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Mathematical Modelling of Vertical Looped Water Slide Using Matlab

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Abstract: A conceptual mathematical model of a water slide with vertical loops is developed. The principle used is the conservation of energy. The thrill experienced by a rider on a water slide is mainly due to the variation of G-force acting on the rider through the course of the ride. The geometry of the slide is developed by plotting G-force variation with the arc length of the loop. The G-force exposure limits should meet with the standards set by the F24 committee on amusement parks and rides. The coordinates of the slide geometry are determined by using Euler's method of discretized equations.
Keywords: G-Force, Centripetal acceleration, Clothoid curve, Weightlessness, Potential Energy, Kinetic Energy

I. INTRODUCTION

The main goal of this paper was to investigate the correlation between geometry of the water slide governing forces acting on structural systems by designing features. To do this, two main objectives were defined. The first objective was to design the shape of the slide such that the riders experience the thrill by feeling their body weight differently at every section of the slide. The second objective was to design the slide such that conforms to the ASTM standards set by F24 committee for amusement riders and equipment to ensure that safety of the passenger is not compromised. The design process is balancing act between safety and enjoyment.

II. DESIGN METHOD

A. Design Considerations

When a water slide is to be designed, various factors are considered and come into play for all aspects of the design. Designing a water slide starts with an idea for the ride, which may or may not be for a specific amusement park or location. During the concept phase, considerations such as the possible classification, components, theming and intended demographic may be taken into account. Other considerations such as the target audience also plays a key role in the developing the concept of the water slide. For e.g., large drops and inversions may not be suitable to younger audience. The main aim of the design is to expose the rider to varying G forces along the ride. The rider feels weightless at one section of the slide. By time the rider reaches the opposite end, he feels twice or sometimes thrice his bodyweight.

B. Safety Standards and Regulations

When designing a water slide, the most important consideration is the safety of the riders and those surrounding the ride. There have been rules and guidelines set to assure the safety of all no matter what the ride entails.

1) *G Forces:* The experience which a rider has while on a roller water slide is the product of specifically designed geometry. Two impactful variables that are designed for the rider's experience are G forces and velocity. G forces are the force resulting from a person's change in acceleration or directions; a g is a value that is multiplied by the acceleration of gravity to achieve the equivalent force felt by the person. The limits of the human body under G forces depend on the direction, type and duration. During vertical G forces, positive g's act on humans from head to toe and negative g's act on human from toe to head. The G force exposure limits are set to sustain a comfortable ride for the average human. If too high of G forces are experienced, then the blood will rush out of the head causing the person to pass out. If the negative G forces reach too large of values, then too much blood will go towards the brain also having detrimental effects on the person. The ASTM F2291 regulations [1] provide a graph to determine the acceptable vertical G forces depending on the duration time, as seen in Figure 1 below. While designing the water slide, the designer should ensure that G-force exposure lies within this limit.

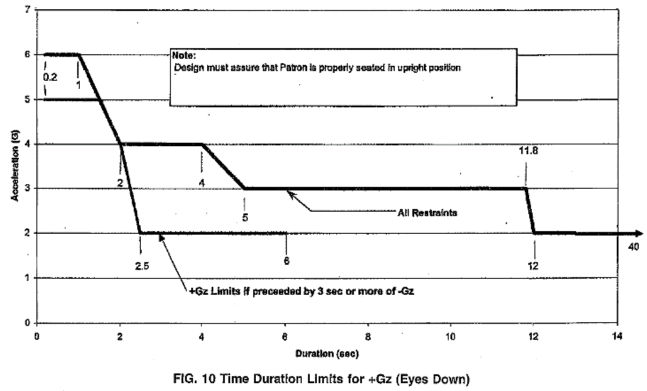

Fig. 1 Time duration limits for +Gz

The geometry of the slide is developed using mathematical calculations. Further, G force exposure for the geometry is plotted to evaluate the comfort and safety. All calculations were done using MATLAB software.

III.METHODOLOGY

A. *Physics of Water Slide*

Consider a water slide which consists of a lift hill and a single loop as in Figure 2 below.

