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Mathematical Study on Palatable Diet Administration with a Reference to Diabetes Mellitus on Varying Body Frames - ANOVA Test with PFC Ratio

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Abstract: A mathematical model is presented in this paper to compute the palatable prescription of chosen diet for diabetes mellitus type-II using Joselin's principle. Two types of portions of the food as red portion and black portionare chosen for palatable diet with 1:1:1gm equivalent to 4:9:4cal.taken as Protein(P), Fat(F) and Carbohydrate(C) respectively. For the analysis, three types of body frames as Small(S), Medium(M) and Large(L) for both men and women aged 19 years and above are considered. A reference tojuvenileis introduced for the palatable diet. Differential equationsin the coupled form are definedto explain the varying glucose level and varying insulin level. Average calorie values are computed in reference to age and height for the estimation of PFC using linear regression model. ANOVA gives the comparison of results of calorie estimation for palatable diet chosen. With the administration of palatable diet, the values of PFC can be varied to meet the required glucose level of diabetes mellitus type-II.
Keywords: Joselin's principle, Protein, Carbohydrate, Fat, ANOVA, Diabetes mellitus type- II

I. INTRODUCTION

Diabetes is the disorder of metabolism causing excessive thirst and the production of large volume of urine containing excess of sugar. Metabolic disorder is of two types:

I)Diabetes insipidus: This is the rare metabolic disorder in which the body passes large quantity of colourless urine that contains more water causing thirsty, dry hands, constipation. This is due to the failure of kidney's function where in the water is to be reabsorbed.

II)Diabetes mellitus:This is the disorder of carbohydrate metabolism in which sugars in the body are not oxidized to produce energy due to lack of pancreatic hormone insulin. The accumulation of sugar leads to cause of hyperglycaemia in the blood. The disorder may be triggered by the various factors including physical stress, weight loss, retinopathy and hardening of the arteries (atherosclerosis). Also, due to unoxidized glucose, the cells in the lining of blood vessels and other organs may get damaged. The series of such events cause the arteries get clogged and resulting in heart attack and stroke. The systolic and diastolic blood pressure shows the increase from 4.77% to 10.35% when compared to normal values 80-120 mm Hg. As aresult the viscosity of blood decreases, the flow rate increases. But due to increase in viscosity, the flow rate decreases in the arteries. Further the micro aneurysms develop in the arteries when the cell function is disturbed. In order to regulate the proper utilization of blood glucose, administration of insulin and palatable diet distribution are necessary for varying body frames to maintain the proper utilization of insulin for the metabolic activities.

A general approach of Gatewood et.al[13] explained the effect of hormone glucose and insulin secreted by pituitary and thyroxin produced by thyroid as simplest mathematical form. Brownlee et. al[9] described glycosylation end products in tissue and the biochemical basis of diabetic complications. They also analysed the fluctuations of chemical compounds in diabetic patients. Kapur[19] presented a compartment model for diabetes mellitus taking the interaction of blood glucose with insulin at different time intervals. Bankroft et.al[5] made a comparative study of dysfunction in men with and without diabetes mellitus. Bartholovistsch et.al[8] studied the behaviour of viscosity of the blood in diabetic cases and non-diabetic cases.

