iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 12 Issue: III Month of publication: March 2024

DOI: https://doi.org/10.22214/ijraset.2024.58966

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 12 Issue III Mar 2024- Available at www.ijraset.com

Mathematical Modelling of Non-Stop Heron's Fountain and its Practical Applications

S. Ali[1], S.M. Zain[2], M. Tariq Shah[3]
National University of Science and Technology, Islamabad, 24090, Pakistan

***Abstract:** Heron's fountain is one of the brilliant ideas given by Heron (Mathematician and Physicist) in which we get free kinetic energy to run our fountain.*

The schematic diagrams of the physical demonstration of the magic fountain are given in this paper. The fundamental work principles of magic fountain are hydraulics and Pneumatics. In engineering, it is important to analyze a system by developing mathematical models, so we used the principle of fluid mechanics and Bernoulli's theorem to develop the equation for the velocity of a free jet. It is a very important thing in engineering to make use of energy, The free jet having kinetic can used for energy production purposes.

In this paper, Mathematical Modelling and some fundamental applications of the non-stop fountain have been discussed. Heron's fountain due to its perpetual nature has thrilling applications in the micro hydropower sector. Moreover, it has keen value in environmental sustainability and air pollution. The contribution of the fountain cannot be underestimated towards the beauty of buildings and relaxation of mind.

***Keywords:** Free jet velocity, Bernoulli's Theorem, Magic Fountain, Hydropower Generation, Pneumatics, RAM Pump*

I. INTRODUCTION

The Non-Stop fountain concept was first given by a Mathematician and Physicist named Heron of Alexandria in the first century AD. Heron's Fountain is an amazing mechanical device involving the principal of hydraulics and pneumatics. The basic schematic diagram is shown in Fig. 1. Mathematics is the beauty of science and if we observe the non-stop fountain with an engineering eye it needs mathematical modeling.

In this paper, we briefly derived the equations for pressure differences, Velocity of the resulting out jet. With the help of these equations, we can critically analyze the effect of one constraint on another one. Basically, the main principle in the heron's fountain is the principle of conservation of energy and the gravitational potential energy is converted into fluid kinetic energy. This paper will show the relationship between a jet's speed and the potential energy of the water tank.

The mathematical analysis contains some assumptions also we will discuss them later. As we discussed earlier about energy conservation here's for the derivations of Heron's fountain equation, we will use Bernoulli's theorem of energy conservation in two dimensions.

In this Heron phenomenon, the free jet's kinetic energy is like a free energy source that we can use for many practical purposes. Due to this kinetic energy non-stop, the fountain has many uses and full applications some of which are under discussion like Air quality improvement, Reduction of noise pollution, Cooling, Hydropower generation, and water feature in buildings.

There are some limitations of the non-stop fountain like in Fig. 1 when the middle tank becomes empty the flow will stop. There should be no loss of air or water because both are working fluids there. We will discuss them and suggest solutions for them.

II. FUNDAMENTAL WORKING PRINCIPLE

The fundamental principles of magic fountains are basically two, the first one is the principal of pneumatic and the second one is the principal of hydraulics.

In pneumatics, we make use of compressed gas or air to transfer mechanical energy from one point to another. In Fig. 1. The pressure at point 1 is downward and pushes the water level in tank A upward and as result the air in free space in tank A and tank B first become compressed (because as the water level moves up in tank A it exerts pressure on air).

The hydraulics principle is that this compressed air exerts pressure on the water in tank B and pushes the water downward and as a result, the water moves out to the jet through the pipe.

©IJRASET: All Rights are Reserved

ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 12 Issue III Mar 2024- Available at www.ijraset.com

Table 1(Consideration for Mathematical Modelling)

NOMENCLATURE				
P_1	Pressure at point 1		P_2	Pressure at point 2
Z_1	The hydrostatic head of point 1 from the water level in tank A		Z_2	The hydrostatic head of point 1 from the water level in tank A
g	Gravitational acceleration constant		V_3	Velocity of jet
D_{H2O}	Density Of Water			

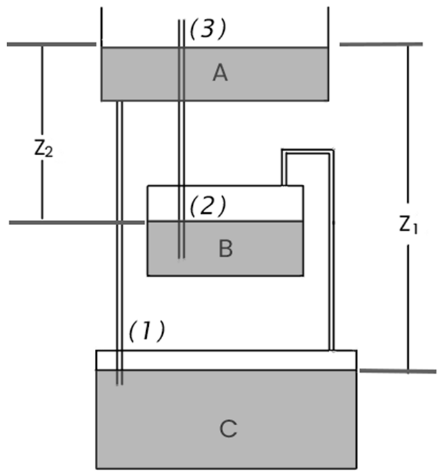


Figure 1 Schematic Diagram of Heron's Fountain

III. MATHEMATICAL MODELLING

The most fundamental principle we used to derive the free jet velocity is Bernoulli's principle. And the consideration of symbols for different constraints is given in Table 1.

