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Mathematical Modelling of Context-Dependent Fuzzy Decision Support Systems for Doctoral Researchers

Prof.(Dr.) Md.Mushtaque Khan
Professor & Head P.G Department of Mathematics, Jai Prakash University,Chapra,Bihar,India

Abstract: Doctoral researchers must constantly make multiple trade-offs related to data collection, writing, and supervisor requests. Choices that seem clear today may change in urgency and context based on fatigue, deadlines, and supervisor requests. This research proposes a context-dependent Pythagorean fuzzy decision support system (CD-PF-DSS) to address these challenges. The proposed decision support system employs a context-dependent softmax function that facilitates a reliability transformation and preserves Pythagorean fuzzy feasibility. Four theorems provide sufficient conditions under which feasibility is preserved, the model is bounded, dominance is consistent, and the model is stable. A sufficient condition for rank preservation is also provided. The model is demonstrated through examples and indicates that the revision of thesis chapters is the preferred action compared to the revision of thesis chapters when deadlines are relaxed, and contextual constraints are in place. The model also demonstrates that the revision of thesis chapters is less preferred compared to the revision of manuscripts when deadlines are high. It is expected that the system will provide operational support to researchers based on context and trade-off decisions.
Keywords: fuzzy decision support; Pythagorean fuzzy sets; contextual weighting; doctoral research management; multi-criteria decision-making; uncertainty modeling

I. INTRODUCTION

Forming decisions during research work tends to happen with very little information available. An academic may realize there is an upcoming deadline for a publication. Yet, they may also be uncertain of the state of the results, the amount of data they have, their supervisor's preferred project, and the toll the continued work may have on them. For these reasons, a researcher cannot create a priority list and adhere to it. What may be the correct decision this week may change dramatically next week.

Research in decision support that will help the doctoral candidate has begun to gear towards the combination of fuzzy modeling systems and artificial intelligence in which a qualitative assessment can be made with quantitative criteria [1]. In the area of fuzzy modeling, if a judgment is both in support and in opposition and is combined with some hesitation, advanced fuzzy modeling may be useful. In addition, there has been research that focuses on preserving preferences in richer environments of fuzzy modeling as well as the uncertainty within the membership functions by the use of interval type-2 and constrained type-2 frameworks [3]-[6]. The area of multi-criteria decision-making shows the importance of fuzzy modeling, non-linear weighting, and rankings, as well as an inclusion of a consistency analysis [7]-[10].

There is also the requirement that the doctoral candidate be sustainable and human. A body of research has defined the factors that affect the progress of doctoral candidates. These factors include workload, deadline pressure, the supervisory relationship, as well as the resources available, career concerns, and overall well-being [11]-[13]. Most importantly for the decision support model, when a deadine approaches, a higher weight should be given to urgency. Constraints on resources and fatigue should be considered as well; however, the weight given for sustainability should also be increased.

A CD-PF-DSS is described with three advances. First, criterion weights become functions of a context vector, meaning they are not constants. Second, contextual assessments of reliability are used without violating the Pythagorean Theorem. Third, there is a ranking score which is both uncertain and has bounds. The system can be used for situations with a discrete set of options such as selecting the next research task, choosing between different approaches, or prioritizing the order of milestones.

II. RESEARCH GAP AND DESIGN PRINCIPLES

Most fuzzy multi-criteria decision models assume criterion weights are fixed during a decision episode. This assumption is reasonable for stable procurement or engineering selection problems, but is too limiting for PhD planning. The meaning of "best next action" differs completely when a submission deadline is approaching, a lab is no longer available, a supervisor is demanding a chapter, or the researcher is overwhelmed with work. Effects of type-2 systems and constrained fuzzy systems improve the representation of uncertainty and interpretability [3]-[6], and some of the recent developments in fuzzy MCDM improve preference consistency and weighting [7]-[10]. Still, these advances do not imply how context of a PhD will change criterion weights and confidence levels.

