iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 **Issue:** IV **Month of publication:** April 2023

DOI: https://doi.org/10.22214/ijraset.2023.50874

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue IV Apr 2023- Available at www.ijraset.com

Mathematical Modelling of Linear Induction Motor Considering End Effects in DQ Frame

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Abstract: In this paper, a mathematical model based on d-q axes equivalent circuit to describe the dynamic behaviour of Linear Induction Motor (LIM) considering the end effects. Because time-varying characteristics like the end effect, core saturation, and half-filled slots make dynamic modelling challenging, we can simplify the problem by employing two axes modelling which reduces the number of variables. The model is simulated using Synchronously rotating reference frame. The model is simulated using MATLAB/ SIMULINK tools as it is the prominent way for implementing the dynamic equations using the function blocks.
Keywords: Linear Induction Motor, Synchronously Rotating Reference Frame, end effects.

NOMENCLATURE

Symbols	Description
V	Voltage (V)
C	Current (A)
Vdp, Vqp	Primary voltage in the d-q axes(V)
Vdr,Vqr	Rotor voltage in the d-q axes (V)
idp,iqp	d-q axis primary current (A)
idr ,iqr	d-q axis rotor current (A)
λdp,λqp	d-q axis primary flux linkages
λdr,λqr	d-q axis rotor flux linkages
Rp,Rr	primary and rotor resistance (ohm)
Lls,Llr	primary and rotor leakage inductance(H)
Lm	magnetizing or mutual inductance (H)
Ls,Lr	primary and rotor self-inductances (H)
P	no. of poles
τ	pole pitch (m)
D	length of the rotor (m)
Q	factor associated with rotor length
v	velocity (m/s)
ω	primary angular velocity (rad/sec)
ωr	rotor angular velocity (rad/sec)

I. INTRODUCTION

Linear Induction Motor produces a rectilinear motion and are widely used for automated systems in industries. The dynamic modelling of these motors is not easier as the presence of end effects leads to complex electromagnetics. The necessity of this kind of modelling is to understand the transient and steady state behaviour of the motors better.

The model for LIM with and without end effects is simulated using synchronously rotating reference frame and results are compared, this helps to understand effect of air gap which causes end effects. The two-axis modelling of LIM is needed to avoid the time varying nature of inductances which leads to complexity in the modelling process. As the number of variables in the dynamic equations is reduced in two-axis modelling, the hybrid computers help in obtaining the solutions easily.

For control purposes, system variables in DC quantities are preferred compared to the available sinusoidal variables. As the operating point is decided by the DC values, it is much easier to model small signal equations compared to non-linear equations. The equivalence in the MMF values of three phase and two-phase quantities has to be maintained while transformation is done using Park's transformation.

For example, if there are Ns turns per phase in three phase winding, then 3Ns/2 turns are needed in two phase winding for MMF equality. A new variable called Zero sequence components is introduced for the inversion of Park's transformation and for handling the unbalanced voltages.

II. END EFFECTS

When the primary moves, a new flux is always generated at the primary entry side, while at the exit side flux will be disappeared. There will be a rapid generation and disappearance of the magnetic lines which produce statically induced currents in the secondary sheet. The air gap flux is affected by the eddy currents. With the increase of speed of the primary, the losses, and the flux-profile become sever this is called End-Effect in LIM. If velocity increases, primary's length decreases this increases end effect which causes reduction of magnetization currents of LIM. For zero velocity the length of the primary is considered as infinite to reduce the end effects.

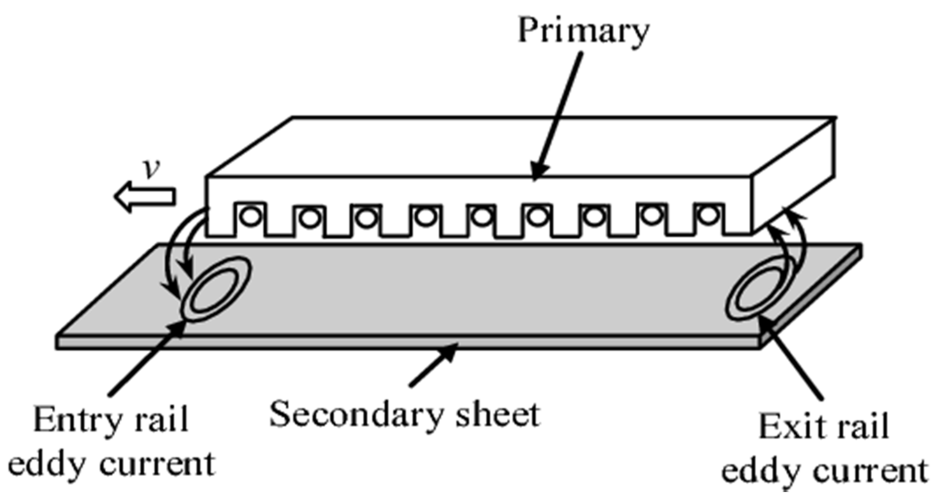

Fig 1: entry and exit rail eddy current.

