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Mathematical Analysis of a SIR Epidemic Model with Vaccination and Awareness

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Abstract: This study focuses to use mathematics to better understand how infectious diseases propagate. It suggests a SIR epidemic model with immunization and awareness campaigns. The population is split into three categories in this model: susceptible, infected, and recovered. While awareness campaigns seek to lower the rate of disease transmission, vaccination attempts to lower the number of susceptible people. The basic reproduction number and equilibrium points are found by analysing the model. The requirements for disease extinction are revealed via stability analysis of the disease-free equilibrium. The effects of vaccination and awareness on disease dynamics are illustrated through numerical simulations. According to the study, raising vaccination knowledge and coverage can successfully lower illness rates.
Keywords: SIR, Vaccination and Awareness, Mathematical Analysis.

I. INTRODUCTION

Infectious diseases remain significant challenges to public health systems around the world. Mathematical modelling plays a crucial role for diseases spread comprehension and controlling measures implementation. Epidemic models allow researchers to study the dynamics of infectious diseases and assess intervention measures like vaccination, treatment, and public awareness campaigns. In the case of epidemic models, one of the most popular ones is Kermack and McKendrick's original SIR. In this model, individuals in a population are separated into three parts: susceptible people who can become infected, infected people who cause infection to others and the recovered people who get immunity after

However, the impact of vaccination campaigns and public awareness, which are essential for limiting disease outbreaks, is not taken into consideration by the traditional SIR model. While awareness programs promote preventive behaviours including hygienic habits and social distancing, which lower the transmission rate, vaccination lowers the number of susceptible individuals. This study suggests a SIR epidemic model that takes vaccination and awareness effects into account in order to address these problems. To find equilibrium locations, the basic reproduction number, and stability requirements, the model is mathematically studied. The findings shed light on how vaccination and awareness campaigns affect illness prevention.

A. Literature Review

According to recent research, raising knowledge has a big impact on how people behave, which lowers the chance of infection and speeds up healing. Under the assumption of a well-mixed infinite population, an analytical technique has been used to build a linked SIR model with an Unaware–Aware (UA) framework (SIR-UA). In order to capture both sickness and awareness stages, the population is split into six compartments: SU, SA, IU, IA, RU, and RA. By including all potential transitions between consciousness and disease processes, this model expands upon earlier paradigms. Phase diagrams and parameter analysis show how awareness affects recovery patterns and epidemic magnitude under different circumstances.

This study uses an SEIR model with memory effects to investigate how media-driven awareness affects the spread of infectious diseases. Equilibrium and bifurcation behaviour are determined using stability analysis and the fundamental reproduction number. The findings demonstrate that infection levels are successfully decreased via fractional-order control, decreased transmission, and raised awareness. In order to decrease the spread of disease, this study creates a SIR-based model that includes media awareness as a control variable. Simulations and stability analysis demonstrate that awareness reduces infection levels and aids in isolating susceptible. The findings demonstrate the effectiveness of media awareness as an intervention, particularly in situations where vaccines or therapies are not available.

In order to examine the combined effects of vaccination and awareness on the dynamics of infectious diseases, this study creates an SVIQR model. Disease-free and endemic equilibria are examined using stability analysis and the fundamental reproduction number. The findings demonstrate that successful immunization and awareness campaigns greatly lower the rate of transmission and promote improved public health initiatives.

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B. Objectives of the Study

The primary goals of this research are:

To create a SIR epidemic model that takes public knowledge and vaccine impacts into account.

1) To examine the suggested model's mathematical characteristics, such as equilibrium points and the fundamental reproduction number.
2) To investigate the stability of the disease-free equilibrium and identify the circumstances that allow the disease to be completely eradicated.
3) To investigate how awareness and immunization affect the dynamics of infectious disease transmission.
4) To use numerical simulations to demonstrate the model's behavior and examine how various factors affect the spread of disease.

II. MODEL FORMULATION

Think about a population that is split up into three groups:

- S(t): People who are susceptible
- I(t): People who are infected
- R(t): People who have recovered
- N(t) = S(t) + I(t) + R(t) is the entire population.

The parameters listed below are defined:

β is the transmission rate.

γ is the recovery rate, and v is the vaccination rate.