The proposed model is built upon five principles. Context responsiveness states that weights should be updated using smooth functions as opposed to ad hoc, discontinuous updating. Feasibility preservation requires that all context transformations should be restricted to the admissible Pythagorean domain. Uncertainty honesty states that weak or stale evidence should not be converted to support or oppose decisions with increased confidence. Decision monotonicity states that no worse alternatives should remain no worse. Human governability states that as a critic, a score should be explained with a confidence value and stability, which should be subject to criticism by the supervisor.

The unit of decision has intentional flexibility. Alternatives can be tasks or sequencing of actions, design, data collection, software, or journals. Criteria can also be combined. Thus, this model has a layer of mathematics rather than a set of rules specific to a discipline. Knowledge from the discipline will enter as PFNs, activation functions, and reliability. The guarantees of analysis and aggregation will remain unchanged. This architecture is designed to support diverse research, especially in the fields of laboratory, fieldwork, computational, clinical, humanities, and social science doctoral research.

III. MATHEMATICAL PRELIMINARIES AND PROBLEM DEFINITION

Let the set of alternatives be $A = \{A_1, \dots, A_m\}$ and the criteria be $C = \{C_1, \dots, C_n\}$. For doctoral planning, representative benefit criteria are scientific value, deadline urgency, feasibility, supervisory alignment, and workload sustainability. Cost criteria can be converted to benefit form by interchanging membership and non-membership values.

A Pythagorean fuzzy number (PFN) assigned to alternative A_i under criterion C_j is

$$\tilde{p}_{ij} = (\mu_{ij}, \nu_{ij}), \qquad 0 \le \mu_{ij}, \nu_{ij} \le 1, \qquad \mu_{ij}^2 + \nu_{ij}^2 \le 1. \tag{1}$$

Here, μ_{ij} measures support, ν_{ij} measures opposition, and the hesitation degree is

$$\pi_{ij} = \sqrt{1 - \mu_{ij}^2 - \nu_{ij}^2}. \tag{2}$$

The current doctoral situation is represented by a normalised context vector

$$\mathbf{x} = (d, r, s, f, c)^{\top} \in [0,1]^5, \tag{3}$$

where d is deadline pressure, r is resource constraint, s is supervisory demand, f is fatigue or workload strain, and c is career or publication pressure. Additional dimensions may be appended without changing the model.

IV. CONTEXT-DEPENDENT FUZZY DECISION MODEL

A. Contextual Criterion Weights

Let $b_j > 0$ be the baseline importance of criterion C_j, $\lambda_j \ge 0$ its context sensitivity, and $\phi_j(\mathbf{x}) \in [0,1]$ a criterion-specific activation function. The context-dependent weight is

$$w_j(\mathbf{x}) = \frac{b_j \exp\{\lambda_j \phi_j(\mathbf{x})\}}{\sum_{k=1}^{n} b_k \exp\{\lambda_k \phi_k(\mathbf{x})\}}. \tag{4}$$

The exponential form permits smooth reallocation of importance. For example, $\phi_{\text{urgency}}(\mathbf{x}) = d$, $\phi_{\text{sustainability}}(\mathbf{x}) = f$, and $\phi_{\text{feasibility}}(\mathbf{x}) = 0.6r + 0.4f$.

B. Contextual Reliability Transformation

Let $\rho_{ij}(\mathbf{x}) \in [0,1]$ denote the reliability or contextual compatibility of assessment $\tilde{p}_{ij}$. A logistic specification can be used:

$$\rho_{ij}(\mathbf{x}) = \frac{1}{1+\exp\left[-\left(a_{ij} + \mathbf{q}_{ij}^{\top}\mathbf{x}\right)\right]}. \tag{5}$$

The context-adjusted PFN is defined by

$$\tilde{p}_{ij}(\mathbf{x}) = \left(\sqrt{\rho_{ij}(\mathbf{x})}\,\mu_{ij}, \sqrt{\rho_{ij}(\mathbf{x})}\,\nu_{ij}\right). \tag{6}$$

Low reliability contracts both support and opposition, transferring mass into hesitation rather than falsely strengthening either side.