III. DYNAMIC MODEL OF LIM.

A. D-Q Axes Equivalent Circuit

During the start of the LIM and changes in the load conditions, large currents are drawn by the motor which causes increase in oscillations, voltage dips and injection of harmonics on the supply side. The d-q axis equivalent circuit of LIM with end effects are shown in "Fig. 1 and Fig. 2". In synchronously rotating reference frame, ω is considered as 314 rad/sec.

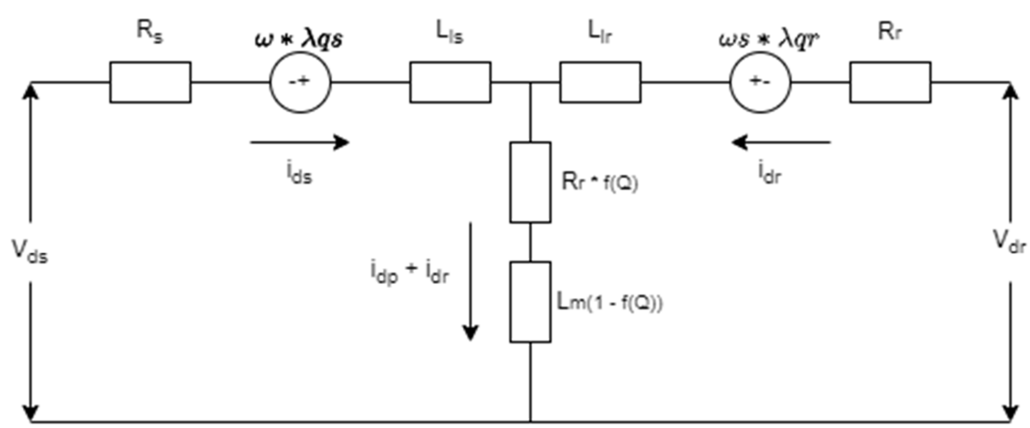

Fig.1 : D-Axis Equivalent Circuit

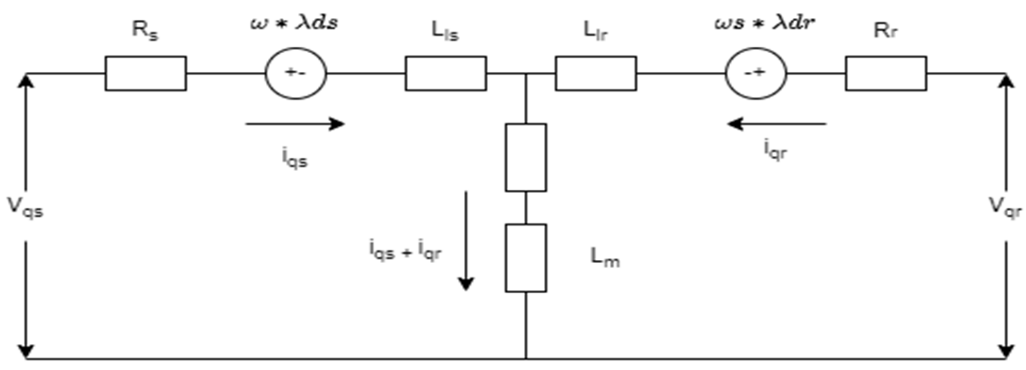

Fig.2 : Q-Axis Equivalent Circuit

B. Equations

The d-q axes primary and the rotor voltage for LIM with end effects from "(1) to (4)" is obtained by applying KVL for the circuits in Fig. 1 and Fig. 2. For modelling of LIM without end effects the term f(Q) is considered as zero.

$V_{dp} = Rp*I_{dp} + R_r*f(Q)*(I_{dp} + I_{dr}) + ldp' - \omega lqp$ (1)

$V_{qp} = Rp*I_{qp} + lqp' + \omega * ldp$ (2)

$V_{dr} = R_r*I_{dr} + R_r *f(Q)(I_{dp} + I_{dr}) + ldr'-(\omega - \omega_r) * lqr$ (3)

$V_{qr} = R_r*I_{qr} + lqr' + (- \omega_r) ldr$ (4)

But for a singly-fed machine, such as a cage motor, $V_{dr} = V_{qr} = 0$

The term f(Q) in the above equations is expressed as

$$f(Q) = \frac{1-e^{-Q}}{Q} \quad (5)$$

The Q factor associated with the length of the rotor is given by "(6)". The velocity (v) of the rotor is inversely proportional to the length of the rotor. Initially when the velocity is zero, the rotor length is considered as infinite and thus the end effects doesn't come into existence. With the increase in velocity, the rotor length decreases and the end effects become obvious. Thus, the Q factor quantifies the end effects to some extent.

$$Q = \frac{D*R_r}{(L_m+L_{lr})*v} \quad (6)$$

The primary and the rotor flux linkages in the d-q axes are given by

$ldp = L_{lp}*I_{dp} + Lm * (I_{dp} + I_{dr})$ (7)

$lqp = L_{lp}*I_{qp} + Lm' * (I_{qp} + I_{qr})$ (8)

$ldr = L_{lr}*I_{dr} + Lm * (I_{dp} + I_{dr})$ (9)