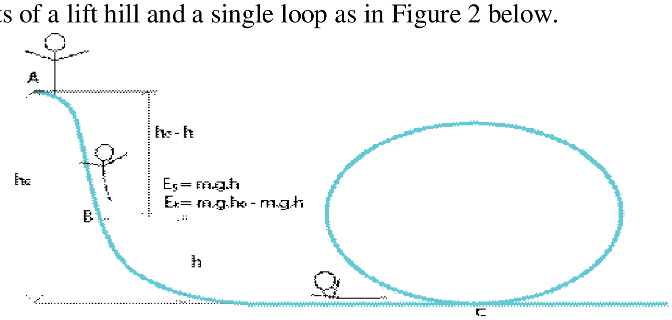

Figure. 2 Schematic diagram of water slide with a single vertical loop

At the top of the slide, the rider gains some amount of potential corresponding to the height of the slide. As the rider slides down with velocity, the potential energy is gradually converted into kinetic energy. At the bottom of the slide, all of the potential energy possessed by the rider at the top of the slide is converted into kinetic energy.

Referring to Figure 2, let the bottom of the loop be the reference point, 'h_0' be the height to the top of the slide at point A and 'h' be the height of any point B on the slide.

The potential energy of the rider of mass 'm' at the top of the slide at point A is given by,

E_g = m. g. h (1)

The potential energy at point B is given B,

E_g= m.g.h_0 (2)

Therefore, the amount of kinetic energy is given by,

E_k= m.g.(h_0-h) (3)

Let v be the velocity with which the rider is sliding down the slide. The relation between the velocity and the kinetic energy is given by,

E_k= ½. m. v^2 (4)

From equations (3) & (4),

The velocity of the rider is given by,

$$V = \sqrt{[2.g.(h_0 - h)]} \quad (5)$$

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B. Analysis of a Circular Loop

1) *Centripetal Acceleration:* Consider a circular loop of radius ‘r’ in Figure 3. As the rider approached the loop, the height h = 0 and the velocity of approach is given by,

$$V_0 = \sqrt{(2.g.h_0)} \qquad (6)$$

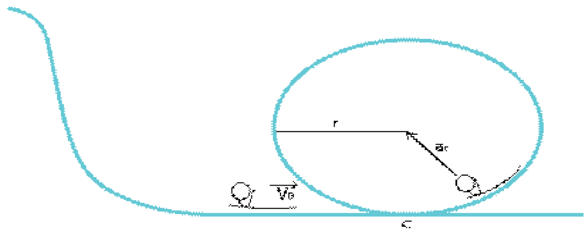

Fig. 3 Schematic diagram of a rider approaching the loop

This equation does not account for the energy losses due to friction and air resistance that will cause the rider to lose speed as he travels along the slide. For the time being, these energy losses are neglected.
As the rider enters the loop, he experiences a centripetal acceleration towards the Centre of the loop. The magnitude of the acceleration is given by,

$$a_c = V^2/r$$

2) *Force Analysis*

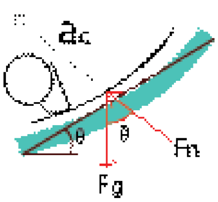

Fig. 4 Free body Diagram of the rider at an arbitrary position on the slide

From the free body diagram, the two forces acting on the rider are

- Force of gravity – F_g
- Normal Force – F_n

The normal force F_n acts at an angle θ to the normal, which is also equal to the angle of the track at that point.

From Newton’s 2nd Law of motion,
$\Sigma F = m.a$

where ‘a’ is the acceleration of a moving object of mass m. Considering the forces in the direction of the normal force,

$$F_n - F_g \cos(\theta) = m.a_c$$

$$F_n - F_g \cos(\theta) = \frac{mv^2}{r} \qquad (8)$$

Using $F_g = m.g$, where g is the acceleration due to gravity, we get

$$F_n - m.g\cos(\theta) = \frac{mv^2}{r}$$

$$\mathbf{F_n = m.g\cos(\theta)} + \frac{mv^2}{r} \qquad (9)$$

Dividing, F_n by mass time the acceleration due to gravity yields “G-Force” that the rider will experience as he travels around the loop.

$$\frac{F_n}{m.g} = \cos(\theta) + \frac{v^2}{r.g} \qquad (10)$$

$$\mathbf{G = \cos(\theta)} + \frac{v^2}{r} \qquad (11)$$

Using the expression for v in eq (5), we write eq (10) as

$$G = \frac{2.g.\,(h_0-h)}{r.g} + \cos(\theta)$$

$$\mathbf{G} = \frac{\mathbf{2.\,(h_0-h)}}{\boldsymbol{r}} + \cos(\theta) \qquad (12)$$

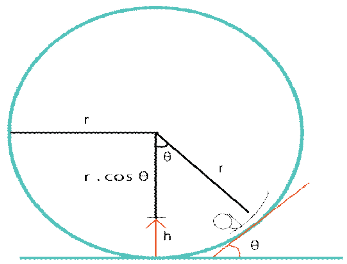

Fig. 5 Geometry of a circular loop

For a circular loop, the angle of the track at any given point is equal to the angle travelled by the rider along the loop measured from the Centre.