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C. Uncertainty-Penalised Utility

For $0 \le \eta \le 1/2$, define the criterion utility

$$q_{ij}(\mathbf{x}) = \frac{1+\rho_{ij}(\mathbf{x})\left(\mu_{ij}^2-\nu_{ij}^2\right)}{2} - \eta\left[1-\rho_{ij}(\mathbf{x})\left(\mu_{ij}^2+\nu_{ij}^2\right)\right]. \quad (7)$$

The first term is a normalised Pythagorean score. The second subtracts a penalty for contextual hesitation. The overall score and confidence are

$$S_i(\mathbf{x}) = \sum_{j=1}^{n} w_j(\mathbf{x}) q_{ij}(\mathbf{x}), \quad (8)$$

$$H_i(\mathbf{x}) = 1-\sum_{j=1}^{n} w_j(\mathbf{x})\sqrt{1-\rho_{ij}(\mathbf{x})\left(\mu_{ij}^2+\nu_{ij}^2\right)}. \quad (9)$$

Alternatives are ranked primarily by S_i and, when scores are equal within a chosen tolerance, by the larger H_i.

V. ANALYTICAL PROPERTIES

Theorem 1 (Pythagorean feasibility preservation). For every valid PFN $\tilde{p}_{ij}$ and every $\rho_{ij}(\mathrm{x}) \in [0,1]$, the transformed value in (6) is a valid PFN.

Proof. From (1), $\mu_{ij}^2 + \nu_{ij}^2 \le 1$. Therefore,

$$\left(\sqrt{\rho}\mu_{ij}\right)^2 + \left(\sqrt{\rho}\nu_{ij}\right)^2 = \rho\left(\mu_{ij}^2+\nu_{ij}^2\right) \le \rho \le 1.$$

Both transformed components remain in $[0,1]$; hence the Pythagorean condition is preserved. ▫

Theorem 2 (Normalisation and bounded score). The weights in (4) satisfy $w_j(\mathrm{x}) > 0$ and $\sum_j w_j(\mathrm{x}) = 1$. Moreover, for $0 \le \eta \le 1/2$, $0 \le q_{ij}(\mathrm{x}) \le 1$ and $0 \le S_i(\mathrm{x}) \le 1$.

Proof. Positivity follows from $b_j > 0$ and the exponential function; division by their sum gives unit normalisation. Put $a = \mu_{ij}^2$, $b = \nu_{ij}^2$, and $\rho = \rho_{ij}(\mathrm{x})$. Equation (7) becomes

$$q = 1/2 - \eta + \rho[(1/2+\eta)a + (\eta - 1/2)b].$$

Since $a, b \ge 0$, $a + b \le 1$, and $0 \le \rho \le 1$, the minimum occurs at $a = 0, b = 1, \rho = 1$, giving $q = 0$; the maximum occurs at $a = 1, b = 0, \rho = 1$, giving $q = 1$. Equation (8) is a convex combination of values in $[0,1]$. ▫

Theorem 3 (Contextual dominance consistency). If $q_{aj}(\mathrm{x}) \ge q_{bj}(\mathrm{x})$ for every criterion j, then $S_a(\mathrm{x}) \ge S_b(\mathrm{x})$. Strict inequality in at least one criterion implies $S_a > S_b$.

