a few unique numbers of rank n that arespecial Polygonal numbers, Star number and Gnomonic numbers. Keywords: Three-variable quadratic equation, integral solutions, Polygonal numbers, Star number and Gnomonic number.


## I. INTRODUCTION

There are various distinct kinds of quadratic Diophantine equations. Anyone can look at [1,7] for an in-depth review of a broad spectrum of concerns and a substantial survey of the body of literature. The non-trivial integral solutions to the quadratic Diophantine problem of the form $l x y+m(x, y)=z^{2}$ have been examined in [8]. The integral solutions of an distinct Pythagorean triangular problem have been investigated in [9,10]. Two parametric non-trivial integral solutions to the quadratic homogeneity Diophantine problem $X^{2}+P X Y+Y^{2}=Z^{2}$, for which P constitutes a non-zero constant, are given in [11]. The non-trivial integral solutions of the quadratic homogeneity equation, $l \alpha\left(x^{2}+y^{2}\right)+b x y=4 l \alpha^{2} z^{2}$ have been analysed in [12]. This formof quadratic Diophantine equation, $(x-y)(x-z)+y^{2}=0$ is examined in [13] for its integral solutions at various angles, and their parameterized representations are identified.
we examine an additional fascinating quadratic equation, $z^{2}=170 x^{2}+y^{2}$, and derive distinct patterns of integral solutions that are non-trivial. Furthermore, a few fascinating relationships between the solutions special Polygonal number and Gnomonic numbers are illustrated.

## II. NOTATIONS

$T_{m, n}=\left[1+\frac{(n-1)(m-2)}{2}\right]=$ Polygonal Number with rank n and sides m .
$\operatorname{Star}_{n}=6 n(n-1)+1=$ Star number of rank n .
Gno $_{n}=(2 n-1)=$ Gnomonic number of rank n .

## III. METHOD OF ANALYSIS

The Diophantine equation of quadratic has to be computed for its non-zero integral solution is

$$
z^{2}=170 x^{2}+y^{2}(1)
$$

Assuming,
$z=z(a, b)=a^{2}+170 b^{2}$
where a and b are non-zero integers.

## A. Pattern: 1

Equation (1) can be written as

$$
z^{2}-170 x^{2}=y^{2}(3)
$$

Assuming $y=y(a, b)=a^{2}-170 b^{2}(4)$
We get,
$\left(a^{2}+170 b^{2}\right)^{2}=\left(170 x^{2}+y^{2}\right)$
Using the factorization method, we have
$z+\sqrt{170} x=(a+\sqrt{170} b)^{2}(5)$
$z-\sqrt{170} x=(a-\sqrt{170} b)^{2}$
Comparing the rational and irrational factors,
$z=z(a, b)=a^{2}+170 b^{2}$
$x=x(a, b)=2 a b$
The corresponding non-zero distinct solutions are
$x=x(a, b)=2 a b$
$y=y(a, b)=a^{2}-170 b^{2}$
$z=z(a, b)=a^{2}+170 b^{2}$
Observations:

1. If $a=b$ and $a$ is even, then $a$ is divisible by 2 .
2. If $a$ is odd and $b$ is even, then $z$ is divisible by 3 .
3. $x(b, b)+y(b, b)+z(b, b) T_{10, b} \equiv 0(\bmod 3)$
4. $x(1, a)-y(1, a)+z(1, a)-68 T_{12, a}-137 G n O_{a} \equiv 0(\bmod 137)$
5. $y(1,1)+z(1,1)+x(1,1)$ is a perfect square.
6. $y(2,2)+z(2,2)$ is a Deficient number.
7. $11 x(b, b)+y(b, b)+z(b, b)$ which represents a Nasty number.
8. For any values of a and $\mathrm{b}, z-x-y$ is divisible by 2 .

## B. Pattern: 2

Equation (1) can be written as
$170 x^{2}+y^{2}=z^{2} * 1$
Assuming
$z=z(a, b)=a^{2}+170 b^{2}$
and write
$1=\frac{(13+i 12 \sqrt{170})(13-i 12 \sqrt{170})}{24649}(8)$
Using factorization method, equation (7) can be written as
$(y+i \sqrt{170} x)(y-i \sqrt{170} x)=\left[\frac{13+i 12 \sqrt{170}}{157}\right]\left[\frac{13-i 12 \sqrt{170}}{157}\right]\left[(a+i \sqrt{170} b)^{2}(a-i \sqrt{170} b)^{2}\right]$
we get
$(y+i \sqrt{170} x)=\frac{1}{157}(13+i 12 \sqrt{170})(a-i \sqrt{170} b)^{2}(9)$
$(y-i \sqrt{170} x)=\frac{1}{157}(13+i 12 \sqrt{170})(a+i \sqrt{170} b)^{2}(10)$
Comparing real and imaginary parts, we get
$x=\frac{1}{157}\left[12 a^{2}-2040 b^{2}+26 a b\right]$
$y=\frac{1}{157}\left[13 a^{2}-2210 b^{2}-4080 a b\right]$
Since finding integer solutions is what we specialize in, we have chosen and appropriately such that and are integers.
Consider $a=157 A$ and $b=157 B$
The integer solutions are
$x=1884 A^{2}-320280 B^{2}+4082 A B$
$y=2041 A^{2}-346970 B^{2}-640560 A B$
$z=24649 A^{2}+4190330 B^{2}$