Sathia [28] analysed the treatment of Alpha-Lipoic Acid (ALA) for the improvement of glucose effectiveness in lean and obese patients with type-II diabetes mellitus (non-insulin-dependent diabetes mellitus (NIDDM)). The variations were observed on whole blood viscosity, haematocrit, shear rate and the plasma viscosity. They also explained the fundus appearance in hypersensitive and diabetic, lack of exercise, a poor diet, current smoking and abstinence from alcohol use were all associated with significantly increased risk factors of diabetes. Gary et.al[12] presented the effect of body weight of a free 76 kilojoules (320 calories) daily supplement of almonds for six months. Katiyar and Basavarajappa et.al[20] analysed the diabetes mellitus under palatable composition of quantitative diet with varying body frames. Allick Gideon et.al[1] compared the effects of a caloric high carbohydrate and high fat improves glucoregulation in type- II diabetes mellitus by reducing post absorptive glycogenolysis.Venkatapuram et.al[30] explained metabolic syndrome is considered to be a metabolic precursor of type – II diabetes mellitus and is an independent risk factor in the pathogenesis of atherosclerosis. MeenaVerma et. al[22] presented the effect of increasing duration of diabetes mellitustype –II on glycated haemoglobin and insulin sensitivity. Villegas et.al[31] analysed that high intake of foods with a high glycemic index and load, especially rice, the main carbohydrate contributing food may increase the risk of type- II diabetes mellitus in Chinese women.

Keeping in view of the investigations made by various authors, the present paperconcerns the study of mathematical model for diabetes mellitus type-2. The study of palatable composition of quantitative diet using system of coupled linear differential equations is considered in the analysis. Required calorie against the height of human beings is studied as a special case using linear regression model. It is aimed at achieving the required total calorie as the ratio of Ca vs. small, Ca vs. medium and Ca vs. large body frames for men and women with age 25years and above.Studyis focused to improve the outcome of palatable composition in maintaining the blood glucose and insulin levels during fasting (pre-prandial: no caloric intake for the previous 8 hours) and after consuming the breakfast (postprandial: calorific intake in every four hours). Three types of body frames small, medium and large have been considered with an average age variation rate. The initial concentrations of blood glucose and insulin levels are documented at the time of meal-1, meal-2, mean-3 and meal-4 in a successive interval of four hours. The ratio of calorie vs height using regression equation has been computed for various PFC values for three different body frames.Palatable diet under equally divided meals will help the usage of reasonable doses to achieve the closeness of normal blood glucose levels. We have considered three different body frames with different age and height in the regression modelfor computing the total calorie of PFCusing Joslin's principle.

II. FORMULATION

Model refers to the quantitiesas 'x' for blood glucose level, 'y' for blood insulin level, 'z' for food input, 't' for time and 'I' for insulin input. Under the assumption of the normal fasting level of blood glucose with 70-100 mg/100ml before breakfast and 120-140 mg/100ml following a meal, the gradients of blood sugar (x) and the insulin (y) are modelled as,

$$\frac{dx}{dt} = -\alpha xy + \beta x_0 H(x_0 - x) - \beta x H(x_0 - x) + \gamma Z(t) \qquad (1.1)$$

$$\frac{dy}{dt} = (\varphi x) \times H(x - x_0) - (\varphi x_0) H(x - x_0) - \psi y_0 + \xi I(t) \qquad (1.2)$$

where α, β and γ are the positive constants called sensitivity values for insulin, the low blood sugar level, high blood sugar level and the input level respectively for the sugar level (gradient). We consider φ, ψ and ξ as the positive constants and are taken as the sensitivity values for high blood sugar level, insulin level and the input level respectively, 'H' is the unit step function which controls the quantity of food intake sensitivity of the diet plan of four different meals.

The input to the blood sugar level is via food source. The food store can be assumed to be filled periodically and the contents at any stage are reduced in a simple exponential manner.

The source term Z(t) in terms of quantity of food can be expressed as,

$$\xi = Z(t) = \begin{cases} 0 & , t < t_0 \\ Qe^{-\delta(t-t_0)} & , t \geq t_0 \end{cases} \qquad (1.3)$$

Where Q - quantity of meal, δ - delay parameterand t_0- time of the meal. Then we take the maximum effect to mean leakage rate as I(t),

The food intake for the first meal is modelled as,

$$I(t) = \frac{\rho t_1}{t_1 - t_0} + bt + k \qquad (1.4)$$

Let $t_1 - t_0 = \Delta t = 3$ hours

When t= 3 hours, the food intake I(t) is given by,

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$I(t) = \rho + 3b + k$ (1.5)

$z(t) = Qe^{-\delta(t_1 - t_0)}$ (1.6)

Linear regression analysis between the input as variation of height associated with varying body frames give rise to total calorie. Regression coefficients control the change in blood glucose level for the chosen palatable diet for four different meals. The quantities of four different meals in the form of variations of calorie v/s height for various body frames has been taken into consideration in the following regression equation.