$lqr = L_{lp}*I_{dr} + Lm' * (I_{qp} + I_{qr})$ (10)

Leakage Inductance + Mutual Inductance = Self Inductance

Llp + Lm = Lp (11)

Llr + Lm = Lr (12)

The thrust obtained is given by

$F = \frac{3\pi p}{4\tau} * (\text{ldp.Iqp - lqp,Idp})$ (13)

Substituting flux equations (7), (8), (9), (10) in voltage equations (1), (2), (3), (4) respectively we get

$Idp' = \frac{1}{(\mathbf{Llp+Lm'})}$ * (Vdp – Idp(Rp + Rr. f(Q)) – Idl(Rr.f(Q)) – Idl'(Lm') + Iqp(ω(Lm+Llp)) + Iql(ω.Lm))

$Idr' = \frac{1}{(\mathbf{Llr+Lm'})}$ * (Vdr – Idr(Rr + Rr.f(Q)) – Idp (Rr.f(Q)) – Idp'(Lm')+ Iqr(ω- ωr).(Llr +Lm)+ Iqp(Lm(ω-ωr)))

$Iqp' = \frac{1}{(\mathbf{Llp+Lm})}$ * (Vqp – Iqp(Rp) – Iql'(Lm) - Idp(ω(Lm+Llp)) - Idl(ω.Lm))

$Iqr' = \frac{1}{(\mathbf{Llr+Lm})}$ * (Vqr – Iqr(Rr) – Iqp'(Lm) - Idr((ω- ωr)*(Lm+Llp)) - Idp((ω − ωr).Lm'))

For modelling of LIM considering end effects the total thrust (F) is given by

$F = \frac{3\pi p * Lm\,(1-f(Q)}{2\tau 2 * Llr + Lm(1-f(Q))} * (\text{ldr*Iqp} - \frac{Llr\,(f(Q)}{Lr\,(1-f(Q)} * \text{Idp.Iql})$ (18)

IV. 3- PHASE TO 2- PHASE TRANSFORMATION

By Clark's transformation the input 3-phase voltage quantities Va, Vb and Vc are converted to Vα, Vβ in stationary reference frame. Then Vα, Vβ are converted into rotating reference frame as 2-phase voltage quantities Vd and Vq using Park's conversion.

$V_\alpha = 2/3*[Va - 0.5(Vb + Vc)]$

$V_\beta = 2/3*[\sqrt{3/2} * \{Vb - Vc\}]$

$Vd = V_\alpha * \cos\rho + V_\beta * \sin\rho$

$Vq = V_\beta * \cos\rho - V_\alpha * \sin\rho$

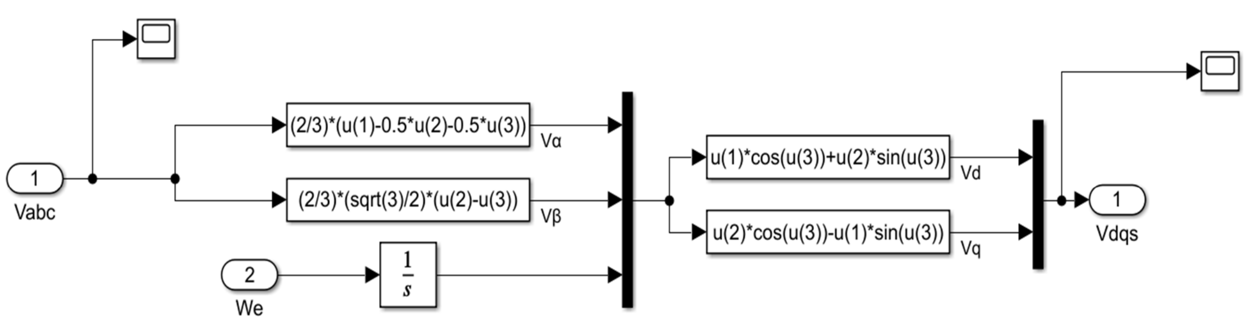

Fig 3. Vabc to Vdq for stator convertion

V. SIMULATION MODEL OF LIM.

The simulation models of LIM with and without considering end effects using synchronously rotating reference frame where ω_s is considered as 314 rad/s are shown in Fig. 3 and Fig. 4. This d-q axes model requires d-q axes input voltages namely, Vd and Vq. This can be

obtained by Clark's and Park's transformation. Stator and Rotor flux are obtained and rotor and stator currents. A function block is used for finding f(Q), but for without end effects f(Q) is considered as zero. The subsystem of LIM simulation model with end effect using synchronously rotating reference frame is shown which gives force and speed of the motor as the result.

