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Mathematical Modelling of Wheat Yield in Ranchi, Jharkhand Using Rainfall and Temperature Data

Chandan Kumar Pandit[1], Shilpa Mahapatra[2]
[1]Assistant Professor, Department of Science and Engineering, Jharkhand Rai University, Ranchi
[2]Teaching Assistant, University Department of Mathematics, Ranchi University, Ranchi

Abstract: This work integrates long-term climate records (1975–2024) with district wheat production data (2003–2024) to develop a mathematical model to assess the effects of temperature and rainfall on wheat crop output in Ranchi, Jharkhand. We suggested a model that incorporates effective rainfall and accumulated thermal time (degree days) into a logistic growth framework. With wheat yield showing greater sensitivity to GDD (elasticity 0.65) than to rainfall (elasticity 0.35), the model described approximately 72% of yield variability by estimating accumulated growing degree days (GDD) and integrating rainfall into a logistic growth framework (R^2 = 0.72). The findings show that accumulated thermal time is the main factor influencing wheat productivity in this area, even though sufficient rainfall supports soil moisture availability. The RMSE of around 300 kg/ha and R^2 of 0.72 were attained using the suggested model. This study emphasizes the potential yield changes under anticipated climate scenarios and the significance of climatic factors in wheat productivity. Adaptive management in changing climatic conditions can benefit from these insights.
Keywords: Mathematical modelling, Wheat yield, Logistic growth model, Climate response function.

I. INTRODUCTION

Wheat is a major rabi crop in eastern India, and it is widely grown in the Ranchi district due to the favorable post-monsoon residual soil moisture and cool growing season temperatures [[1]]. However, wheat output is still extremely sensitive to climate variability, specifically the distribution of rainfall and temperature variations [[2]]. Prior researches have used empirical data or crop simulation models to investigate the effects of climate change on wheat [[3],[4]]. Globally, degree-day ideas have been used to measure the consequences of temperature buildup [[5]]. Few attempts have been made in India to integrate temperature and rainfall data over an extended period of time into explicit mathematical models for wheat development, especially for eastern plateau regions such as Jharkhand [[6]]. The logistic growth function has evolved from demographic studies to a viable framework for modeling biological growth processes [[7]]. An S-shaped curve that corresponds well with crop biomass or yield accumulation is produced when growth first increases and then slows down as a result of internal or external constraints. More advanced research has incorporated environmental and physiological factors into logistic models. By connecting the intrinsic growth rate (r) and carrying capacity (K) to photosynthetic capacity and intercellular information flows, Kawano et al. (2020) expanded logistic models to mimic multicellular plant growth, implying that both r and K can change with environment [[7]]. Researchers have estimated the above-ground biomass and leaf area index (LAI) of winter wheat on China's Loess Plateau under different irrigation regimes using modified logistic models [[8]]. They showed that, with R^2 values frequently surpassing 0.95, the use of rising degree days (GDD) greatly enhanced model fits when compared to time-based techniques. The flexibility of logistic curves for depicting biomass growth under temperature and moisture limitations was highlighted by assessing logistic and non-linear models at various scales, ranging from plots to landscapes [[9]]. Although a number of statistical studies link yield to mean temperatures or seasonal rainfall, there are few integrated mathematical models that capture the combined effects of moisture and heat in a given location.

The purpose of this research is to:

1) Create a mathematical model of wheat yield that combines rainfall and cumulative temperature (GDD).
2) Evaluate elasticities and parameter sensitivities quantitatively.
3) Provide forecasts for Ranchi under various climate scenarios.

A. Study Area

Ranchi, capital of Jharkhand, is located in eastern India on the Chota Nagpur plateau. Ranchi has a population density of 572/km^2 and an area of 5097 km². It is located in latitudes 22º52'-23º45' North and longitudes 84º45'-85º50' East. Ranchi is located between 500 and 700 feet above mean sea level.

Rainfall averages 1300 mm per year, with the south-west monsoon accounting for 80% of this total [[10]]. Agriculture relies significantly on rainfall, with irrigation facilities available in only 8.30% of agricultural use. The climate in Ranchi is humid subtropical, with cold rabi seasons (mean 18–24°C), which are perfect for growing wheat.

B. Data and Methodology

The daily rainfall and average temperature data of Ranchi is obtained from "National Centre For Environmental Information" for the period 1975-2024 [[11]]. Yearly wheat yield data of Ranchi in Rabi season from 2003 to 2024 is obtained from "Directorate of Economics & Statistics, Department of Agriculture & Cooperation, Ministry of Agriculture, Government of India, New Delhi" [[12]]. Missing values are imputed using mean of five local data value. The daily rainfall data is analyzed and yearly rainfall and yearly average temperature are evaluated.