A. *Bernoulli's Principal*

It is the theorem that explains the conservation of mechanical energy of a system published by Swiss Physicist Daniel Bernoulli in 1738.

The theorem states that a fluid in a closed flow with high internal pressure will have low velocity and a fluid with low internal pressure will have high velocity.

When deriving this theorem Bernoulli assumes some things that are:

1) Flow must be streamlined.
2) Fluid must be inviscid.
3) Fluid must be incompressible.
4) Density of the fluid must be the same at every point.

The mathematical form of the theorem between any two general points 1 and 2 is:

$$P_1 + \frac{1}{2}Dv_1^2 + DgZ_1 = P_2 + \frac{1}{2}Dv_2^2 + DgZ_2$$

B. *Derivation Of Equations*

Let's consider Figure 2, The pressure at points 1 and 2 will be hydrostatic pressure due to the weight of water from the top having heights Z_1 and Z_2 respectively.

The pressure at point 1 is: $P_1 = D_{h_2o}gZ_1$

The pressure at point 2 is: $P_2 = D_{h_2o}gZ_2$

©IJRASET: All Rights are Reserved | SJ Impact Factor 7.538 | ISRA Journal Impact Factor 7.894 |

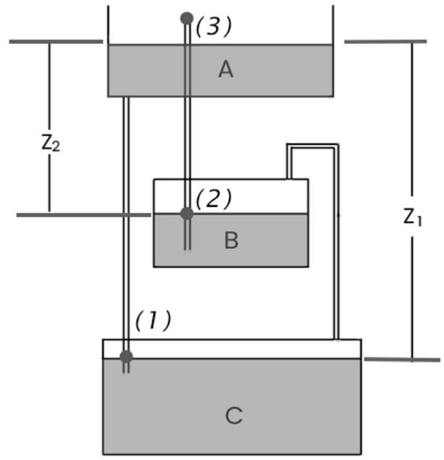


Figure 2. Selected points to apply Bernoulli's Theorem

First Air between tanks C and B will be compressed and then it will transform energy from tank C to tank B. This energy will result in the flow of water.

The pressure difference between 1 and 2 is:

$\Delta P = D_{h_2o} g Z_1 - D_{h_2o} g Z_2$

=> $\Delta P = D_{h_2o} g (Z_1 - Z_2)$ ------------*(1)*

(1) is the resultant pressure at point 2 in tank B, which will tend the water to move upward towards the fountain.

To find the jet's velocity let's consider point 3 just above the jet and consider point 4 just at the water surface in tank A. Apply Bernoulli's theorem between points 3 and 4:

Potential energy per unit volume of water at 3: $D_{h_2o} g Z_2$

Potential energy per unit volume of water at 4: $D_{h_2o} g Z_1$

By Applying Bernoulli's equation, we get.,

$P_4 + \frac{1}{2} D v_4^2 + D g Z_1 = P_3 + \frac{1}{2} D v_3^2 + D g Z_3$ -------*(2)*

Here, $P_4 = P_3 = P_{atm}$ => $P_3 = P_4 = 0$ (in terms of gauge)

Equation (2) => $v_3^2 = \frac{2Dg(Z_1 - Z_2)}{D}$

Substitute (1) in above equation =>

$v_3^2 = \frac{2\Delta P}{D}$

Taking square on both sides,

$v_3 = \sqrt{\frac{2\Delta P}{D}}$ ------------------------*(3)*

Equation (3) is the velocity of the water jet just outside the pipe.