From Fig-5, using trigonometry, we get

$$h = r - r.\cos(\theta)$$

$$h = r\,(1 - \cos(\theta)) \qquad (13)$$

Using the expression for h in eq.11, we get

$$G = \frac{2.\,(h_0 - r.(1-\cos(\theta)))}{r} + \cos(\theta) \qquad (14)$$

$$\mathbf{G} = \frac{\mathbf{2.}\boldsymbol{h_0}}{\boldsymbol{r}} + \mathbf{3.cos(\boldsymbol{\theta})\ \text{-}2} \qquad (15)$$

Here, the height of the slide 'h_0' and the radius of the loop 'r' are constant. Hence, we can determine the G-force acting on the rider at every point on the loop as long as we know h_0 and r.

3) *Variation of G-Force along the circular loop:* In order to plot graph along the we can rewrite eq. (15) in terms of initial velocity of the rider before entering the loop V_0. This is particularly important if we want to account for the energy losses.

Using the third equation of motion, we can write:

$$h_0 = \frac{(V_0)2}{2.g} \qquad (16)$$

Using eq. (14) in eq. (13), we get

$$\mathbf{G} = \frac{(V_0)^2}{r.g} + 3.\cos(\theta) - 2 \qquad (17)$$

In this equation, we can write θ in terms of arc length of the loop and its radius from the arc length equation of a circle.

$S = r.\,\theta$

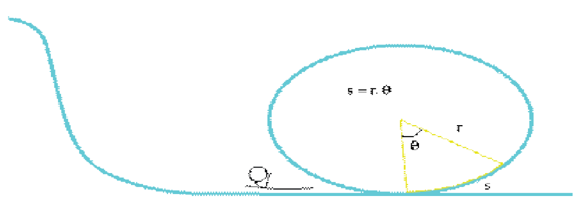

Fig. 6 Arc of the loop

We can finally plot the G vs S graph for the circular loop using the equation,

$$\mathbf{G} = \frac{(V_0)^2}{r.g} + 3.\cos\frac{s}{r} - 2 \qquad (18)$$

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4) *Minimum velocity required to clear the loop:* As depicted in Figure-7, there are basically two forces acting on the rider: the gravitational force and the normal force exerted by the slide on the rider. The normal reaction at the bottom of the slide as 'N_B' and the normal reaction force at the top of the slide is denoted as 'N_T' while the gravitational force F_g = m. g.

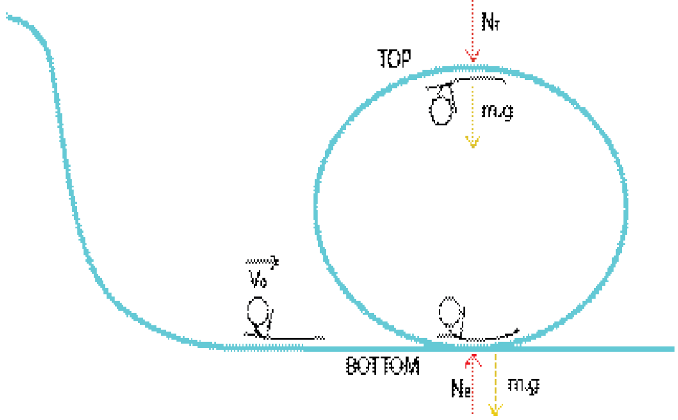

Fig. 7 Force analysis at the top and bottom of the loop

At the bottom of the loop, both the normal reaction force acts opposite the force of gravity. Using Newton's second law of motion, we can write

$N_B - mg = m.\ a_c$ (19)

Using the expression for a_c from eq. (7) we get,

$\mathbf{N_B - mg} = \frac{m.(V0)2}{r}$ (20)

Therefore, the normal force at the bottom of the loop is given by:

$N_B = \frac{m.(V0)2}{r} + m.g$ (21)

At the top of the loop, both the normal reaction force acts along the force of gravity. Hence, we get

$N_T + mg = \frac{m.(V0)2}{r}$ (22)

Therefore, the normal force at the bottom of the loop is given by:

$\mathbf{N_T = m.(V_0)^2/r - m.g}$ (23)

However, since the weight of the rider is acting away from the slide, it does not exert any normal reaction on the rider. In this case, the rider experiences "weightlessness" and tends to fall down from the top of the slide which is not desirable. In order to avoid this, we need to determine the entry velocity with which the rider should enter the loop to avoid weightlessness at the top of the loop.
If N_T is equated to 0, we can get the expression for the velocity at which the rider experiences weightlessness at the top.

