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Mathematical and Stochastic Growth Models

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Abstract: Many statistical and mathematical models of growth are developed in the literature and effectively applied to various conditions in the existent world involves many research problems in the different fields of applied statistics. Nevertheless, still, there are an equally a large number of conditions, which have not yet been mathematically or statistically modeled, due to the complex situations or formed models are mathematically or statistically inflexible. The present study is based on mathematical and stochastic growth models. The specification of both the growth models is depicted. A details study of newly modified growth models are mentioned. This research will give substantial information on growth models, such as proposed modified exponential growth models and it's specifications clearly motioned which gives scope for future research.

I. INTRODUCTION

Mathematical Models are classified according to their topic area. Mathematical Models in Physics (Mathematical Physics), Mathematical Models in Chemistry (Theoretical Chemistry), Mathematical Models in Medicine (Mathematical Medicine), Mathematical Models in Economics (Mathematical Economics and Econometrics), Mathematical Models in Psychology (Mathematical Psychology), Mathematical Modeling in Engineering (Mathematical Engineering), and so on.

We can also divide Mathematical Models according to the mathematical techniques using in solving them. Like Mathematical Modeling with Classical Algebra, Linear Algebra and Matrixes, and Integral Equations Mathematical modelling using integro-differential equations, as well as mathematical modelling using differential-differential equations. Mathematical modelling based on the maximal principle, that we have Mathematical modeling through ordinary and partial difference equation, etc.

II. REVIEW OF LITERATURE

It is observed that during the past decade the make use of Logistic Growth and Exponential Growth modeling have explored from their original acceptance in epidemiologic research. These growth models are now normally explored in numerous fields. They are not limited only to Biomedical Research, Business and Finance, Criminology, Ecology, Engineering, Forestry, Demography and others. At the same time there has been an identical amount of endeavor in research on all Stochastic aspects of the Exponential growth models too. It is found the Logistic Curve has certain properties which make it useful for the pragmatic representation of development phenomena. Commonly, this curve has three arbitrary constants, which correspond basically to the upper asymptote, the time origin, and the time unit or rate constant. In this curve, the degree of skewness, as measured by the relation of the ordinate at the point of inflection to the distance between asymptote is set.

III. OBJECTIVES OF THE STUDY

The central aim of this research work focused on various Mathematical and Stochastic aspects of growth curves.
The main objectives of the present study are:

1) To develop few new growth curves by using logistic growth models.
2) To describe the modeling of Stochastic Growth by using Mathematical and Stochastic aspects.

IV. TOOLS AND METHOD

A. Growth Curves And Their Applications

A number of procedures have been developed to facilitate the scholars to analyze and quantify and quantify the change. Most important analytic strategies to get the ordinal data include growth curve modes and autoregressive modes. The choice of the model for the study depends on the nature of the study and research question. One of the most important assumptions in growth curve technique is that change is scientifically related to the passage of time, at least across the time range in question. This type of data is referred to be 'balanced on-time' by (burchinl & appel bauro) ware (1985). Evaluating to what extent a particular growth mode is capable of exhibiting the observed pattern of change with respect to time appears to be an important aspect of the growth model testing.

The experience curve, often known as the 'J' curve, is a model of population expansion that occurs when there is no limit to population size. In this situation, the logistic curve, commonly known as the 'S' curve, is used to describe the impacts of limiting and carrying factors in the environment.

In statistics, growth curves are used to determine the type of growth pattern of a quantity, whether linear or cubic. A business can construct a mathematical model to anticipate future sales once the growth type has been defined. (Population overtone is an example of a country's growth curve.)

1) *Exponential Curve:* When the growth rate of a mathematical function value is proportionate to the current function value, the growth is described as an exponential function for the time being. When the growth rate is negative, exponential decay occurs in the same way; this is known as geometric growth or geometric decay. The ratio of the growth rate, the function values form a geometric progress, in either exponential growth of a variable x at the growth rate 'r', as time 't' goes on discrete internals, is

$$x_t = x_0(1+r)^t \quad (1)$$

Where x_0 is the value of x at time 0.

Exponential cure used in Physics, Finance, Biology, Computer technology, Econometrics and so on.