IV. PRACTICAL APPLICATIONS

A. Air's Quality Improvement

Indoor fountains offer more than just aesthetic appeal and a pleasant atmosphere they also contribute to air purification. These fountains capture and retain airborne pollutants, preventing them from circulating in your living space. This purification process is driven by the generation of negative ions, which are then released into the air to combat air contaminants. Breathing in this cleaner air can provide a refreshing experience and enhance mental clarity. Additionally, as water evaporates from indoor water features, it emits negative ions that further purify the air, making it more invigorating to breathe. This improved air quality can also lead to clearer thinking and enhanced focus.

Both wall-mounted and freestanding indoor fountains can double as effective humidifiers, particularly benefiting those dealing with congestion.

B. Reduction Of Noise

Since the sound of water is known as a white noise generator. This usually helps to mask unwanted sounds from the surroundings. This is more practical in urban areas. The schematic explanation has been showed in figure 3.

Acoustics in a room reduce echo and noise pollution because when the sound waves collide, some energy is absorbed by them. Now water is one of the most efficient acoustics because of its weak intermolecular forces (as compared to solids) and the mobility of its molecules. When the sound waves strike with them water reflects less but absorbs more.

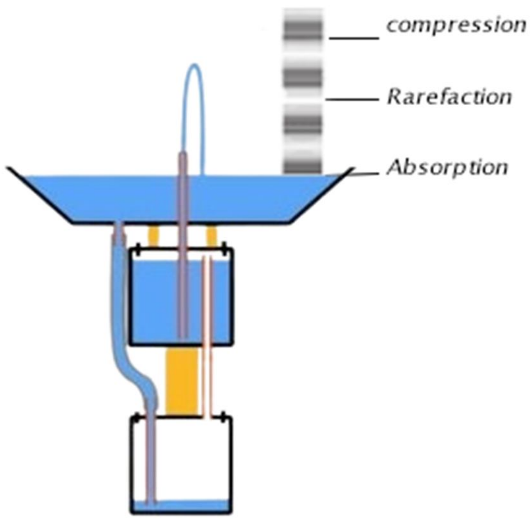


Figure 3. Schematic Representation of how noise is reduced.

C. Cooling

The fountain continuously generates evaporated droplets. this cool surrounding by evaporation. Thus, a fountain can be used to lower ambient temperature, making the area comfortable. This can be practically used in plazas, parks, and outdoor seating areas.

Water is one of the best heat absorbers and have high specific heat capacity. Instead of surface water in reservoirs, Lakes, Pounds, and Canals, the water-cooling effect is much greater in atomized form i.e., droplets form because the surface area is large. And the fountain provides the best spot for cooling a small square area. This can lower air temperature by 0.7ºC to 3ºC (Reducing Heat With Water | Urban Green-blue Grids, n.d.-e).

D. Hydropower Generation

Engineers can design systems to generate electricity from a continuous flow of water from the fountain. It rotates turbines which results in power generation. Heron's fountain is near to the perpetual motion machine which can maintain continuous head.

The water jet having kinetic energy can be converted to the rotational kinetic energy of turbine and then to the electrical energy but here the turbine needs to be design wisely because the head is low. In figure 3 virtual estimation of Heron's fountain is modified with RAM (Reciprocating Air Machine) pump. The RAM pump will maintain the balance of air water in two tanks and will provide continuous flow.

The schematic design of hydropower generation is given in figure 5.

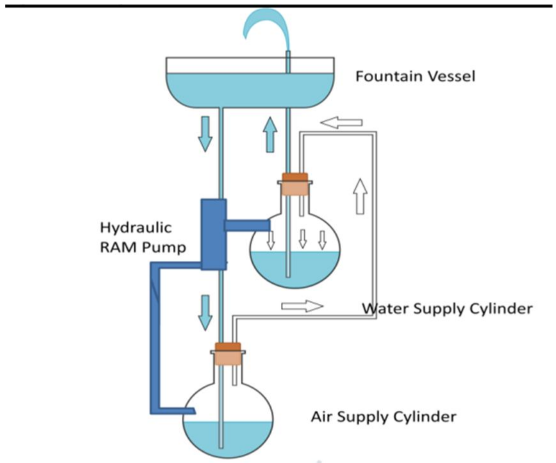


Figure. 4. Modified Heron's Fountain with Hydraulic RAM Pump (Awati & Patil, 2020)