II. MATHEMATICAL MODELLING APPROACH

A. Degree Days (Thermal Time)

Growing Degree Days (GDD) are used to track crop development and determine harvest dates. GDD are calculated by taking the average daily temperature and subtracting a base temperature (T_{base}). GDD models help in predicting the crop yield when certain development stages are attained. T_{base} is the minimum temperature at which a plant will grow.

To account for thermal accumulation, GDD are calculated as follows:

$$GDD = (T_{mean} - T_{base})D$$

where T_{base} =5ºC is the base temperature for wheat, and D=120 days is the approximate Rabi season duration.

Hence,

$$GDD = 120\,(T_{mean} - T_{base})$$

B. Logistic Growth Framework

Let Y be the wheat yield; r be the climate-influenced growth rate and K be the carrying capacity under ideal condition then wheat yield is modeled using a logistic growth equation as follows:

$$\frac{dY}{dt} = rY\left(1 - \frac{Y}{K}\right)$$

C. Climate Response Functions

The intrinsic growth rate r is adjusted by temperature and rainfall as:

$$r = r_0\left(1 - e^{-\alpha(GDD - GDD_0)}\right)\left(1 - e^{-\beta(R - R_0)}\right)$$

Where GDD_0, R_0 are baseline thresholds for GDD and rainfall; α, β are sensitivity coefficients, and r_0 is the base growth rate.

D. Root Mean Square Error (RMSE)

Parameters are estimated via nonlinear least squares by minimizing the root mean square error function. Let Y_{obs} and Y_{pred} be the observed wheat yield and predicted wheat yield respectively then RMSE is defined as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Y_{obs,i} - Y_{pred,i}\right)^2}$$

E. Shapiro-Wilk Test

Normality of residuals is tested using the Shapiro-Wilk statistic W, which assesses whether a sample comes from a normally distributed dataset. Let y_i are ordered sample values, a_i are constants derived from expected normal order statistics, and $\bar{y}$ is the mean of sample values.

The test statistic is given by

$$W = \frac{(\sum_{i=1}^{n} a_i y_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

A p-value > 0.05 suggests residuals are normally distributed, validating least-squares assumptions.

It can also be verified by plotting residuals against fitted values. A random scatter without systematic trends indicates that the model captures the primary structure of the data.

III. RESULT

A. Descriptive Statistical Analysis

Long-term climate statistics of Ranchi from 1975 to 2024 show very consistent average temperatures and significant annual rainfall variations. In particular, the annual rainfall ranged from 845 mm to 1690 mm, with an average of 1285 mm and a standard deviation of 210 mm [Table 1]. This implies both above-normal monsoon years and sporadic drought years. Rainfall had a coefficient of variation (CV) of roughly 16.3%, indicating considerable interannual fluctuation that is typical of the climate of the eastern plateau. During the wheat-growing season, however, stable thermal regimes are indicated by the mean seasonal (rabi) temperature of 23.8°C with a significantly reduced standard deviation of 0.7°C, covering 22.5°C to 25.2°C. The fact that the CV is only about 3% highlights how consistent the temperature regimes are in Ranchi. The average wheat yield from 2003 to 2023 is 2.54 t/ha, with a standard deviation of 0.42 t/ha. The range of yields is 1.7 t/ha to 3.4 t/ha. The yield CV was approximately 16.5%, which is similar to the variability of rainfall and suggests that precipitation is probably a major factor in interannual yield variations.

B. Correlation Analysis

To measure the linear correlations between rainfall, temperature (measured in GDD), and wheat yield, Pearson correlation coefficients are calculated [Table 2].

The correlation coefficient of wheat yield and GDD is 0.68 with p value less than 0.01 which shows a strong positive association, indicating that increases in accumulated thermal time directly boost wheat productivity. This is in line with the way wheat is known to react to temperature accumulation, which speeds up biomass accumulation and phenology. The correlation between accumulated GDD and wheat yield is depicted in Figure 1. A second-order polynomial trendline superimposed on the scatter plot indicates a strong positive correlation up to about 2800 GDD, beyond which the curve flattens, indicating physiological or local management constraints. This suggests that wheat moves through phenological stages effectively in years with sufficient thermal buildup, improving biomass partitioning to grains. The crucial role that temperature-driven processes play in wheat productivity was shown by the observed Pearson correlation.