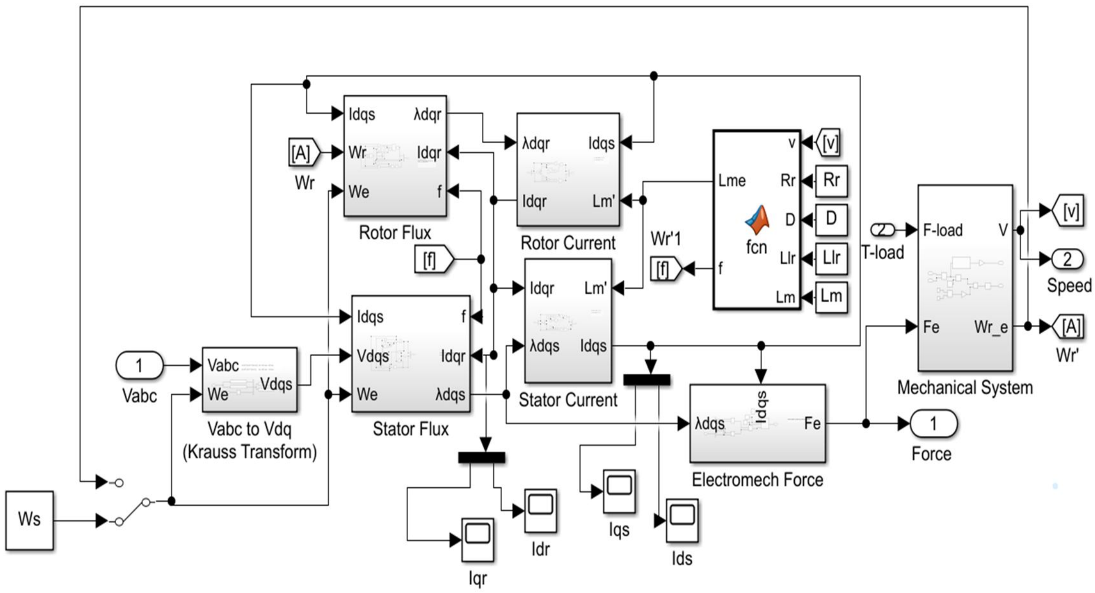

fIg. 4 LIM Simulation Model Using Synchronously rotating reference frame.

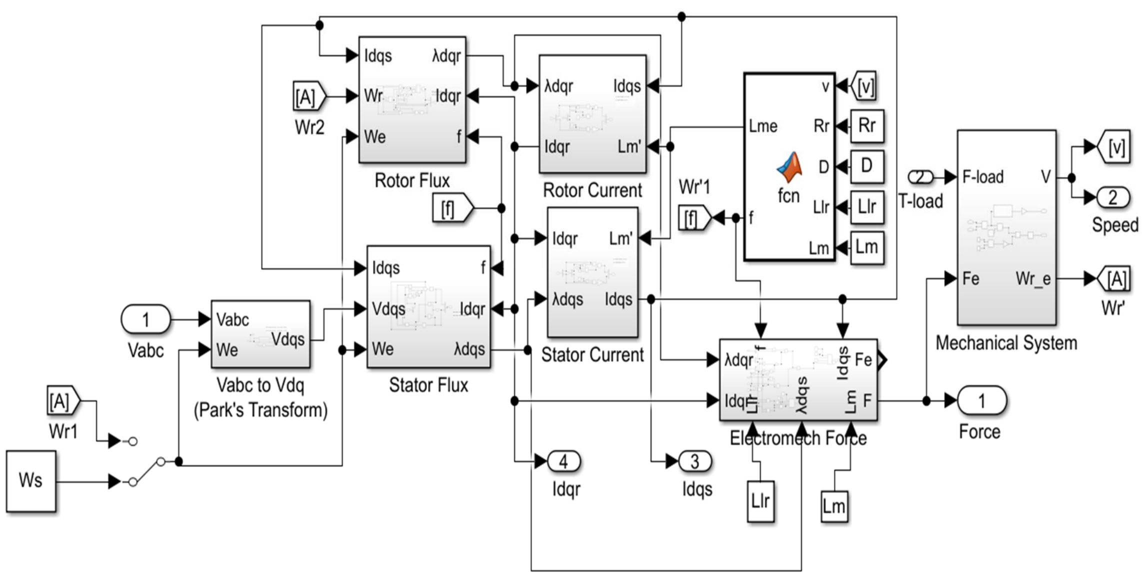

Fig. 5 LIM Simulation Model Using Synchronously rotating reference frame considering end effects.

Where Lm' = Lm*(1- f(Q))

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The parameters for modelling of LIM are given in table 1:

Parameters	Values
Ws	314 rad/s
Vo	220 V
Length of the primary	1 m
Length of the Rotor (D)	0.25 m
Primary Resistance (Rs)	1.298 ohm
Rotor Resistance (Rr)	0.976 ohm
Primary Inductance (Ls)	0.0684 H
Rotor Inductance (Lr)	0.0416 H
Mutual Inductance (Lm)	0.0412 H
No. of slots (s)	8
No. of poles (p)	2
Mass (M)	15 Kg
Moment of Inertia (J)	0.00247 Kg/ m2
Pole pitch	0.027

Table 1: parameters involved in LIM

VI. SIMULATION RESULTS

By modelling the linear induction motor and taking end effects into account, the transient behaviour of the motor was observed when a 30N load force was applied over the course of
1 sec. The waveforms of the linear induction motor's speed, force, thrust, currents, and flux connections are displayed below. In LIM, the existence of an end effect is what causes the oscillations in the thrust force.

