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Intrinsic Solution on Ternary Quadratic Diophantine Equation

$$6x^2 + 6y^2 - 11xy = 72z^2$$

G. Janaki[1], P. Sangeetha[2], B. Pavithra[3]
[1]Associate Professor, PG and Research and Department of Mathematics, Cauvery College for women (Autonomous) (Affiliated to Bharathidasan University), Tiruchirappalli , Tamilnadu, India
[1]Assistant Professor, PG and Research and Department of Mathematics, Cauvery College for women (Autonomous) (Affiliated to Bharathidasan University), Tiruchirappalli , Tamilnadu, India
[3]PG Scholar, Cauvery College for women (Autonomous), Tiruchirappalli, Tamilnadu, India

I. INTRODUCTION

Numerous ternary quadratic equations exist. For a more complete understanding, check [1-7]. The And form of the ternary quadratic Diophantine problem $kxy + m(x+y) = z^2$ has been studied for solving non-trivial integrals. In [9-15], non-zero integral solutions to the various Diophantine equations are investigated. These findings have motivated us to hunt for an infinite number of non-zero integral solutions to the ternary quadratic equation given by, which is another fascinating equation. There are also a few intriguing correlations between the answers, as well as some unique numbers such as triangular, cantered, gnomon, and star. In addition, a Python program is used to code in the quadratic diophantine equation in the five patterns to determine the program's output.

II. NOTATIONS

- $T_{m,n}$ = Triangular Number of rank n.
- Gno_n = Gnomonic Number of rank n.
- $Star_n$ = Star Number of rank n.
- CH_n = Cantered Hexagonal number of rank n.

III. METHODOLOGY

The ternary equation of quadratic Diophantine condition is for its non-zero necessary arrangement

$$6x^2 + 6y^2 - 11xy = 72z^2 \longrightarrow \quad (1)$$

Swapping out linear transformation

$$x = u + v \quad \text{and} \quad y = u - v \longrightarrow \quad \mathbf{(2)}$$

in(1) leads to.

$$u^2 + 23v^2 = 72z^2 \longrightarrow \quad \mathbf{(3)}$$

1) Template: 1

Suppose

$$z = z(a,b) = a^2 + 23b^2 \longrightarrow \quad (4)$$

where a and b are non-zero integers.

$$72 = (7 + i\sqrt{23})\ (7 - i\sqrt{23}) \longrightarrow \quad (5)$$

Operating (4) and (5) and using factorization Method ,

$$(u + i\sqrt{23}v)(u - i\sqrt{23}v) = (7 + i\sqrt{23})(7 - i\sqrt{23})(a + i\sqrt{23}b)^2 (a - i\sqrt{23}b)^2 \longrightarrow \quad (6)$$

Equating like terms and corresponding real and imaginary parts

$$u(a,b) = 7a^2 - 161b^2 46ab$$

$$v(a,b) = a^2 - 23b^2 + 14ab$$

Equating (2) can be solved as an integer by substituting the above values for u and v,

$$x = 8a^2 - 184b^2 - 32ab - 208$$

$$y = 6a^2 - 138b^2 - 60ab - 152$$

$$z = a^2 + 23b^2$$

Observation:

$$x(a,1) + y(a,1) - 2T_{16,a} + 40Gno_a \equiv (\text{mod} - 722)$$

$$x(a,1) + y(a,1) + z(a,1) - 5T_{8,a} - 41Gno_a \equiv (\text{mod} - 700)$$

$$x(a,a) - y(a,a) - 2z(a,a) + 16T_{10,a} + 24Gno_a \equiv (\text{mod} - 80)$$

$$x(a,1) + y(a,1) - 2z(a,1) - 2Star_a + 46Gno_a \equiv (\text{mod} - 776)$$

$$y(a,1) + x(a,1) + 92T_{14,a} + 230Gno_a \equiv (\text{mod} - 438)$$

$$3x(a,a) - 5y(a,a) - 14z(a,a) + 151T_{20,a} + 640Gno_a \equiv (\text{mod} - 1988)$$