$\mu_{E/T} = \beta_0 + \beta_1 x$ (1.7)

where E-calorie T- height,x- variation of height associated with variation of body frame,$\mu_{E/T}$-total calorie with PFC, β_0– regression coefficient for initial set of points(first twomeals)and β_1 – regression coefficient for the normal set of points.

III. ANALYSIS

Equations (1.1) and (1.2) describe the blood sugar and insulin levels. Changing the non-linear term αxyas $\alpha\bar{x}y$ (linear term of x and linear term of y are to be taken as independent Solutions) then,

For $x>x_0$, $H(x_0-x) = 0$ (by unit step function)

For $t \le t_0$, $z(t) = 0$, $I(t) = 0$,

for which, the equations (1.1) and (1.2) become,

$\frac{dx}{dt} = -\alpha\bar{x}y$ (1.8)

$\frac{dy}{dt} = \varphi x - \varphi x_0 - y$ (1.9)

The solution the equations (1.1) and (1.2) give the numerical computation to carry out up to 2nd decimal places since, the third & fourth decimals in each computation is sensibly same.

Equations (1.8) and (1.9) have beenanalytically solvedrespectively for x(t) and y(t), using equation (1.7) with $D = d/dx$. Then,

$x(t) = C_1 e^{m_1 t} + C_2 e^{m_2 t} + x_0$ (1.10)

$y(t) = C_3 e^{m_1 t} + C_4 e^{m_2 t}$ (1.11)

The value of x(t) remains within the fastinglevel x_0(pre-prandial, before meal - 1). The value of y(t) remains as parallel solution response. Then,

$x(t) = C_5 e^{m_1 t} + C_6 e^{m_2 t} + x_0 + \frac{\gamma\mu_{E/T}(\Psi-\delta)e^{-\delta(t-t_0)}}{\delta^2 - \Psi\delta + \alpha\varphi\bar{x}}$ (1.12)

$y(t) = C_7 e^{m_1 t} + C_8 e^{m_2 t} + \frac{\gamma\mu_{E/T} e^{-\delta(t-t_0)}}{\delta^2 - \Psi\delta + \alpha\varphi\bar{x}}$ (1.13)

we redefine the physiological parameters as,

$x = x^{PD}, y = y^{PD}, C_1 = C_1^{PD}, C_2 = C_2^{PD}, Q = Q^{PD} = \gamma\mu_{E/T}{}^{PD}$

For PFC by introducing regression model.

We obtain the net variation of Z(t) as food source, then,

$\int_{t_0}^{\infty} z(t)\,dt = \int_{t_0}^{\infty} \mu_{E/T} e^{-\delta(t-t_0)}\,dt$ (1.14)

For food source z(t) at 4 different meals at the average rate of [1482.76396 –2550] calories for men aged 25 years and above.

For food source z(t) at 4 different meals at the average rate of [1484.76396 –1953.66] calories for women aged 25 years and above.

For food source z(t) at 4 different meals at the average rate of [1514.76396 –2034.77] calories for juvenile aged ½ year and 19 years.

By referring the selection of food with normal time for consumption, let a person eats half meal in 10 minutes (1/6 hours), by equation (1.15), we obtain,

$\frac{1}{2}\left(\frac{\mu_{E/T}}{\delta}\right) = \mu_{E/T} \int_{t_0}^{t_0+\frac{1}{6}} e^{-\delta(t-t_0)}\,dt$ (1.15)

Required calorie limits are computedas [1482.76396 – 2256.277703] for men, [1484.76396 – 2041.277703] for women, [1514.76396 – 2431.277703] for juvenile using Joslin's Principle [15].