A. *Speed Waveform*

Considering the motor with end effects, the study state is attained after 0.6 sec. When the motor is at steady state it runs with a speed of 6 m/s, suddenly when a 30N load force is applied at a time period of 1 sec the speed reduces from 6 m/s to 5.9 m/s.
But when we compare it by not considering the end effects, we can observe that the steady state is attained after 0.2 sec which is much quicker. This change is due to air flux which causes end effects in Linear Induction Motor.

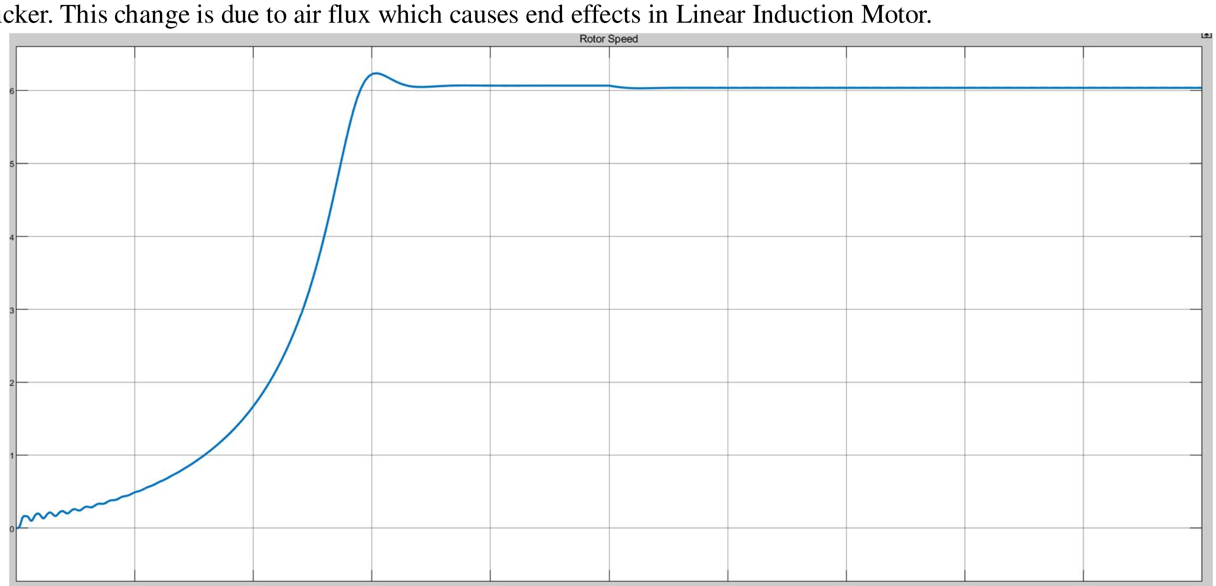

Fig 6. Speed waveform of LIM considering end effects

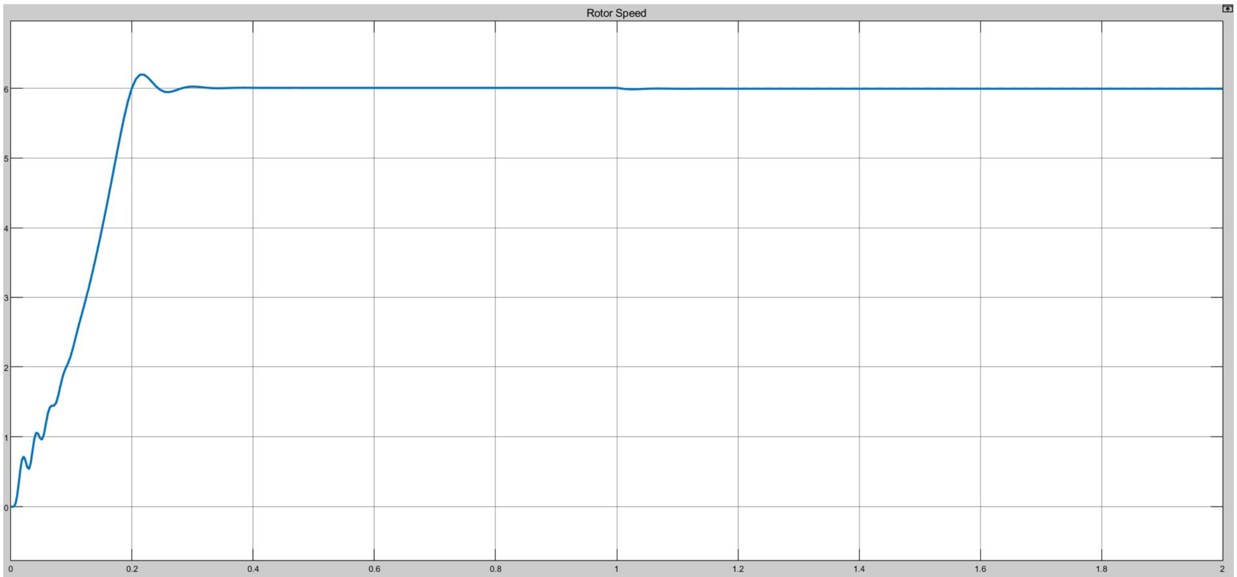

Fig 7. Speed waveform of LIM without considering end effects

B. Thrust Waveforms

The initial thrust is 1100 N for LIM model without end effect and around 500 N for LIM model with end effect. This reduction is due to the attenuation thrust caused due to end effects. Once the rated velocity is reached by the motor which is attained quicker when end effects are not considered, the thrust becomes almost equal to zero. At Is, due to the load thrust change of 30N, the velocity reduces and the thrust increases slightly.