2) Template: 2

Consider (3) as ,

$$u^2 - 49v^2 + 72v^2 = 72z^2 \longrightarrow \quad (7)$$

Compute equation (7)

$$u^2 - 49v^2 = 72(z^2 - v^2) \longrightarrow \quad (8)$$

Equation (8) in the form of ratio as

$$\frac{u + 7v}{z + v} = \frac{72(z - v)}{u - 7v} = \frac{A}{B}$$

The two equations that follow are identical to this,

$$Bu + (7B - A)v - Az = 0 \longrightarrow \quad (9)$$

$$-Au - (72B - 7A)v + 72Bz = 0 \longrightarrow \quad (10)$$

Apply the cross multiplication,

$$u = 7A^2 + 504B^2 - 144AB$$

$$v = A^2 - 72B^2$$

$$z = A^2 - 14AB + 72B^2$$

After, Operating the value of u ,v in eqn (2) and the corresponding integer solution (1),

$$x = 8A^2 - 144AB + 432B^2$$

$$y = 6A^2 - 144AB + 567B^2$$

$$z = A^2 - 14AB + 72B^2$$

Observation:

1. $y(A,1) + x(A,1) - 2T_{16,A} + 138\,Gno_A \equiv (\text{mod } 870)$
2. $x(1,1) + y(A,1) - 2T_{8,A} + 70\,Gno_A \equiv (\text{mod } 820)$
3. $x(A,A) + y(1,1) - 2T_{10,A} - 69\,Gno_A \equiv (\text{mod } -75)$
4. $x(A,A) + z(1,1) - 148\,T_{6,A} + 74\,Gno_A \equiv (\text{mod } 133)$
5. $x(A,A) - y(A,A) - 2z(A,1) + 12\,T_{26,A} + 52\,Gno_A \equiv (\text{mod } -196)$

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3) Template: 3

Equation (3) as,

$$u^2 = 72z^2 - 23v^2$$

By consider the linear transformation

$$z = x + 72T \ \& \ v = x + 23T \longrightarrow (11)$$

Substituting (11) in (3)

$$u^2 = 49(X^2 + 1656T^2) \longrightarrow (12)$$

write $u = 7U$ $\longrightarrow$ (13)

As, $U^2 = X^2 + 1656T^2$ $\longrightarrow$ (14)

The corresponding solution of (14) is

$$T = 2ab$$

$$U = 1656a^2 - b^2 \longrightarrow (15)$$

$$X = 1658a^2 + b^2$$

Substituting (15) in (11) and (13) we get,

$$u = 7(1656a^2 - b^2)$$

$$v = 1656a^2 + b^2 + 144ab$$

$$z = 1656a^2 + b^2 + 46ab$$

After, Operating the value of u ,v in eqn (2) and the corresponding integer solution (1),

$$x = 13248a^2 - 6b^2 + 144ab$$

$$y = 9936a^2 - 8b^2 - 144ab$$

$$z = 1656a^2 + b^2 + 46ab$$

Observation

1. $x(a,a) - y(a,a) + 4z(a,a) - 38132\ T_{3,a} - 9533\ Gno_a \equiv (\text{mod } -9533)$
2. $x(a,1) - y(a,1) - 400\ T_{20,a} - 1600\ Gno_a \equiv (\text{mod } 1602)$
3. $3y(a,1) - 4x(a,1) + 12z(a,1) + 314\ T_{26,a} + 1727\ Gno_a \equiv (\text{mod } 1715)$
4. $x(a,a) - y(a,a) - 2z(a,a) - 1598\ T_{6,a} - 799\ Gno_a \equiv (\text{mod } 799)$
5. $y(a,1) - 6z(a,1) + 70\ Star_a \equiv (\text{mod } -14)$
6. $x(a,a) - y(a,a) - 21942152\ \ Star_a \equiv (\text{mod } 0)$