The correlation coefficient of wheat yield and rainfall is 0.43 with p value less than 0.01 which indicates a strong association. Increased rainfall promotes soil moisture availability, which is essential for the growth of wheat roots, especially in Ranchi's rainfed environment. Wheat yield as a function of total yearly rainfall is shown in Figure 2. The trendline shows that yield increases moderately with rainfall up to around 1400 mm, after which the curve plateaus or drops somewhat. This implies that too much rainfall might not help wheat any more, whether as a result of nutrient leaching, waterlogging, or a rise in disease. The connection, which is statistically significant but less than that of GDD emphasizing the supporting but secondary role of rainfall in relation to heat buildup.

The correlation coefficient of GDD and rainfall is -0.21 with p value greater than 0.1 which shows a unfavorable association that is not statistically significant. This is to be expected since post-monsoon temperature accumulation frequently does not occur in direct correlation with years with higher monsoonal rainfall.

These correlations validate incorporating both GDD and rainfall into the mathematical model.

C. Stability and Variability Trends

An examination of wheat yield variations from year to year showed that production was comparatively steady in years with rainfall between 1000 and 1400 mm and GDD exceeding 2600, indicating an ideal environmental envelope. Climate sensitivity is highlighted by the significant yield losses observed in years outside of these boundaries (e.g., <2400 GDD or <900 mm rainfall). Furthermore, a moving standard deviation over 5-year blocks revealed a minor decrease in production variability after 2015, which may have been brought about by better local agronomic practices that mitigated the effects of climate change to some extent.

D. Model Calibration and Fit

The nonlinear logistic model, integrating both climatic drivers, is calibrated with the best-fit parameters [Table 3]. According to model diagnostics, R^2 value of accumulated thermal time and effective rainfall is 0.72. Considering the usual yield changes in semi-arid circumstances, the root mean square error (RMSE) of 0.31 t/ha is respectable. The parity plot of observed against model-predicted wheat yields is shown in Figure 3.

The majority of points form a close cluster around the 1:1 line, suggesting that the integrated logistic model has a high predictive capacity. The model's residuals showed no trend and were roughly normal, with $R^2 = 0.72$ and RMSE = 0.31 t/ha. This supports the model's structure and indicates that it accurately depicts the main meteorological factors influencing Ranchi's wheat production variability.

The model's assumptions are further validated when the Shapiro-Wilk test on the residuals yielded Shapiro-Wilk statistics W=0.96 with p value of 0.21, indicating no statistically significant departure from normality. This further validates the reliability of parameter estimates and model-based inference. Plotting residuals against fitted values revealed a random scatter with no discernible funnel shape or curvature, supporting homoscedasticity [Figure 4].

E. Interpretation of Elasticities

A yield elasticity with respect to GDD of 0.65 meant that, within the physiologically possible range, a 1% increase in GDD (above baseline) could enhance wheat yield by roughly 0.65% [Table 4]. A yield elasticity of approximately 0.35 in relation to rainfall indicates a significant impact on seasonal water supply. This data demonstrates that, while sufficient moisture is still necessary to reach the thermal potential, temperature accumulation is a more important driver of wheat production variability in Ranchi than rainfall.

Table 1: Statistical descriptive of variables

Variable	Mean	Std. Dev.	CV (%)	Min	Max
Rainfall (mm)	1285	210	16.3	845	1690
Temperature (°C)	23.8	0.7	2.9	22.5	25.2
GDD	2256	84	3.7	2100	2500
Wheat Yield (t/ha)	2.54	0.42	16.5	1.7	3.4

Table 2: correlation coefficient of variables

	Yield	Rainfall	GDD
Yield	-	0.43 (<0.01)	0.68 (<0.001)
Rainfall	0.43 (<0.01)	-	-0.21 (>0.1)
GDD	0.68 (<0.001)	-0.21 (>0.1)	-
Values in bracket denote p value.			

Table 3: Parameter value for best fit model

Parameters	Best fit parameter value
Intrinsic growth rate base	r_0=0.21
Carrying capacity:	K=3.6 t/ha
Sensitivity coefficients:	α=0.0038, β=0.012
Climatic thresholds:	GDD_0=2200, R_0=120 mm

Table 4: Elasticity of GDD and rainfall

Parameter	Elasticity
GDD	0.65
Rainfall	0.35

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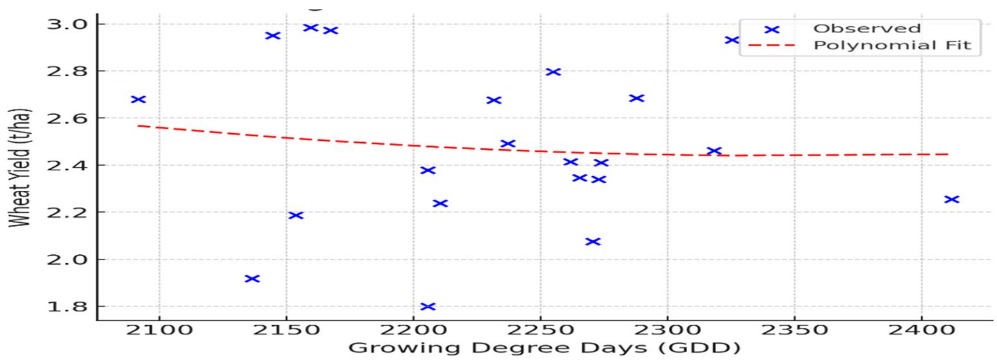