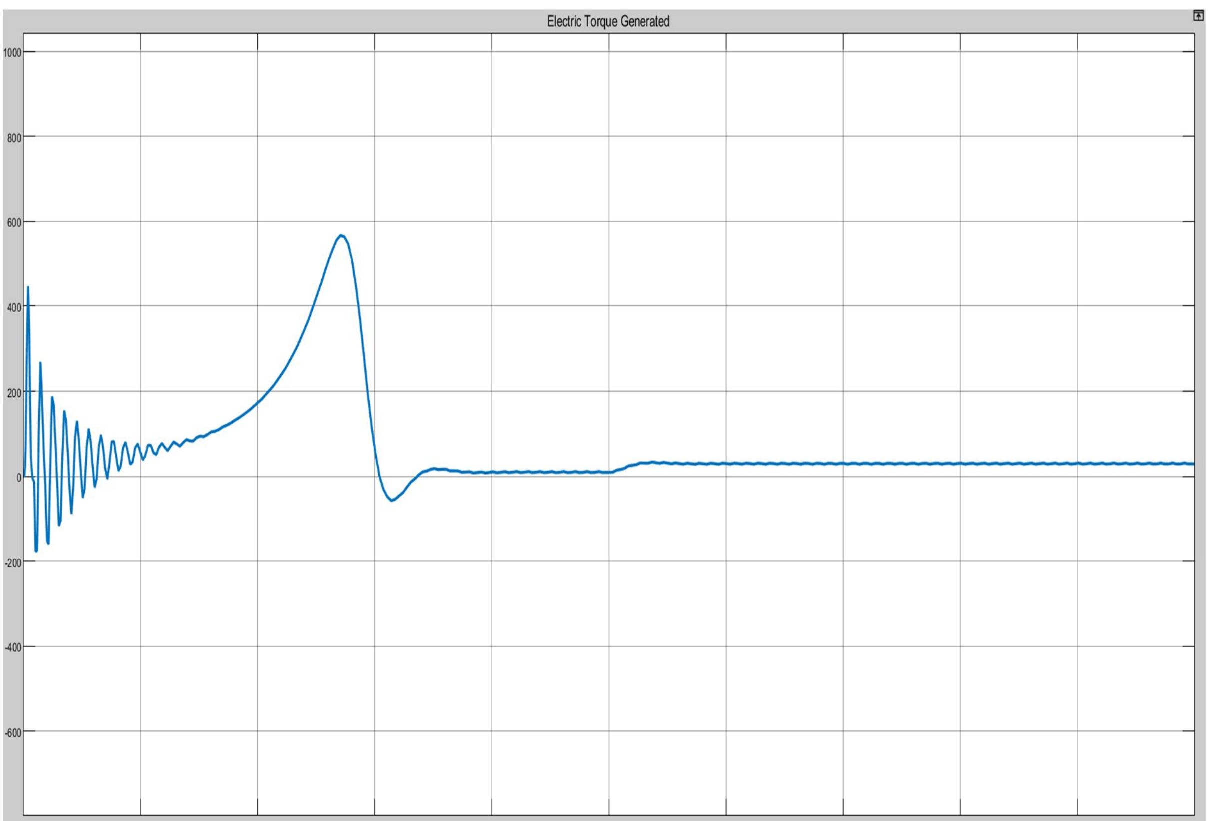

Fig. 8. Thrust waveforms of LIM considering end effects

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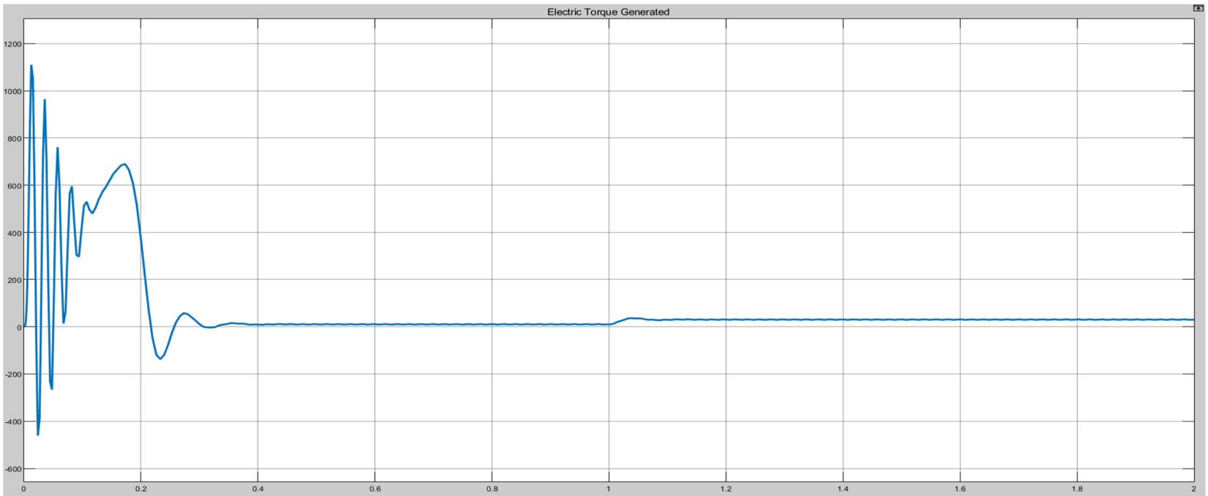