4) Template: 4

(3) can be written as $u^2 + 23v^2 = 72z^2$

$$u^2 + 23v^2 = 72z^2 \longrightarrow (16)$$

"72" can be written as

$$72 = \frac{(5 + i7\sqrt{23})(5 - i7\sqrt{23})}{16} \longrightarrow (17)$$

Employing (17) in (16) and proceeding as in like template 1. We get the non-zero distinct integer solution as

$$u + i\sqrt{23}v)(u - i\sqrt{23}v) = \frac{(5 + i7\sqrt{23})(5 - i7\sqrt{23})}{16}\ (a + i\sqrt{23}b)^2 \longrightarrow (18)$$

Equating like terms and corresponding real and imaginary parts,

$$u = 5a^2 - 115b^2 - 322ab$$

$$v = 7a^2 - 161b^2 + 10ab$$

Since finding integer answer is what we are interested in, pick a and b choose wisely as $a = 4A \,\&\, b = 4B$ Considering (2), the whole number arrangement of (1) are give by

$$X = 48A^2 - 1104B^2 - 1248AB$$

$$Y = 8A^2 - 184B^2 + 1328AB$$

$$z = 16A^2 + 368B^2$$

Observation

1. $z(1,0) = 16 = 4^2 = perfectsquare$
2. $y(1,0)$ *is a cubic root*
3. $x(1,B) + 184T_{14,B} - 164Gno_B \equiv (\mathrm{mod}\ 212)$
4. $2[y(1,1)] = 2304(Ducknumber)$
5. $5[z(A,A) - x(1,1)] - 160T_{26,A} - 880Gno_A \equiv (\mathrm{mod}\ 12400)$

5) Template: 5

(3) can be written as $u^2 + 23v^2 = 72z^2$

$$u^2 + 23v^2 = 72z^2.1 \longrightarrow \quad (19)$$

'1' can be written as

$$1 = \frac{(11 + i\sqrt{23})(11 - i\sqrt{23})}{144}$$

Adapting it in equation (19) and proceeding as in "template1", we get

$$(u + i\sqrt{23}v)(u - i\sqrt{23}v) = (7 + i\sqrt{23})(7 - i\sqrt{23})(a + i\sqrt{23}b)^2 \frac{(11 + i\sqrt{23})(11 - i\sqrt{23})}{144}$$

$$= (7 + i\sqrt{23})\ (a + i\sqrt{23}b)^2 \frac{(11 + i\sqrt{23})(11 - i\sqrt{23})}{144} \longrightarrow \quad (20)$$

Equating the eqn (20) like terms and corresponding real and imaginary parts,

$$u = \frac{1}{12}(54a^2 - 1242b^2 - 828ab)$$

$$v = \frac{1}{12}(18a^2 - 414b^2 + 108ab)$$

Since finding integer answer is what we are interested in, pick a and b choose wisely as $a = 12A \,\&\, b = 12B$. Considering (2), the whole number arrangement of (1) are give by