Proof. Subtracting the two scores gives

$$S_a - S_b = \sum_{j=1}^{n} w_j(\mathbf{x})\left[q_{aj}(\mathbf{x}) - q_{bj}(\mathbf{x})\right].$$

Every term is non-negative because $w_j > 0$. If at least one utility difference is positive, the corresponding weighted term is positive. ▫

Theorem 4 (Lipschitz stability). Assume $\theta_j(\mathrm{x}) = \log b_j + \lambda_j \phi_j(\mathrm{x})$ is L_θ-Lipschitz and every $\rho_{ij}(\mathrm{x})$ is L_ρ-Lipschitz. Then

$$|S_i(\mathrm{x}) - S_i(\mathrm{y})| \le L_S \parallel \mathrm{x}-\mathrm{y} \parallel, \qquad L_S = 2L_\theta + (1/2+\eta)L_\rho. \quad (10)$$

Proof. Softmax differentiation gives $\nabla w_j = w_j\left(\nabla\theta_j - \sum_k w_k \nabla\theta_k\right)$; hence $\sum_j \left|w_j(\mathrm{x}) - w_j(\mathrm{y})\right| \le 2L_\theta \parallel \mathrm{x}-\mathrm{y} \parallel$. From (7), the coefficient multiplying ρ has absolute value no greater than $1/2+\eta$, so $\left|q_{ij}(\mathrm{x}) - q_{ij}(\mathrm{y})\right| \le (1/2+\eta)L_\rho \parallel \mathrm{x}-\mathrm{y} \parallel$. Combining both bounds with $0 \le q_{ij} \le 1$ yields (10). ▫

Corollary 1 (Rank robustness). Let $\Delta_{ab}(\mathrm{x}) = S_a(\mathrm{x}) - S_b(\mathrm{x}) > 0$. The order $A_a \succ A_b$ is preserved for every perturbation δ satisfying $\parallel \delta \parallel < \Delta_{ab}(\mathrm{x})/(2L_S)$.

This result converts a score margin into an interpretable stability radius. Small margins should trigger supervisory review rather than automatic selection.

VI. DECISION ALGORITHM

Algorithm 1: CD-PF-DSS ranking procedure

1) Specify alternatives, benefit-oriented criteria, baseline weights □□, and context activations □□.
2) Elicit PFNs (□□□, □□□) from the researcher, supervisor, or a consensus process.
3) Record the current context vector □ and compute □□(□) using (4).
4) Estimate contextual reliabilities □□□(□) using (5) or a validated rule base.

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5) Compute $□_{□□}$, $□_{□}$, and $□_{□}$ from (7)-(9).
6) Rank alternatives by descending $□_{□}$; use $□_{□}$ as a tie-breaker.
7) Report the leading score margin and conduct a one-at-a-time or local sensitivity analysis.

The computational cost is $□(□□)$ after context functions are evaluated, making the method suitable for frequent use.

VII. CALIBRATED NUMERICAL ILLUSTRATION

Four actions are considered: $□_1$ complete data collection, $□_2$ write a journal manuscript, $□_3$ attend a methodology workshop, and $□_4$ revise a thesis chapter. The criteria are scientific value, urgency, feasibility, supervisory alignment, and sustainability. Baseline weights are $(0.26, 0.22, 0.20, 0.17, 0.15)$ and $□ = 0.20$.

Alternative	Scientific value	Urgency	Feasibility	Alignment	Sustainability
A_1 Data collection	(0.80, 0.30)	(0.70, 0.45)	(0.75, 0.35)	(0.78, 0.30)	(0.55, 0.60)
A_2 Journal manuscript	(0.88, 0.25)	(0.90, 0.20)	(0.70, 0.45)	(0.85, 0.25)	(0.60, 0.55)
A_3 Methodology workshop	(0.68, 0.45)	(0.45, 0.70)	(0.80, 0.30)	(0.65, 0.50)	(0.78, 0.30)
A_4 Thesis revision	(0.76, 0.35)	(0.72, 0.42)	(0.82, 0.28)	(0.90, 0.20)	(0.74, 0.38)

In Context I, deadline and publication pressure are high: $□^{(1)} = (0.90, 0.20, 0.70, 0.50, 0.80)$. Equation (4) gives weights $(0.262, 0.330, 0.132, 0.149, 0.127)$. The action reliabilities are $(0.75, 0.95, 0.70, 0.88)$. In Context II, resource constraint and fatigue dominate: $□^{(2)} = (0.40, 0.90, 0.60, 0.85, 0.40)$, producing weights $(0.195, 0.186, 0.264, 0.146, 0.209)$ and reliabilities $(0.30, 0.65, 0.55, 0.95)$.