2) *Logistic Growth Curve:* A logistic growth curve is an s-shaped curve that can be used to describe functions that increase slowly at start, faster in the middle, and slowly towards the end, eventually reaching their maximum value over a period of time. The limited part of the curve is exponential; the rate of growth rises as it approaches the mid-point of the curve; at the mid-point (k/2), the growth slows but continues to expand until it reaches the arympitolo, 'k,' which is known as the carrying aptitude for the environment.

This type of curve is frequently used for biological growth patterns, where there is an initial phase of exponential growth, followed by a levelling down of the US/Indians, as more of the population becomes infected, or as food supplies or other circumstances limit further growth. The symmetric logistic growth function has the following shape:

$$y = k/(1+\exp(a+b*x)) \quad (2)$$

Where k, a, b are parameters that shape and scale the function. The value of 'b' is negative fits a logistic curve to the number of new cures of AIDS reported in United States during the period from 1981 to 1992. The computed function fits the data remarkably showing well that the AIDS infection rate was followed by a classical logistic curve, which was leveled off at about 47500 new cases per year in United States

In this modeling of population growth, the Logistic curve, in reality, provides good fits on material showing an inflection about midway between the growth models methodology has been extensively practiced. Since , growth of living things is normally nonlinear, it is responsible to explore the use of nonlinear growth model to population growth.

Preferably, a nonlinear growth model is selected based on hypothetical considerations from the subject matter field. Possibly, the best known category of nonlinear growth models are Logistic and Exponential growth models. These models are used to explain how something grows with changes in a regressor namely time variable. Typical applications are in biology, where plants and organisms grow with time, but there are also a lot of applications in Economics, Business, Engineering and Demography.

a) Liner Growth Models: $y_t = \alpha_0 + \alpha_1 t + \varepsilon$
b) Second Degree Polynomial Growth Models: $y_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \varepsilon$
c) *m*^th^ Degree Polynomial Growth Model: $y_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + - + \alpha_n t^n + \varepsilon$
d) Exponential Growth Models; (i) $y_t = \alpha_0 \alpha_1^t \varepsilon$

$$y_t = \alpha_0 e^{\alpha_1^t} \varepsilon$$

e) Second Degree Polynomial Growth Model: $y_t = \alpha_0 \alpha_1^t \alpha_2^{t^2} \varepsilon$

V. RESULTS AND DISCUSSIONS

A. *Specification Of A Newly Modified Exponential Growth Model*

Consider an organism like homosapiens or bacteria in a culture flask with overlapping generations and continual breeding. All ages are displayed at the same time, and the population size fluctuates gradually in small increments as individuals are born and die at any given period. The use of a differential equation to explain this continuous population growth with immediate rates determined across relatively short time intervals is useful.

If N= population size
b= instantaneous birth rate per female
d= instantaneous death rate per female
then population growth is given as

$$\frac{dN}{dt} = (b-d)N \tag{3}$$

When per capita birth and death rates are combined into a single metric, r= b-d, often known as the intrinsic rate of rise or exponential growth, the following results are obtained:

$$\frac{dN}{dt} = rN \tag{4}$$

Population increase is proportional to N and the instantaneous growth rate r, as seen in the above expression. Individuals just substitute themselves when r=0 birth and death rates are balanced, and population size remains constant. When r=0, the population shrinks toward extinction, while when r>0, it grows. To forecast future population sizes, we use the differential form of this continuous growth model

Despite the fact that r is an instantaneous rate, its numerical value can only be determined over a finite range. $N(t) = N(0)e^{n}$

(4)

If this rate remains constant, we may forecast future population size, N(t), based on the constant growth rate (r), the current population size, N (0), and the time span over which growth happens (t).

VI. CONCLUSION

The different mathematical and Stochastic growth models are developed in the literature and successfully applied to analyse the time series data concerning to a large number of situations in the actual worlds. The central aim of the modeling of growth is to envisage the future development of the situation Stochastic growth model of economy may be applied to predict the future trends and it provides a basis of policy decision.

In the current research work, an effort has been made to develop the models to depict the mathematical and Stochastic aspects of growth and draw from some new Stochastic growth models by using logistic and Exponential growth models.

First, a Logistic growth model has been specified as a number of family of generalized linear models. The concepts of odds ratio and model Deviance is defined and a method is described to estimate the odds ratio. The maximum method of estimation is discussed to estimate the parameters of the multiple logistic growth model. The validity of the integrity of fit of the logistic growth model is tested by using the model deviance.