Figure 1: Scatter plot of wheat yield and GDD

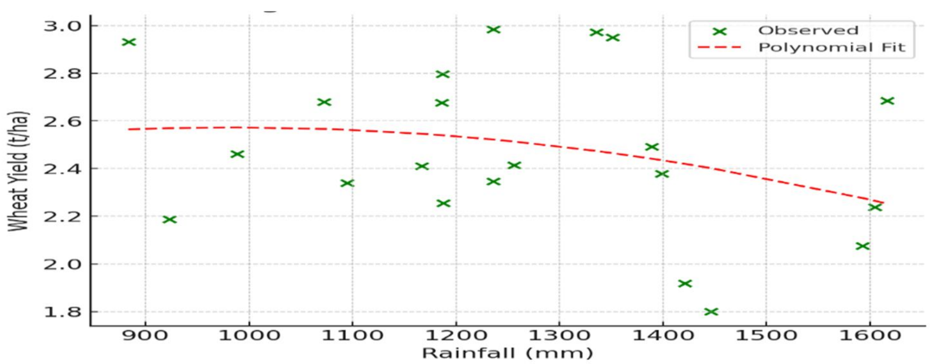

Figure 2: Scatter plot of wheat yield and rainfall

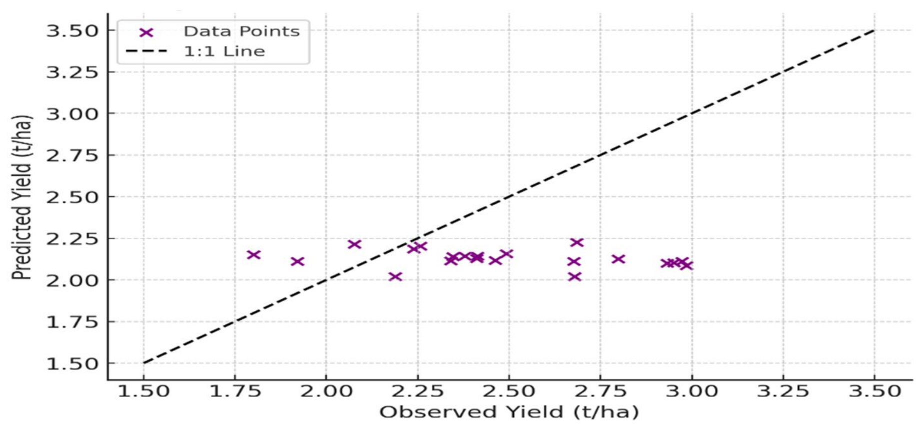

Figure 3: Observed and predicted wheat yield plot

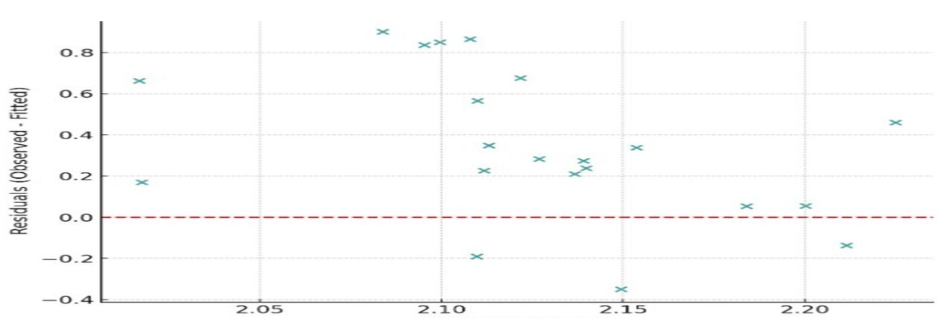

Figure 4: Scatter plot of residuals and fitted wheat yield

IV. CONCLUSIONS

This study created a climate-integrated mathematical model that effectively represented the main factors influencing Ranchi's wheat production fluctuation over a 22-year period. With elasticities of roughly 0.65 and 0.35, respectively with GDD and rainfall, the results showed that accumulated growing degree days (GDD) have a greater impact on wheat yields than annual rainfall. With a strong R^2 value of 0.72, the logistic model structure aligns well with accepted physiologic and agronomic concepts and supports past