δ is the awareness rate that lowers transmission

The system describes the updated SIR model with immunization and awareness:

$$\frac{dS}{dt} = -\beta(1-\delta)SI - \nu S$$

$$\frac{dI}{dt} = \beta(1-\delta)SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I + \nu S$$

While awareness lowers the effective transmission rate, vaccination immediately transfers vulnerable individuals to the recovered class.

III. EQUILIBRIUM POINTS

Equilibrium points occur when

$$\frac{dS}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

A. Disease-Free Equilibrium

When there are no diseased people, the disease-free equilibrium (DFE) is reached.

$$E_0 = (N, 0, 0)$$

As of right now, there is no sickness and the entire population is at risk.

B. Endemic Equilibrium

When the disease continues to exist in the population, the endemic equilibrium is reached. It is acquired by resolving the system's equilibrium equations. The model parameters determine the values.

IV. BASIC REPRODUCTION NUMBER

The average number of secondary infections caused by a single sick person in a population that is fully susceptible is represented by the basic reproduction number R_0.

For the proposed model,

$$R_0 = \frac{\beta(1-\delta)}{\gamma + \nu}$$

Interpretation:
If $R_0 < 1 \longrightarrow$ infection dies out
If $R_0 > 1 \longrightarrow$ infection spreads
Vaccination and awareness reduce the value of R_0, thereby helping to control disease transmission.

V. STABILITY ANALYSIS

The stability of the disease-free equilibrium is studied using the Jacobian matrix.
We compute the Jacobian matrix:

$$J = \begin{pmatrix} -(v+\mu) & -\beta S^* & 0 \\ 0 & \beta S^* - (\gamma+\mu) & 0 \\ v & \gamma & -\mu \end{pmatrix}$$

Eigenvalues are:

$$\lambda_1 = -(v+\mu)$$
$$\lambda_2 = \beta S^* - (\gamma+\mu)$$
$$\lambda_3 = -\mu$$

Thus,

$$\lambda_2 < 0 \text{ if and only if } R_0 < 1$$

Therefore:

- If $R_0 < 1$, DFE is locally asymptotically stable.

and unstable when

- $R_0 > 1$

Thus, vaccination and awareness programs play a crucial role in stabilizing the disease-free state.

VI. NUMERICAL SIMULATION

The following system is used for numerical evaluation in order to demonstrate the behaviour of the suggested model:

$$\frac{dS}{dt} = -\beta(1-\delta)SI - \nu S$$
$$\frac{dI}{dt} = \beta(1-\delta)SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I + \nu S$$

The simulation takes into account the following parameter values:

$$\beta = 0.5, \gamma = 0.25, \nu = 0.15, \delta = 0.3$$

The effective transmission rate is therefore equal to

$$\beta(1-\delta) = 0.5(1-0.3) = 0.35$$

$S(0) = 900, I(0) = 100, R(0) = 0$ are the starting population values.

The following formula is used to determine the derivatives at $t = 0$ are calculated as follows:

$$\frac{dS}{dt} = -0.35(900)(100) - 0.15(900)$$
$$= -31500 - 135 = -31635$$
$$\frac{dI}{dt} = 0.35(900)(100) - 0.25(100)$$
$$= 31500 - 25 = 31475$$
$$\frac{dR}{dt} = 0.25(100) + 0.15(900)$$
$$= 25 + 135 = 160$$

The approximate values using Euler's numerical method with step size h=0.01 are

$$S(0.01) = 900 + 0.01(-31635) = 583.65$$
$$I(0.01) = 100 + 0.01(31475) = 414.75$$
$$R(0.01) = 0 + 0.01(160) = 1.6$$

The simulation results demonstrate that raising vaccination and raising awareness shortens the peak number of infected individuals.

VII. RESULTS AND DISCUSSION

The suggested model emphasizes how crucial immunization and awareness campaigns are to the management of infectious diseases. While knowledge lowers the effective transmission rate, vaccination directly lowers the susceptible population. Both elements have a substantial impact on the dynamics of disease, according to the examination of the reproduction number. The value of R_0 falls as vaccination and awareness levels rise, ultimately resulting in the eradication of the illness. Combining vaccination with awareness campaigns has a greater control effect than employing either tactic alone, according to numerical models.