$\frac{m.(V0)2}{r} = m.g$

$\mathbf{(V_0)^2 \propto r.g}$ (24)

Therefore, in order to go around the loop without falling off, the rider must enter the velocity for which,

$\mathbf{(V_0)^2 > K.\ r.g}$ (25)

Where K is an integer. The value of K is found graphically. Typically, the loop is designed such that the value of K gives a positive value of G close to 0 so that the rider experiences not entirely but close to weightlessness.

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5) *Plotting the G vs S Graph:* The graph for eq. (19) is plotted using the following MATLAB code.

```
s = linspace(-5,10,40000);
G1 = 5 + 3*cos(5*s)  - 2;
G2 = 5.5 + 3*cos(5*s) - 2;
hold on
plot(s,G1)
hold on
plot(s,G2)
xlim([0 1.25])
ylim([-1 7])
plot (s,0)
set(gca,'ytick',linspace(-5,10,16))
grid on
legend({'(V0)^2=5.r.g','(V0)^2=6.r.g'},'Location','northeast','Orientation','horizontal')
title('G VS S - Circular Loop')
xlabel('Arc Length of the Loop "S"')
ylabel('Acceleration in G')
```

Fig. 7 MATLAB code for eq. (19)

Here, $(V_0)^2 = 5.r.g$, for which the rider has just enough energy for weightlessness and $(V_0)^2 = 5.5.r.g$ for which the rider clears the loop. Following is the graph showing G force exposure variations for the above two cases.

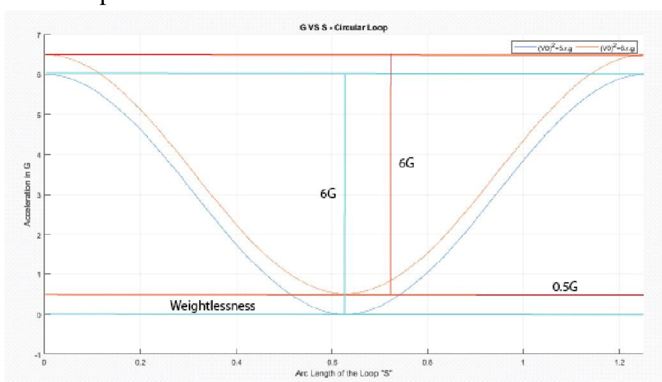

Fig. 8 Variation of G force along the length of the loop

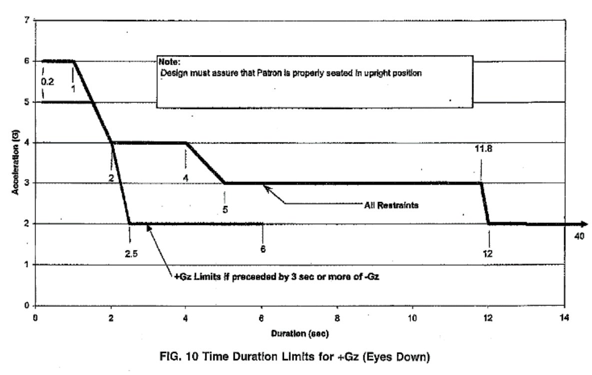

Fig. 9 Time duration limits for different levels of G force exposure

From the graph in Figure-8, we can see the fundamental issue with circular loops. Regardless of the initial velocity and the radius of the loop, there will always be a 6G's difference between the bottom and the top of the loop. If the rider is travelling with an initial velocity just enough to clear the loop, his velocity at the top of the loop will be 0 and will experience -1 G and will experience a force of +5G at the bottom of the loop. If the rider has an initial velocity such that he experiences weightlessness at the top of the loop, then he will experience a force of +6G at the bottom of the loop. The F24 committee of amusement rides and devices has created ASTM standards for the design of safe amusement attractions and they provide limits for G force exposure which includes magnitude, direction and duration.

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For the +Gz forces that rider experiences on the vertical loop, the G force exposure limits set by the F24 committee is shown in Figure-9. If the passengers are going to experience a force of 4G's then the duration of that force must not exceed 4 seconds and this duration decreases as higher G forces are reached. This is one of the reasons why most amusement rides are designed to have maximum G- force exposure of 4 G's.