Alternative	Score, Context I	Confidence, Context I	Score, Context II	Confidence, Context II
A_1 Data collection	0.542	0.310	0.393	0.110
A_2 Journal manuscript	0.728	0.507	0.550	0.288
A_3 Methodology workshop	0.444	0.280	0.452	0.214
A_4 Thesis revision	0.637	0.401	0.680	0.451

The transition from drafting manuscripts to revising theses suggests that this change is not about Turbulence. It demonstrates the response to a given external situation. The first external situation had some value for publication. The situation we respond to now has value for both feasibility and sustainability and radically decreases the reliance on data collection. The confidence levels also demonstrate that A_1 is weakly justified in Context II.

VIII. DISCUSSION

The model differs from a weighted sum in two ways. The importance of criteria can change with respect to the Doctoral case as can the Evidential Reliability of Alternatives that are independent of their original Fuzziness.

The model is also auditable. Each recommendation is traceable to baseline weights, context activation, PFNs, Reliability and the Uncertainty Penalty. Theorems 1-4 ensure that the model is incapable of producing invalid fuzzy states, does not result in unbounded outputs, does not demonstrate dominance reversals, and is not overly sensitive to the assumptions of regularity. It should be noted that in order for the model to conduct expertise within the bounds of the data, there should be a balance of past and future data with care not to interpret a lack of surveillance data as a state of being, with an ethics of care. This is especially valid when rank margins are narrow and potential mercy is lacking.

A. Calibration, Validation and Governance

Rapid protocols form preliminary operational deployments. The researcher and supervisor iteratively examine a limited number of options and identify between five and seven criteria. Criteria can also be weighted using several methods: a direct allocation, fuzzy best-worst, or context sensitivity, if, for example, a retrospective decision is made weekly. PFNs (Polarity Framework Numbers) can be defined verbally (e.g., “strongly supported with low opposition” can be defined to have a high level of membership and a low level of non-membership). Reliability can be evaluated by the recency of the data, the extent the resource has been confirmed, satisfiability, and the level of agreement among assessors.

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Focus on the future when doing the validation. The system will collect recommendations for a number of planning cycles, even though they will not be enforced. After that, researchers will be able to study and compare what they believe should be the highest priority and what was actually prioritized, along with what was achieved, how much work was done repeatedly, and whether deadlines were met, along with what was believed to be the workload. Researchers should keep in mind that coming to an agreement on what is considered the highest priority may not be sufficient. A good system may offer other options that were ignored. Hence, calibration should consider other aspects, for instance, whether recommendations are understood, and if the scores offer a distinction among options. When the model indicates a tendency to consistently weigh some aspects that are more important above others (e.g., the importance of work to be done in a timely manner vs. the importance of work to be done in a sustained manner), these aspects should be adjusted.

Governance controls should also be put into place. In most cases, self-reporting of context entries will be required (e.g., what was accomplished). There should not be secretive methods to monitor productivity or what is said, written, etc. The security should not be used to evaluate performance or to give evidence for disciplinary actions. Users should be given the option to explain why they are not following an advice, and to make changes to the rule base. This should help ensure that the model remains a deliberative tool, and that they will be able to override the advice.

IX. CONCLUSION

This study developed a context-dependent pythagorean fuzzy decision support system for doctoral researchers. This system converts contexts of deadlines, resources, supervision, workloads, and careers into criterion weights and reliability adjusted fuzzy evaluations. A research priority framework is possible based on the bounded score, a preference measure, formal theorems, the robustness condition, and linear computational complexity. The focus of future studies could be on estimating context functions from longitudinal doctoral planning data; assessing and comparing choices of different fuzzy structures; and evaluating the improved models to attain better completion, with no increase in the same, or less researcher autonomy.