findings. The random residual patterns of predicted wheat yield and Shapiro-Wilk test support model robustness. This study underscores the primary role of accumulated temperature in wheat productivity, while rainfall ensures moisture sufficiency up to a saturation point, beyond which additional rain offers diminishing returns. Yield improvements with rising temperature are feasible but bounded by physiological limits. The model warns against over-reliance on rainfall due to its plateauing effect beyond a threshold. In order to enhance resilient agricultural planning for eastern India, future research could improve this strategy by adding cultivar-specific characteristics, soil moisture dynamics, and climate projections.

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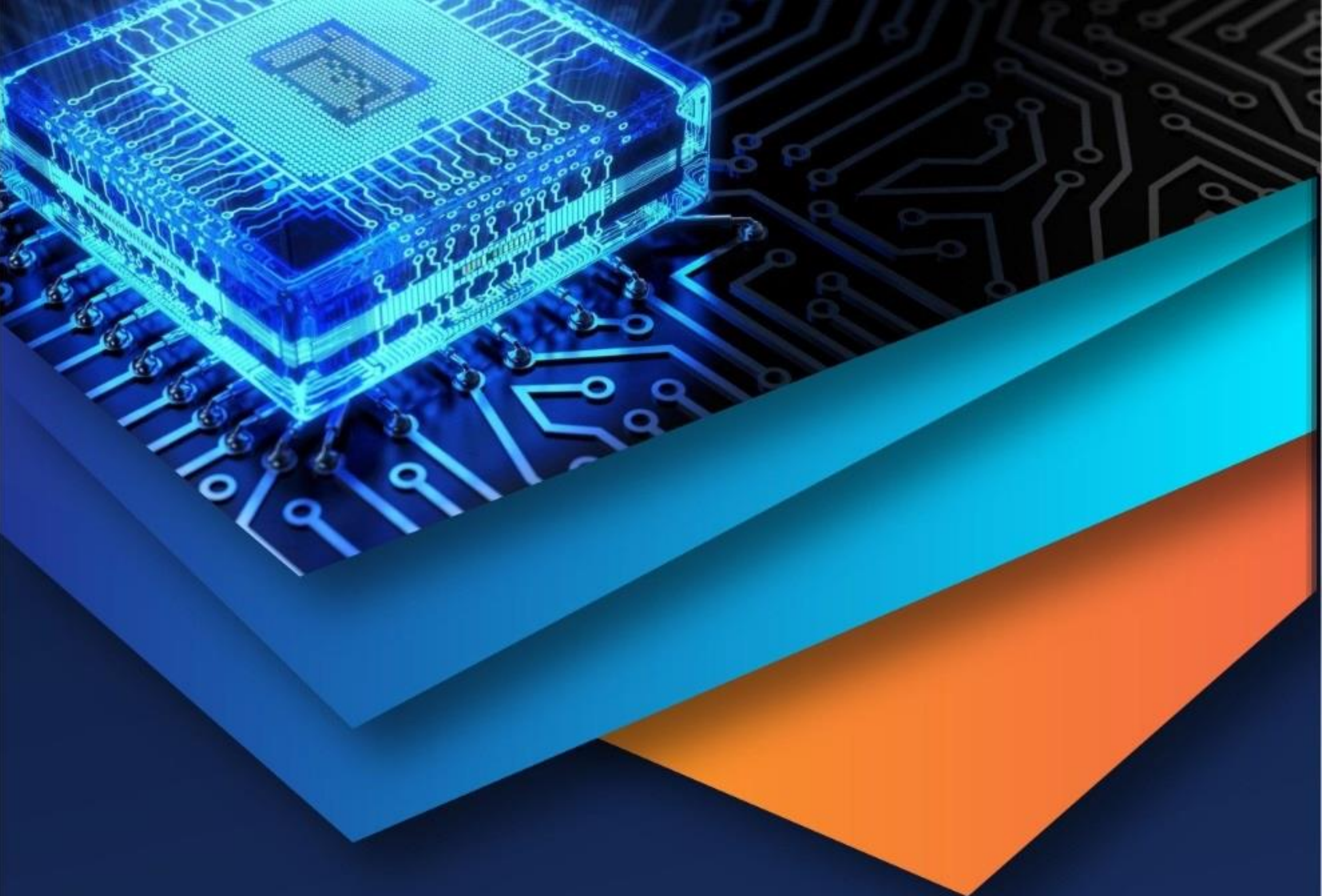

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