6) *Drawbacks of a circular loop:* On a circular loop, the passengers are not only exposed to intense G forces, but the magnitude of the G force changes rapidly. As the passenger approaches the loop from the horizontal section of the slide, the G force will change almost instantaneously from 1G to more than 5G because there is no transition from the infinite radius of the straight path to the radius of the loop. The G force will decrease by the 6G as the rider approaches the top of the loop and then again increase by 6G in the second half of the loop before dropping to 1G instantaneously as the rider exits the loop into another straight section of the track. Hence, it is physically impossible to construct a water slide with vertical loop that is both thrilling and comfortable for the riders.

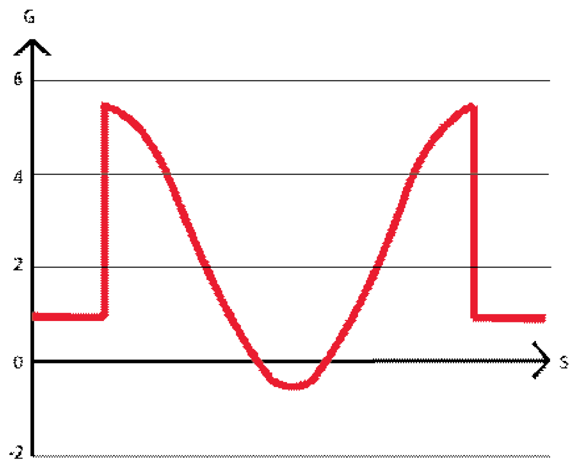

Fig. 10 G force plot of a water slide having a circular loop following a straight path

IV. ANALYSIS OF NON-CIRCULAR LOOPS

In order to design a vertical loop that is safe for the riders, water slide engineers first decide what G forces they want the riders to experience on the vertical loop and they reverse engineer a shape that produces these forces.

1) *Constant acceleration loop:* We know that the expression for centripetal acceleration of the rider around the loop is given by,

$a_c = V^2/r$ (26)

where 'g' is the acceleration due to gravity and C is a constant which gives the magnitude of the centripetal acceleration in G's.

From eq.(25) the expression for the radius of the loop is given by:

$r = V^2/C.g$ (27)

From the second equation of motion, we can write

$V^2 = (V_0)^2 + 2.g.h$ (28)

Plugging in the expression for V^2 in eq. (26), we get

$r = \frac{(V0)2 + 2.g.h}{C.g}$ (29)

In order to plot the geometry of the slide on a x-y coordinate system, 'h' is replaced by 'y' which represents the height of the loop from the reference point.

$\mathbf{r} = \frac{(V0)2 + 2.g.y}{C.g}$ (30)

Consider an extremely small segment of the curved slide with arc length Δs inclined at an angle θ. The length of the segment is so small that the radius 'r' of the segment is constant over its length. This segment can be treated as a sector of the circle with radius 'r' and angle '$\Delta\theta$'.

Using the arc length equation for a circle,

$$\Delta s = r.\ \Delta\theta \tag{31}$$

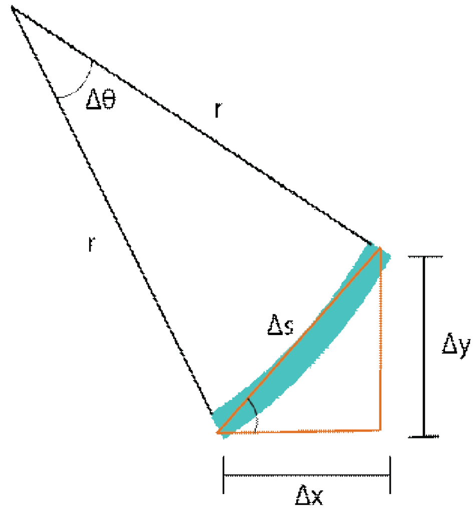

Fig. 11 Section of constant acceleration loop

Let the height of the segment be Δy and the horizontal length be Δx. Since Δs has negligible length, the it can be approximated as a straight line to complete a right-angled triangle.

Using trigonometry,

$$\cos(\theta) = \frac{\Delta x}{\Delta s} \tag{32}$$

$$\sin(\theta) = \frac{\Delta y}{\Delta s} \tag{33}$$

Taking the limits as Δs approaches 0, we obtain a set of three differential equations:

$$\frac{\partial\theta}{\partial s} = \frac{1}{r} \tag{34}$$

$$\cos(\theta) = \frac{\partial x}{\partial s} \tag{35}$$

$$\sin(\theta) = \frac{\partial y}{\partial s} \tag{36}$$

Putting the expression of ‘r’ in eq. (34),

$$\frac{\partial\theta}{\partial s} = \frac{C.g}{(V0)2 + 2.g.y} \tag{37}$$

This set of equations is solved using Euler’s method. Let the starting point of the loop be n and the subsequent points be n+1, n+2 and so on. We get the following discretized equations:

$$\theta_{n+1} = \theta_n + \frac{C.g}{(V0)2 + 2.g.Yn}.\ \Delta s \tag{38}$$

$$x_{n+1} = x_n + \cos(\theta_n).\ \Delta s \tag{39}$$

$$y_{n+1} = y_n + \sin(\theta_n).\ \Delta s \tag{40}$$

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The calculations were carried out on MATLAB using the following code:

```
function plot_test()
data=input('Please enter how many numerical solution going to plot: ');
for i=1:data
    h=input('Enter the Step size: ');
    N=input('Number of Steps: ');
    t(1)=0;
    y(1)=0;
    x(1)=0;
    for n=1:N
    t(n+1)= t(n)+h*3/(13.33-2*y(n))
    y(n+1)= y(n)+h*sin(t(n));
    x(n+1)=x(n)+h*cos(t(n));
    end
    plot(x,y,'color', 'red' );
    title('XY cordinated - Constant Acceleration Loop')
    xlabel('X in metres')
    ylabel('Y in metres')

    xlim([0 5])
    ylim([0 6])

    hold on;
end
```

Fig. 12 MATLAB code for discretization

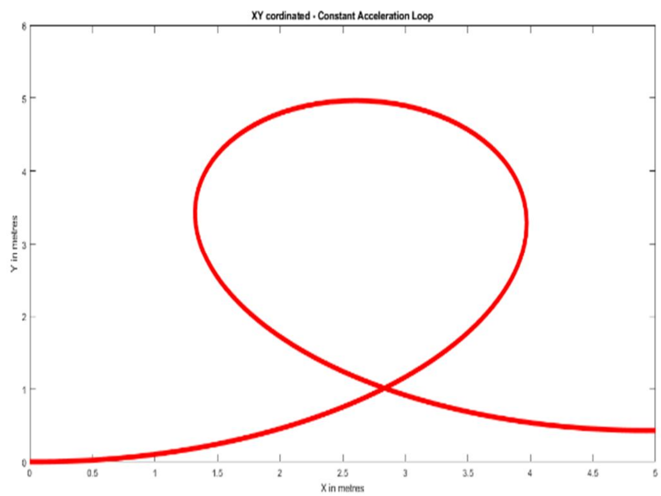

Fig. 13 Geometry of constant acceleration loop

This particular loop was designed to give a constant centripetal acceleration of 3G's when the rider has an initial velocity of 20 m/s.

The following input was given:

- Number of Numerical solutions = 1
- Step size = 0.1 m
- Number of steps = 900

The final shape of the loop looks like an upside-down teardrop.
Now that the geometry of the loop is determined, the comfort of the ride is evaluated by analyzing the G-force exposure graph.
G Force plot along the path is generated using the same principles which were used for a circular loop.

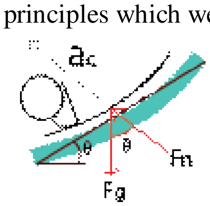

Fig. 14 Free body Diagram

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From eq.(11), the expression for G-force is:

$G = \cos(\theta) + V^2/r\,.g$

From eq.(26),

$V^2 = r.C.g$

Using the expression for V^2 in eq. 10, we get

$G = \cos(\theta) + C$ (41)

Expressing θ in terms of 's' and 'r', we get

$\mathbf{G = \cos(s/r) + C}$ (42)

It is important to note that the radius of the loop is not constant and varies continuously with height. However, since the multiple of the cosine function is 1, the difference between the values of G force at the top and bottom of the loop is 2 : G= C+1 at the bottom($\theta=0^o$) and G=C-1 at the top ($\theta=180^o$), irrespective of the value of ratio 's/r'. The value of 's/r' will have an impact on the time period of the function, i.e how soon the limits are reached. For the time being, this factor is ignored.