$$X = 864A^2 - 19872B^2 - 8640AB$$

$$Y = 432A^2 - 9936B^2 - 11232AB$$

$$Z = 144A^2 + 3312B^2$$

Observation

$$1.\ x(1,1)+y(A,1)-72Star_A+5616Gno_A\equiv(\text{mod}-43200)$$

$$2.\ 2z(A,1)-3x(1,1)-24T_{26,A}-132Gno_A\equiv(\text{mod}-81372)$$

$$3.\ 2[y(A,A)-x(A,A)]-1536T_{20,A}-6144Gno_A\equiv(\text{mod }6144)$$

$$4.\ y(A,A)-x(A,A)-1152T_{14,A}-2880Gno_A\equiv(\text{mod }2880)$$

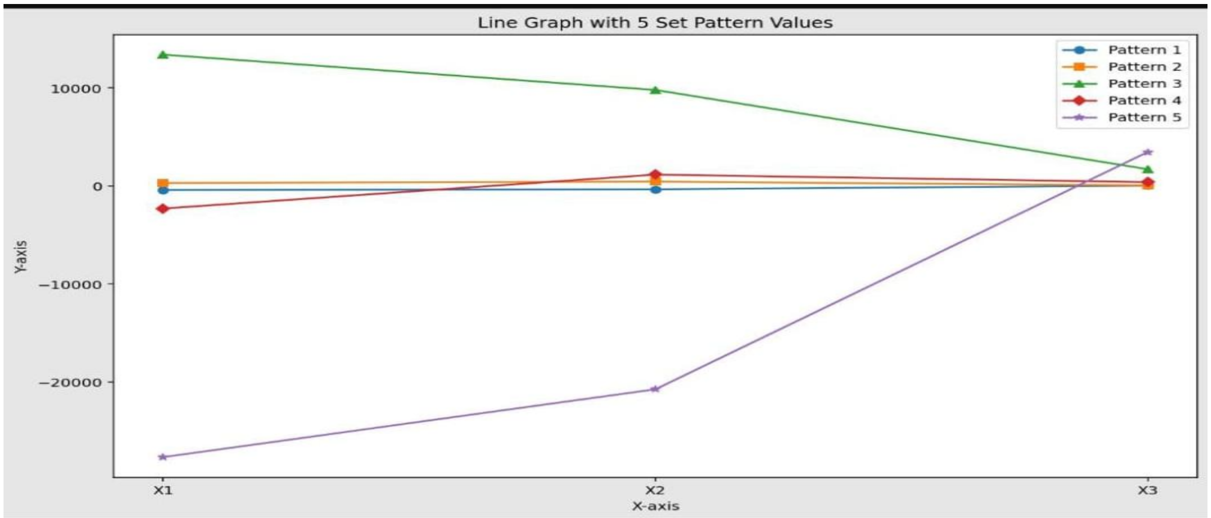
$$5.\ x(1,B)-6y(1,B)-4416T_{20,B}-47040Gno_B\equiv 0(\text{mod }45312)$$

$$6.\ z(A,1)-24T_{14,A}-60Gno_A\equiv 0(\text{mod }3372)$$

$$6x^2+6y^2-11xy=72z^2$$

Python program for Solving Quadratic Diophantine Equation:
Program coding for line graph.

```
import matplotlib.pyplot as plt
import numpy as np
#Define the data
x = np.arange(3)
y1 = [-416, -344, 24]
y2 = [296, 438, 59]
y3 = [13386, 9784, 1703]
y4 = [-2304, 1152, 384]
y5 = [-27648, -20736, 3456]
#Create the figure and axis
fig, ax = plt.subplots()
#Create the line graph
ax.plot(x, y1, marker='o', label='Pattern 1')
ax.plot(x, y2, marker='s', label='Pattern 2')
ax.plot(x, y3, marker='^', label='Pattern 3')
ax.plot(x, y4, marker='D', label='Pattern 4')
ax.plot(x, y5, marker='*', label='Pattern 5')
#Set the title and labels
ax.set_title('Line Graph with 5 Set Pattern Values')
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_xticks(x)
ax.set_xticklabels(['X1', 'X2', 'X3'])
#Add a legend
ax.legend()
#Show the plot
plt.show()
```



IV. CONCLUSION

This work presents infinitely many non-zero unique integer solutions to the ternary quadratic diophantine problem $6(x^2+y^2)-11xy=72z^2$, along with some observations about the solutions. For alternative options of ternary quadratic diophantine equations, one could look for additional examples of non-zero whole number remarkable arrangements and their corresponding highlights, or increase the quadratic diophantine equation of five patterns using Python program coding of line graph to find a diagram output.