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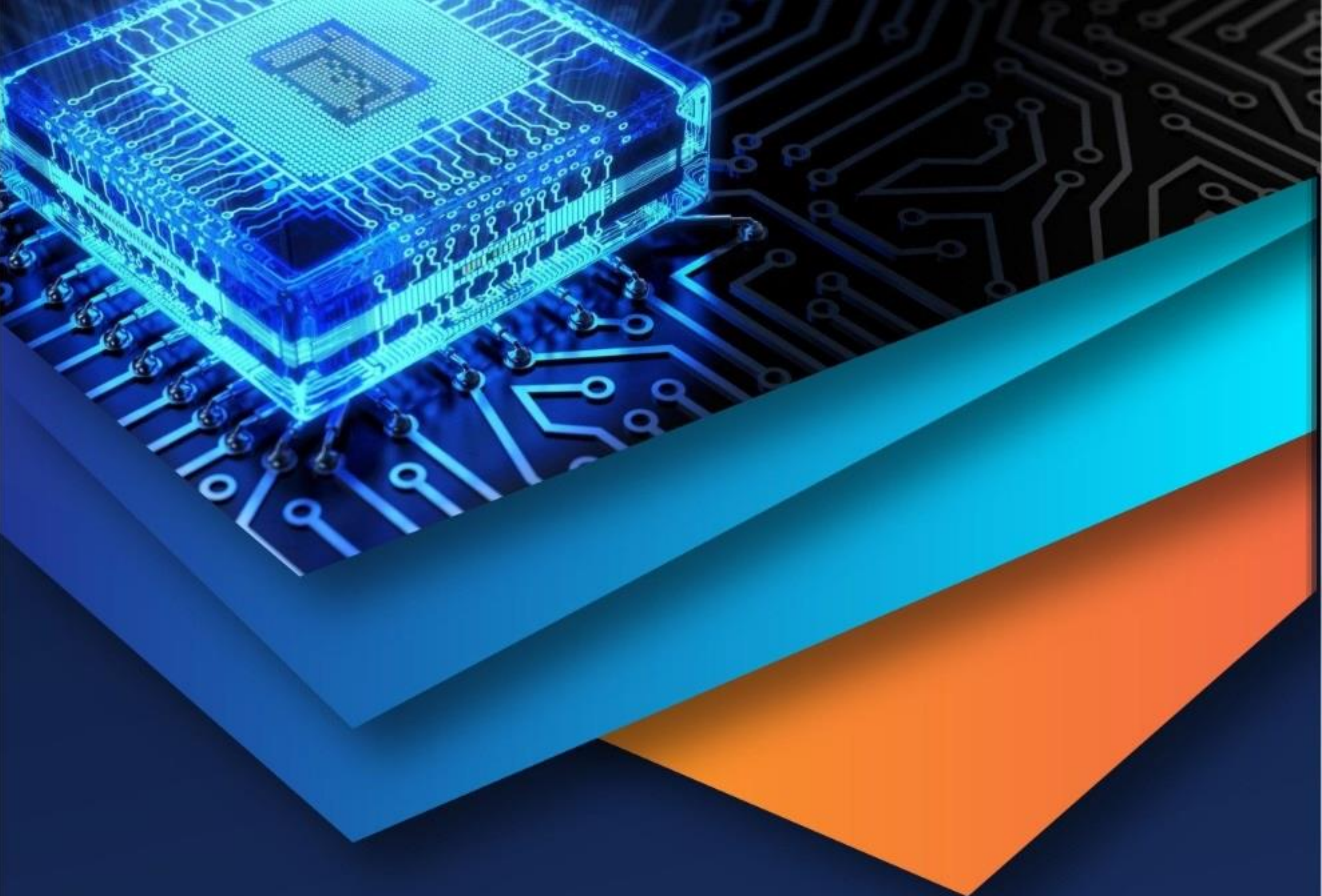

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