Following MATLAB code is used to compute the G force plot.

```
s = linspace(0,20,40000);
G1 = cos(s) + 3;
hold on
plot(s,G1,'linewidth',3,'color','black')
xlim([6 13])
ylim([-1 7])
set(gca,'ytick',linspace(-5,10,16))
set(gca,'XTick',[])
grid on
ay = gca;
ay.FontSize = 20;
title('G VS S - Constant Acceleration Loop','FontSize',20)
xlabel('Arc Length of the Loop "S"','FontSize',20)
ylabel('Acceleration in G','FontSize',20)
```

Fig. 15: MATLAB code for eq. (42)

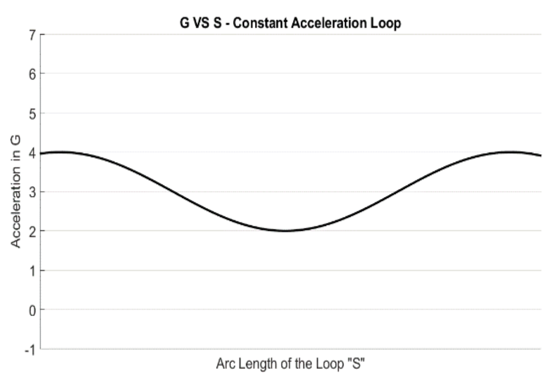

Fig. 16 G vs S plot for constant acceleration loop

It is clear from the graph in Figure-16 that constant acceleration loop is safer and more comfortable than a circular loop as the G forces involved are not as intense.

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2) *Constant G-force loop:* In addition to loops with constant centripetal acceleration, engineers may also want provide loops with constant G force.

From eq.(10),

$G = g.\text{Cos}\ (\theta) + V^2/r$

Rearranging this equation to get a function for r, we get:

$$r = \frac{V^2}{G - g.\cos(\theta)}$$

Using the expression for V^2 , we get:

$$r = \frac{V0^2 - 2.g.y}{G - g.\cos(\theta)} \qquad (43)$$

This expression for 'r' is used in the set of differential equations used to evaluate the previous loops.

$$\frac{\partial\theta}{\partial s} = \frac{G - g.\cos\theta}{V0^2 - 2.g.y} \qquad (44)$$

$$\cos\ (\theta) = \frac{\partial x}{\partial s} \qquad (45)$$

$$\sin\ (\theta) = \frac{\partial y}{\partial s} \qquad (46)$$

Using Euler's method again, we get,

$$\boldsymbol{\theta_{n+1} = \theta_n +} \frac{\mathbf{G - g.cos(\theta n)}}{V0^2 - 2.g.Y(n)} \qquad (47)$$

$$\mathbf{x_{n+1} = x_n + \cos\ (\theta_n)} \qquad (48)$$

$$\mathbf{y_{n+1} = y_n + \sin\ (\theta_n)} \qquad (49)$$

The following MATLAB Code is used to compute the X and Y coordinates of the loop.

```
function plot_test()
data=input('Please enter how many numerical solution going to plot: ');
for i=1:data
    h=input('Enter the Step size: ');
    N=input('Number of Steps: ');
    t(1)=0;
    y(1)=0;
    x(1)=0;
    for n=1:N
    t(n+1)= t(n)+h*3.5-10*cos(t(n))/(400-20*y(n));
    y(n+1)= y(n)+h*sin(t(n));
    x(n+1)=x(n)+h*cos(t(n));
    end
    plot(x,y,'linewidth',5); %plots the first soltuion
    xlim([0 1])
    ylim([0 1])
    set(gca,'XTick',[])
    set(gca,'YTick',[])
    ay = gca;
    ay.FontSize = 20;
    title('XY PLOT - Constant G-Force Loop','FontSize',10)
    xlabel('X','FontSize',10)
    ylabel('Y','FontSize',10)
    hold on;
end
```

Fig. 17 MATLAB code for discretization

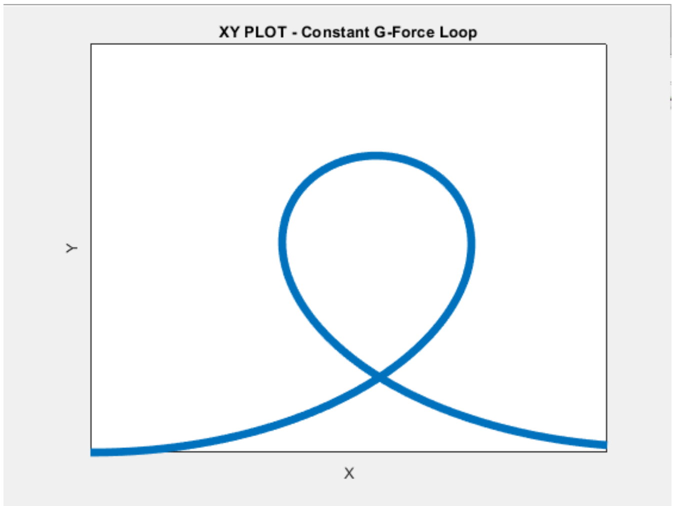

Fig. 18 Geometry of Constant G force loop

The loop is designed for a constant 3.5G's when the rider has an initial velocity of 20m/s. Figure-18 shows the geometry of the constant G-force loop. Even though the loop looks similar to the previous loop, there are subtle differences. The radius of the loop is changing constantly in order to keep the G-force constant.