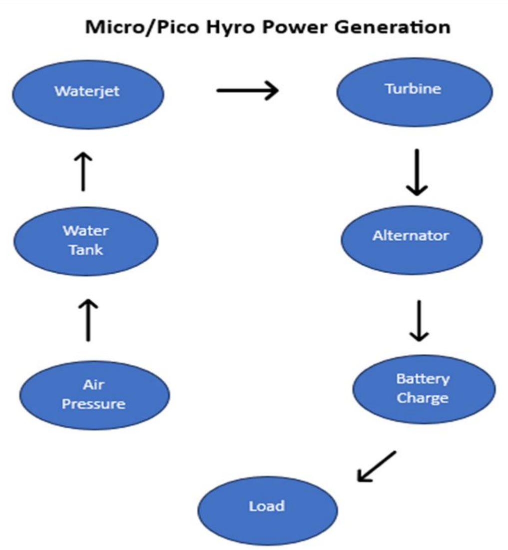


Figure 5 (Micro Hydropower Generation)

E. Water Features in Buildings

Fountains installed inside buildings, hotels, and shopping malls help to create an environment-friendly environment. Also, it helps in improving indoor air quality through evaporation.

Fountains can be designed to attract emotions and make a statement. Also reflect the culture of a country.

V. CONCLUSION

In this article, a critical analysis of a model of Heron's Fountain was done. Mainly focused on the working principle, mathematical modeling, and practical applications of it. The equation for free jet velocity has been derived and its dependence on different parameters has been highlighted. Working principle circulates about two major principles one is the pneumatics and the other is the Bernoulli's equation. Results show that the kinetic energy of the free jet is directly proportional to the under root of the distance between tank A and tank C. This means that the greater the distance the greater the velocity.

Our discussed model of Heron's fountain has many applications because of its non-stop water circulation and a jet with sufficient kinetic energy. Some important applications have been discussed and the engineering behind how to apply Heron's principle for that application.

VI. ACKNOWLEDGMENTS

The authors would like to acknowledge all the friends and Professors who motivated and support our research to work on the magic fountain. We specially want to acknowledge our family members who supported us financially for this research.

VII. CONFLICT OF INTEREST

The authors declare that they have no competing interests.

VIII. AUTHORS CONTRIBUTION

Mainly the paper is comprised of two portions Mathematical modelling and explanation and Practical applications of the magic fountain. S. Ali worked on understanding the history and working principal of Heron's fountain. Moreover, he worked on the developing the mathematical equations and applying Bernoulli's theorem. S.M. Zain along with M. Tariq Shah contributed his efforts towards the practical applications and implementation of the fountain. His work also includes the Graphs, Diagrams, Chats and Tables.

REFERENCES

[1] GevoAbcarian, ZainabAlgharib, OmranHussain, Martín, A. B. B., Villa, A., Villalobos, F. J., TadehZirakian, & Boyajian, D. (2019). Discharge coefficient measurements using Heron's fountain. Journal of Civil Engineering and Architecture, 13(3). https://doi.org/10.17265/1934-7359/2019.03.004

[2] Kezerashvili, R. Ya., & Sapozhnikov, A. (2003, October 9). Magic Fountain. arXiv.org. https://doi.org/10.48550/arXiv.physics/0310039

[3] Lee, H. M., & Lee, H. P. (2020). Noise masking in high population country using sound of water fountain. Applied Acoustics, 162, 107206. https://doi.org/10.1016/j.apacoust.2020.107206

[4] Tanaji, K. A., & Patil, M. D. (2020). Exploring Opportunities in Hydro Electric Power Plant with Heron's Fountain. In Advances in intelligent systems and computing. https://doi.org/10.1007/978-981-15-7234-0_16

[5] GevoAbcarian, ZainabAlgharib, OmranHussain, Martín, A. B. B., Villa, A., Villalobos, F. J., TadehZirakian, & Boyajian, D. (2019). Discharge coefficient measurements using Heron's fountain. Journal of Civil Engineering and Architecture, 13(3). https://doi.org/10.17265/19347359/2019.03.004