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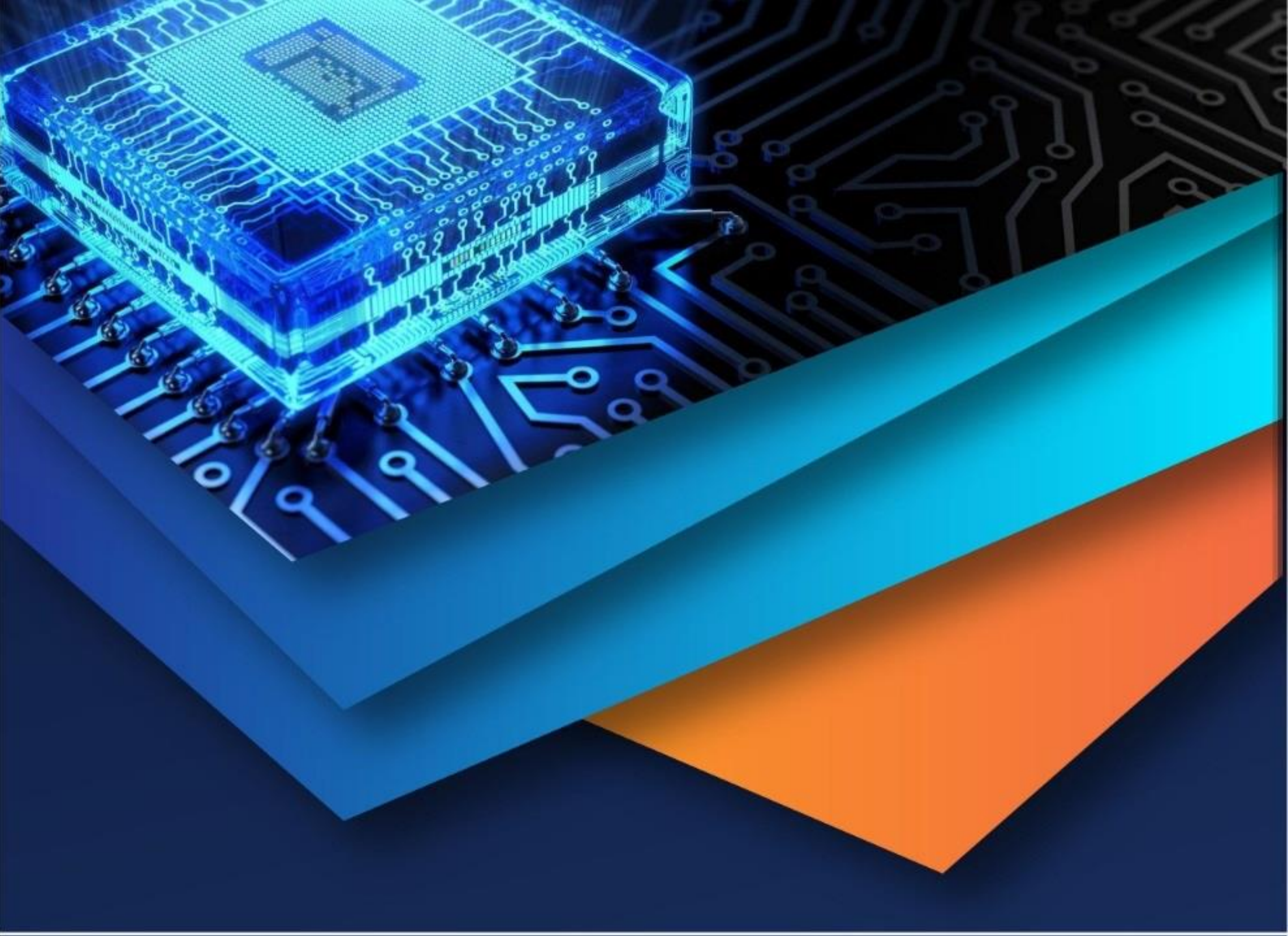

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