Secondly, an Exponential Growth model is specified by using the poisson probability model for count data. Identity link function and log link function are measured under the maximum likelihood estimation of parameters of Exponential Growth model.

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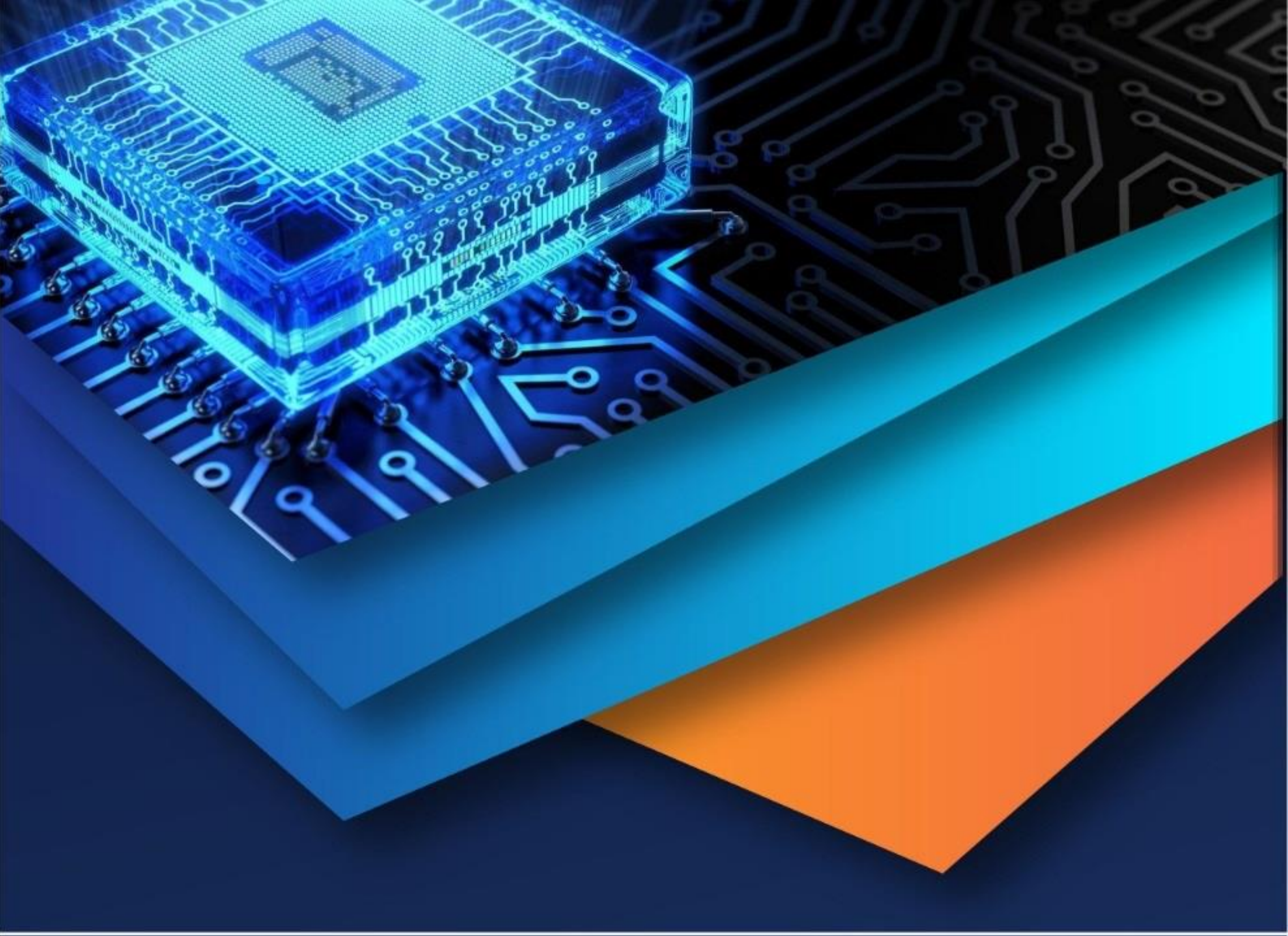

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