V. CONCLUSIONS

Analyzing the mathematical models of various shapes of the vertical loop of the water slide that have been developed, we can conclude that a circular loop will expose the rider to unsafe G-force loads. Whereas non-circular loops which have constant G-force or constant centripetal acceleration will give desired G-force exposure while constantly subjecting the rider to a feeling of weightlessness and multiple G-forces in alternate sequence.

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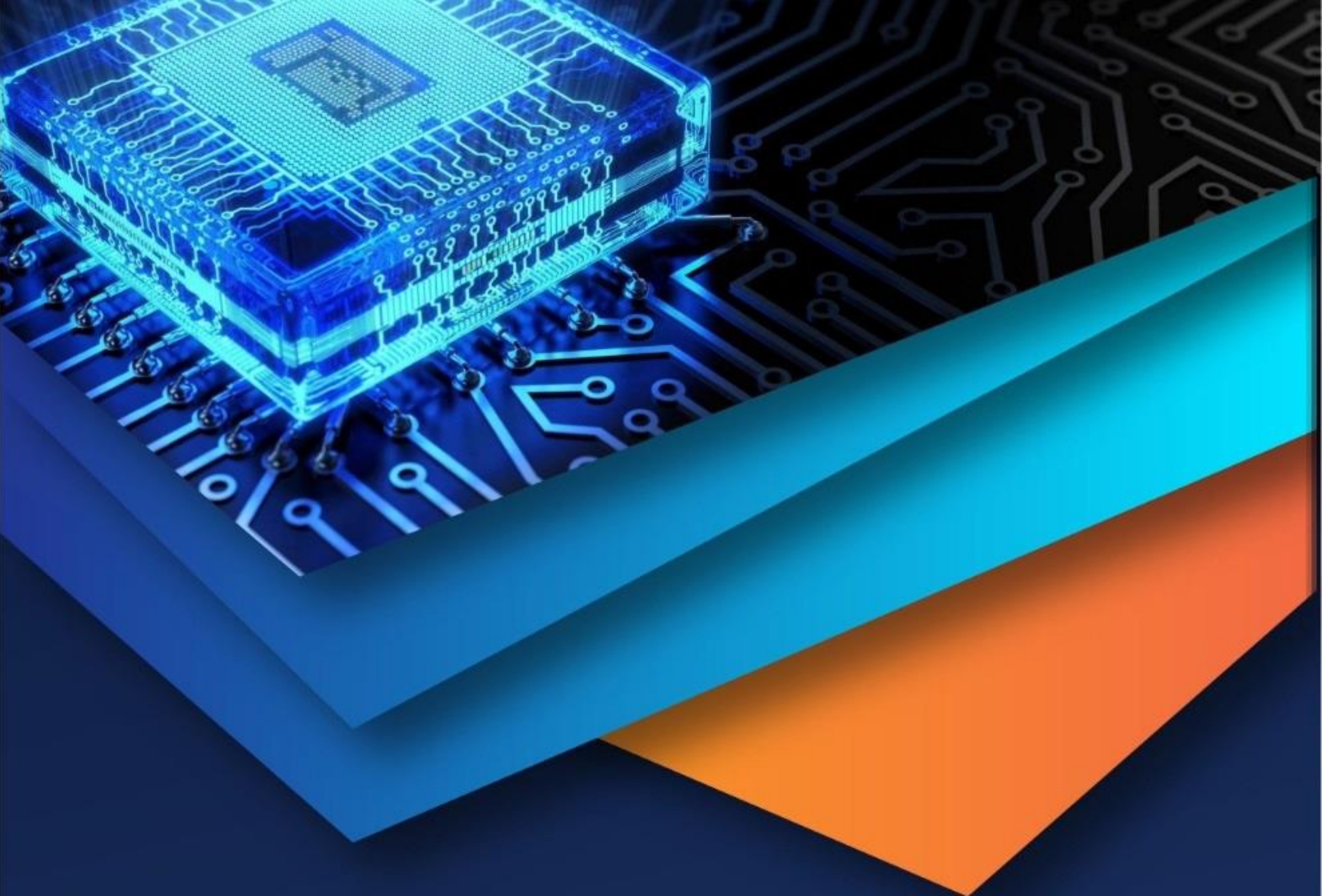

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