Using C_1^{PD} and C_2^{PD}, x (t) and y (t) in terms of PD areobtained as,

$x^{PD} = C_1{}^{PD} e^{-t} sint + C_2{}^{PD} e^{-t} cost + \frac{\gamma\mu_{E/T}{}^{PD}(\psi-\delta)e^{-4.16(t-t_0)}}{\delta^2 - \psi\delta + \alpha\varphi\bar{x}} - \frac{\mu t}{\varphi} + \frac{\psi\mu - \alpha\bar{x}\varphi}{\alpha\bar{x}\varphi^2}$ (1.16)

$$y^{PD} = \left(C_1{}^{PD} + C_2{}^{PD}\right)(0.25)e^{-t}sint + \left(-C_1{}^{PD+}C_2{}^{PD}\right)(0.25)e^{-t}cost + \frac{\varphi\gamma\mu_{E/_T}{}^{PD}e^{-\delta(t-t_0)}}{\delta^{2-}\psi\delta+\alpha\varphi\bar{x}} + \frac{\mu}{\alpha\bar{x}\varphi} \quad (1.17)$$

1) Men aged 25 years and above

$$\mu_{E/_{T_M}}{}^{PD} = \begin{cases} T_{sM}^{Ca} = 964.103896 + (2.6727273)T_{sM}^{PFC} \\ T_{mM}^{Ca} = 244.69 + (4.6036)T_{mM}^{PFC} \\ T_{lM}^{Ca} = -507.86618 + (6.734177)T_{lM}^{PFC} \end{cases}$$

2) Women aged 25 years and above

$$\mu_{E/_{T_W}}{}^{PD} = \begin{cases} T_{sW}^{Ca} = 66.62749 + (5.11392)T_{sW}^{PFC} \\ T_{mW}^{Ca} = 53.679 + (5.0964)T_{mM}^{PFC} \\ T_{lW}^{Ca} = 16.0714286 + (5.1666667)T_{lM}^{PFC} \end{cases}$$

3) Juvenile (children) aged ½ year up to 20 years

$\mu_{E/_{T_{ja}}}{}^{PD} = T_{ja}{}^{PD} = - 227.6591085 + [5.854162]\ T_{ja}{}^{PFC}$ (1.18)

For general case of liquid diet (LD) and house diet (HD):

$Q_{LD}^{PD} = T_{LD}^{Ca} = -2650.0 + (29.0)T_{LD}^{PFC}$ (1.19)

$Q_{HD}^{PD} = T_{HD}^{Ca} = 77.5 + (4.0)T_{HD}^{PFC}$ (1.20)

Where $Q_{LD}^{PD} = \mu_{E/_{T_{LD}}}{}^{PD}$ and $Q_{HD}^{PD} = \mu_{E/_{T_{HD}}}{}^{PD}$

Regression equation explains that the variation of height affects the intake of calorie in the palatable diet. As a result the total calorie varies corresponding to the ratio of $\mu_{E/_T}$, where $\mu_{E/_T}$ denotes the total calorie with, E-calorie T- height.

Values of Q_{LD} and Q_{HD} are compared with new model equations consisting of height of the individual as input and calorie as the corresponding output studied through linear outputValues of $\mu_{E/_{T}LD}$ and $\mu_{E/_{T}HD}$

Values of blood sugar x(t) and insulin y(t) are obtained for four different cases using Joslin's principle [15].

IV. RESULTS AND DISCUSSION

By referring to the palatable composition as Protein: Fat: Carbohydrates (P: F: C), it is observed that, glucose place an important role as metabolic fuel. In generalthe percentage of carbohydrate ratio is more than protein and fat for the digestion process in relation to the oxidation of P: F: C. Because the input of the carbohydrate will be well utilized by the brain, insufficient quantities to meet the energy demand. The measurements of blood glucose levelx(t) and insulin level y(t) from equations (1.16) and (1.17) are functions of time t. The graphsreferred to solution of system of differential equations using linear regression model explain the possible(close to thenormal calorie range) for the maintenance of normal values of x(t) in response to corresponding values of y(t).