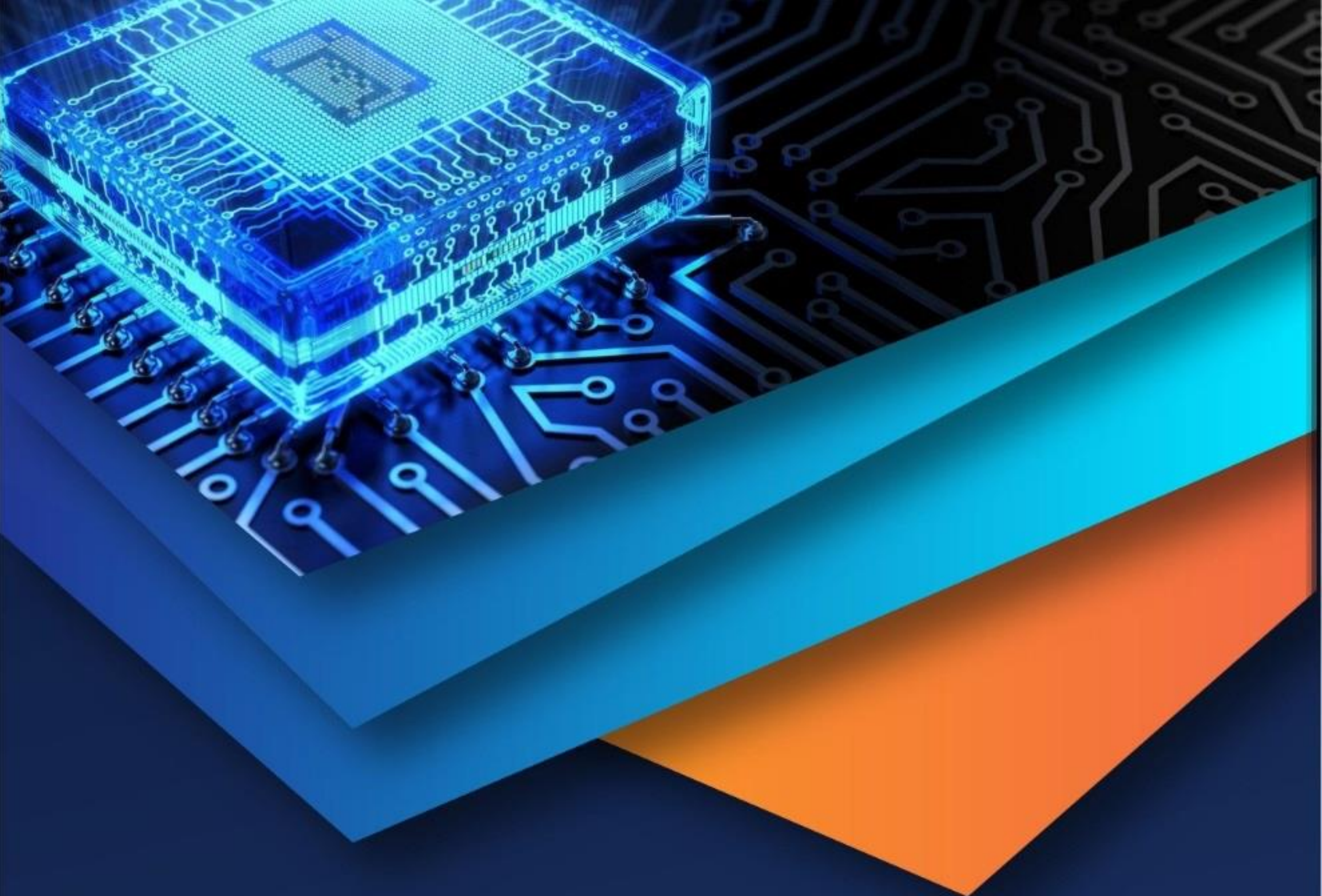
[6] Awati, M., & Patil, L. S. (2020b, May). Exploring opportunities for hydro power generations with Heron's Fountain. JETIR. https://www.jetir.org/view?paper=JETIR2005020

[7] Gerhart, P. M., Gerhart, A. L., & Hochstein, J. I. (2017). Munson's fluid mechanics. Wiley. https://www.wiley.com/enie/exportProduct/pdf/9781119248965

[8] Science Size. (2017, October 18). Infinite Water Fountain - Perpetual Motion - Motion of Water [Video]. YouTube. https://www.youtube.com/watch?v=QE9wHXigokc

[9] Reducing heat with water | Urban green-blue grids. (n.d.a). https://www.urbangreenbluegrids.com/heat/reducing-heat-with-water/

[10] Fountains--Water Wasters or Works of Art? | Water Resources Research Center | The University of Arizona. (n.d.). https://wrrc.arizona.edu/publication/fountains-water-wasters-or-works-art



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 (24*7 Support on Whatsapp)