Observations

1. If $\mathrm{A}=\mathrm{B}$, then $\mathrm{x}-\mathrm{y}-\mathrm{z}$ is divisible by 2 .
2. $x(A, A)-y(A, A)-134235 T_{12, A}-268470 G n O_{A} \equiv 0(\bmod 268470)$
3. $z(B, B)-y(B, B)-x(B, B)-393913 T_{30, A} \equiv 0(\bmod 5120869)$
4. $z(A, 1)+x(A, 1)-2041 T_{28, A}-14287 G n O_{A} \equiv 0(\bmod 3884337)$
5. $y(B, B)+z(B, B)-293590 T_{24, A} \equiv 0(\bmod 2935900)$
C. Pattern: 3

Equation (1) can be written as
$z^{2}-y^{2}=170 x^{2}$
and we get
$(z+y)(z-y)=170 x \cdot x$
a) Case 1

Equation (1) can be written as
$\frac{z+y}{170 x}=\frac{x}{z-y}=\frac{P}{Q}(12)$
From equation (12), we get two equations
$-170 P x+Q y+Q z=0$
$Q x+P y-P z=0$
Applying cross ratio method, we get the integer solutions are
$x=x(P, Q)=-2 P Q$
$y=y(P, Q)=Q^{2}-170 P^{2}$
$z=z(P, Q)=-Q^{2}-170 P^{2}$

Observations
1.For all values of P and $\mathrm{Q}, \mathrm{x}+\mathrm{y}-\mathrm{z}$ is divisible by 2 .
2. $2 y(1, P)-x(1, P)-2 z(1, P)-2 T_{6, P}-2 G n O_{P} \equiv 0(\bmod 2)$
3. $x(P, 1)-4 y(P, 1)-4 z(P, 1)-170 T_{18, P}-594 G n O_{P} \equiv 0(\bmod 594)$
4. Each of the following expressions
(i) $z(Q, Q)+2 x(Q, Q)-y(Q, Q)$
(ii) $T_{30, P}-3 x(Q, 1)+T_{18, P}$
represents a nasty number.
b) Case 2

Equation (11) can be written as
$\frac{z+y}{x}=\frac{170 x}{z-y}=\frac{P}{Q}$
From equation (13), we have two equations
$-P x+Q y+Q z=0$
$170 Q x+P y-P z=0$
Applying cross ratio method, we get the integer solutions are
$x=x(P, Q)=-2 Q P$
$y=y(P, Q)=170 Q^{2}-P^{2}$
$z=z(P, Q)=-P^{2}-170 Q^{2}$
Observations

1. $y(1, Q)-x(1, Q)-9 z(1, Q)-680 T_{7, Q}-511 G n O_{Q} \equiv 0(\bmod 519)$
2. $10 x(1, Q)+3 y(1, Q)-85$ Star $_{Q}-245 G n O_{Q} \equiv 0(\bmod 257)$
3. $y(Q, Q)-6 x(Q, Q)-z(Q, Q)-44 T_{18, Q}-154 G n O_{Q} \equiv 0(\bmod 154)$
4. $3 T_{16, P}-3 x(P, 1)$ represents a Nasty number.
5. $y(1,1)-x(1,1)$ represents a Palindromic number.
6. $z(1,1)+y(1,1)-5 x(1,1)$ is a Dudeney number.
D. Pattern: 4

Equation (1) can be written as
$z^{2}-170 x^{2}=y^{2} * 1$
and write
$1=(\sqrt{170}+13)(\sqrt{170}-13)$
Assuming,
$y=y(a, b)=a^{2}-170 b^{2}$
We have
$z^{2}-170 x^{2}=\left(a^{2}-170 b^{2}\right)[(\sqrt{170}+13)(\sqrt{170}-13)]$
Using factorization method, we get
$z+\sqrt{170} x=(a+\sqrt{170} b)^{2}[(\sqrt{170}+13)](16)$
$z-\sqrt{170} x=(a+\sqrt{170} b)^{2}[(\sqrt{170}+13)]$
From equation (16), we get
Thus, the corresponding non-zero distinct integer solutions are
$x=x(a, b)=a^{2}+170 b^{2}+26 a b$
$y=y(a, b)=a^{2}-170 b^{2}$
$z=z(a, b)=13 a^{2}+2210 b^{2}+340 a b$

## Observations

1. $z(b, b)-x(b, b)-169 T_{30, b} \equiv 0(\bmod 2197)$
2. $z(a, 1)-y(a, 1)+x(a, 1)-T_{28, a}-189 G n O_{a} \equiv 0(\bmod 2739)$
3. $z(b, b)-y(b, b)-x(b, b)-507 T_{12, b}-1014 G n O_{b} \equiv 0(\bmod 1014)$
4. $x(1,1)+y(1,1)$ represents a perfect number.
5. $x(1,1)-y(1,1)$ representsa even composite number.

## IV. CONCLUSION

The Diophantine quadratic equation using special polygonal numbers has been presented. Futhermore, one can look for integer solutions of the quadratic diophantine equations with other special numbers.

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