Fig. 9. Thrust waveforms of LIM without considering end effects

C. Current Waveforms

1) Idqs Current

Even in the current wave forms of stator we can observe that steady state is attained quicker when end effects are not considered and when the load is applied after 1 sec, we can see change in current only in d axes but not in q axes.

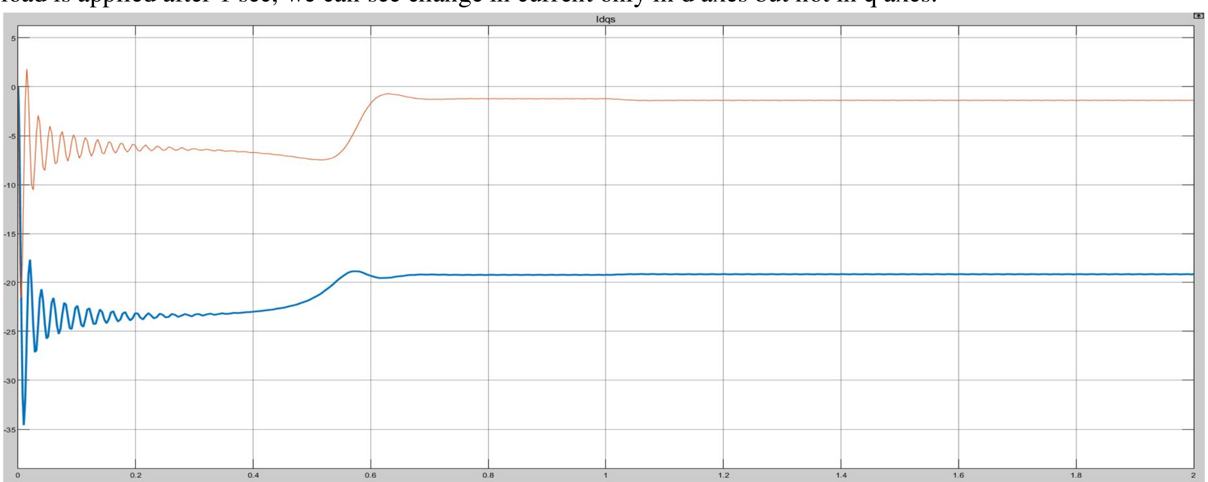

Fig. 10. Current waveform of stator in d-q frame considering end effects

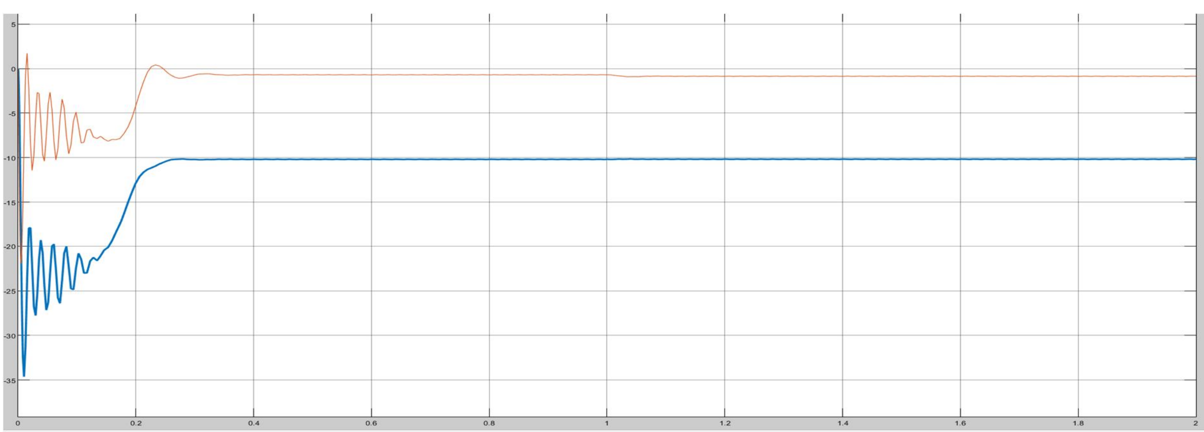

Fig. 11. Current waveform of stator in d-q frame without considering end effects

2) Idqr Current

Even in the current wave forms of rotor we can observe that steady state is attained quicker when end effects are not considered and when the load is applied after 1 sec, we can see change in current only in d axes but not in q axes.