Joslin's Principle used in the present model:

Protein [P] : 1 gm. = 4 calories
Fat[F] : 1 gm.= 9 calories
Carbohydrate [C] : 1gm.= 4 calories

Men

T_{sm}^{PFC}	T_{sm}^{Ca}	T_{mm}^{PFC}	T_{mm}^{Ca}	T_{lm}^{PFC}	T_{lm}^{Ca}
288	1735	305	1649	328.3	1703.3
313.3	1808.3	331.6	1771.6	360	1916.3
343.3	1881.6	361.6	1909.6	393.3	2141
385	1993.3	403.3	2101.3	430	2388
415	2073.5	440	2270	470	2657.5

Table 1.1 : Total PFC for Men(M), with small(s), medium(m) and large(l) body frames

Women

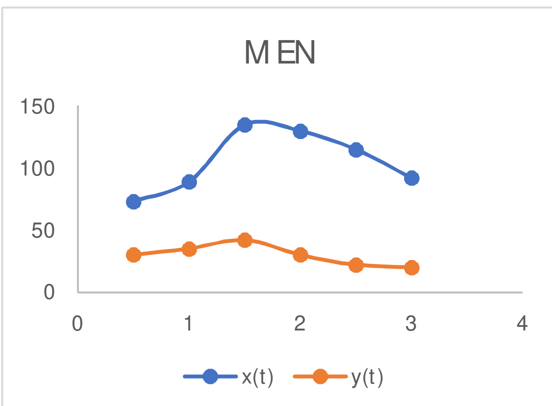
T_{sm}^{Ca}	T_{sm}^{Ca}	T_{mm}^{PFC}	T_{mm}^{Ca}	T_{lm}^{PFC}	T_{lm}^{Ca}
255	1370.67	273.3	1446.67	288.3	1506
275	1473	291.67	1540.3	323.3	1687.67
298.3	1592.3	353.3	1719	348.3	1816
323.3	1720	343.3	1803.67	373.3	1945
345	1831		1939	385	2005

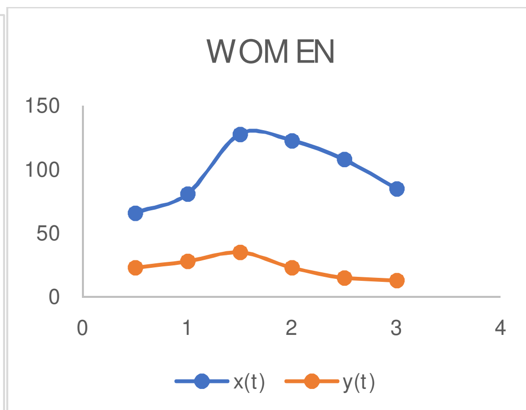
Table 1.2 : Total PFC for Women(W), with small(s), medium(m) and large(l) body frames

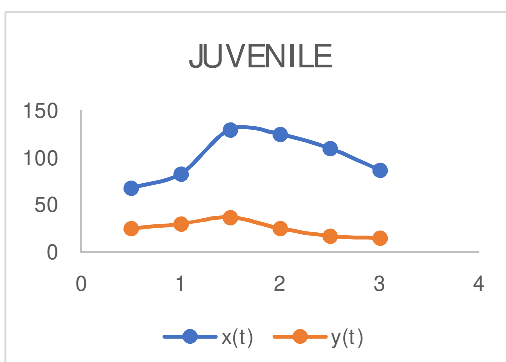
Juvenile

$_{ja}^{PFC}$	J_{ja}^{Ca}
172	779.3
239.67	1175.66
293	1488
352	1831.3
399.5	2111

Table 1.3 : Total PFC for Juvenile with average(a) body frames







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