VIII. CONCLUSION

An SIR epidemic model that included vaccination and awareness effects was created and examined in this study. Stability conditions, equilibrium analysis, and model design were investigated. The findings demonstrate that when the reproduction number is less than one, raising vaccination coverage and awareness considerably lowers the spread of disease and aids in its eradication.

The model can help public health officials and policymakers develop strategies that effectively manage infectious diseases. More intricate models including exposed populations, treatment plans, and time-dependent characteristics might be included in future research.

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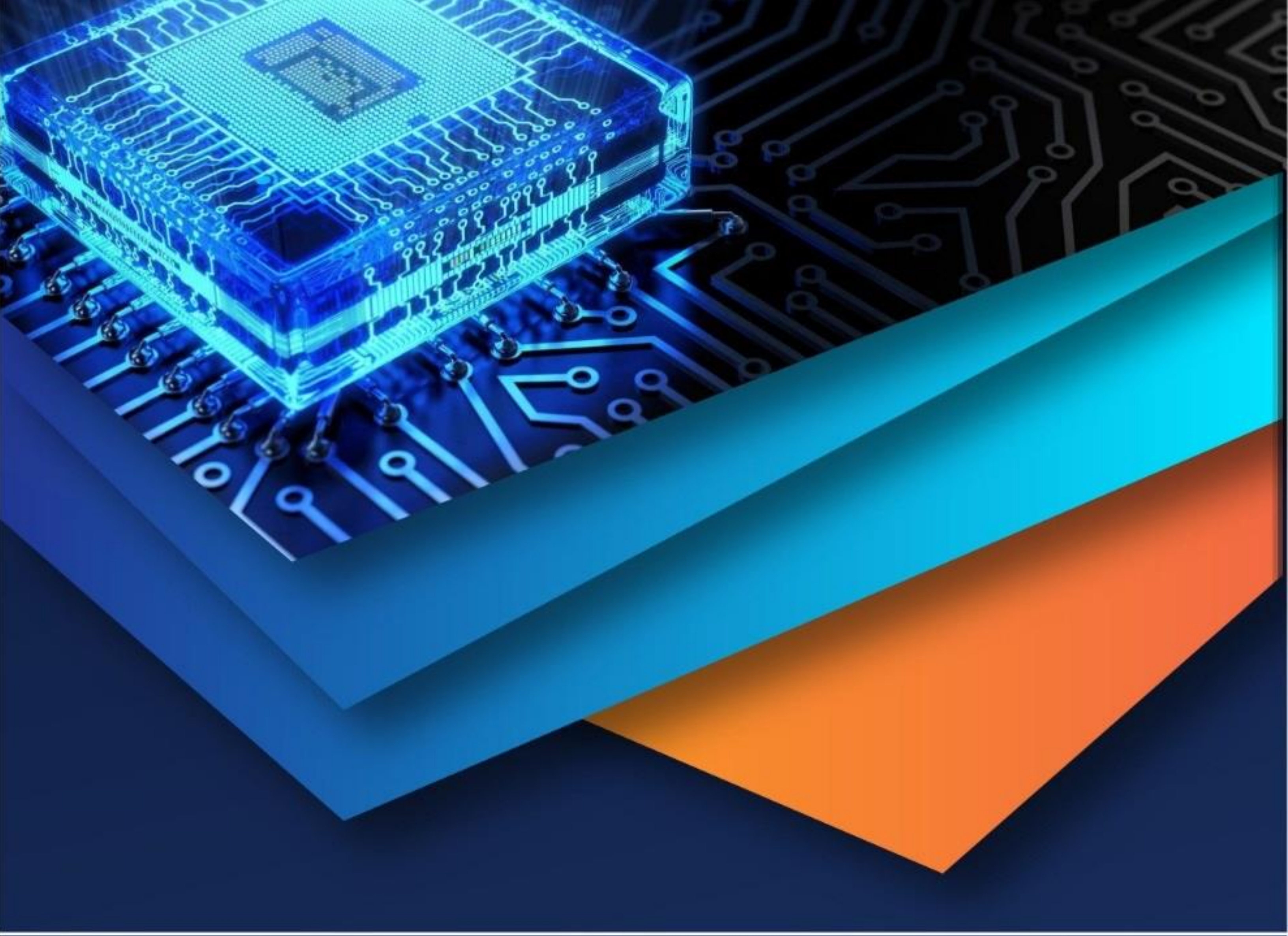

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