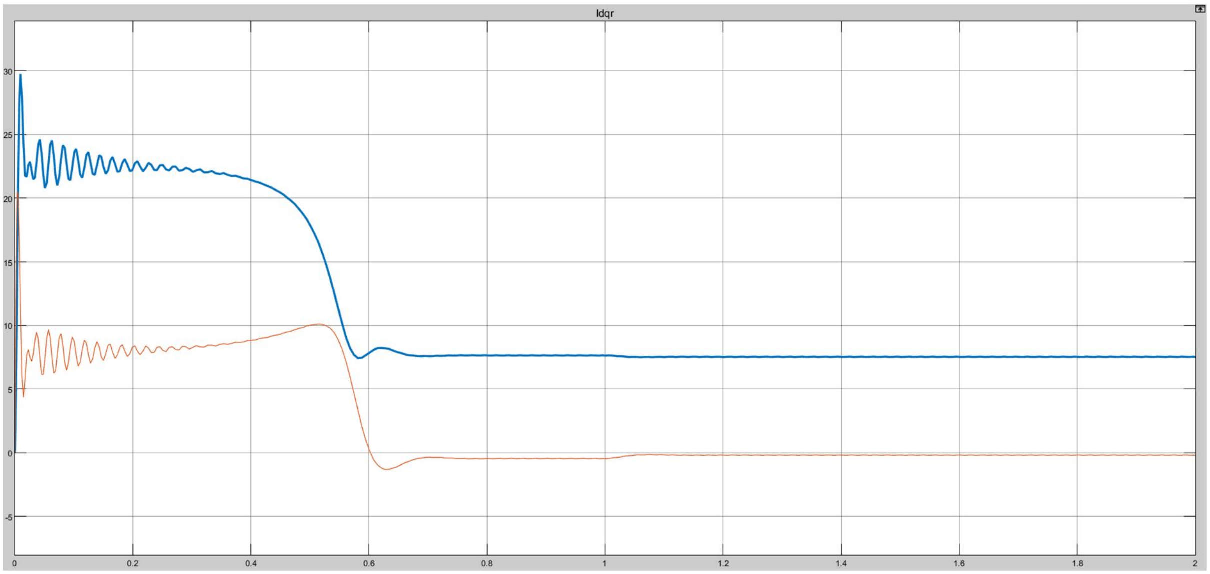

Fig. 12. Current waveform of rotor in d-q frame considering end effects

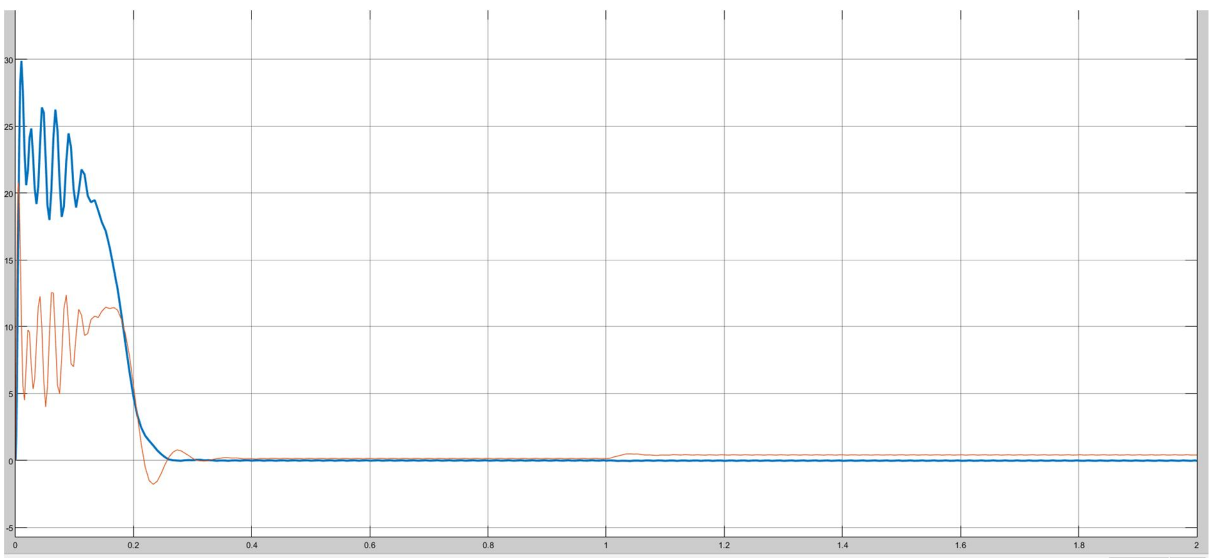

Fig. 13. Current waveform of rotor in d-q frame without considering end effects

VII. CONCLUSION

In this paper, the transient and steady state behaviour of the LIM with and without considering end effects using synchronously rotating reference frame are compared. From the results obtained it is concluded that:

1) In Linear Induction motor, there is attenuation in original thrust (force) due to the end effects.
2) Steady state is delayed to the end effects which are caused because of large air gap.
3) At steady state, the d-q axes primary and the rotor variables become DC quantities using a synchronously rotating reference frame. This reference frame is advantageous for studying multi-machine system.
4) The thrust force behaviour without involvement of the end effects does not have oscillations.

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