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Methodology for Designing Kinetic Planar Surface through Tessellation

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Abstract: This paper illustrates the creation of kinetic planar surface by tessellation method. Tessellation is a traditional technique to usually cover a plane without gaps and create a decorative pattern. As we talk about kinetic design, it is an attractive strategy to solve the adaptive and responsive environmental architectural problems. Mathematical tessellation are the techniques to be used to design a surface. The purpose of this study is to develop a methodology for designing the kinetic planar surface with the help of regular tessellation technique in the area of architecture, mechanical and mathematical interdisciplinary approach. Her the method comprises of repetition of interconnected geometry, whose design derive from the symmetry that is taken from the original reference tessellation.

Keywords: tessellation, regular tessellation, planar surface, planar mechanism, kinetic planar surface, kinetic architecture, mobility.

I. INTRODUCTION

This paper deals with the study to develop a methodology for designing of kinetic planar surface by the use of mathematical tessellation in the area of architecture and mechanical an interdisciplinary approach. As surface design is one of the important interest for both artist and architects throughout the history. Static architecture has been the only medium of relation between architecture and mathematics. But due to rapid change in modern activities and society the need of adaptation has emerged and kinetic architecture has gained importance b y developing in technical and construction properties. As the result of literature review, researchers and designers have considered a particular mechanism type in kinetic architecture. Therefore, the mechanism structure , number of joints and the links can be easily controlled at the same time. However, researchers design varies building with many control elements. As kinetic architecture needs a methodology to enhance the ability on various elements of building in a more easy and rapid way. This paper develops the relation between design process and architecture by kinetic means upon developing the method for kinetic planar surface. Tessellation is a common mathematical technique that is use to cover a plane without any gaps or overlaps. As it has been used to design planar surface as the facades of building. In literature tessellation usually means "tilling". For example "portraying human begins and natural scenes are the concern of Mediterranean peoples in intricate mosaics" "(Grunbaum and shepherd, 1986). The definition " in mathematical approach, tilling means that accountability condition excludes families in which every tile has zero area(such as point or line segment) but nevertheless the definition admits tilling in which some tiles have bizarre shapes and properties"[1]. In mathematical approach first remarkable study was conducted by Johannes kepler in the book"Harmonice Mundi in (1619). Another important study was done by the Russian crystallographer E.S.Fedorov in 1891; he proved that every tessellation of the plane is constructed in accordance to one of the seventeen different groups of isometrics (Thinkquest, 2010). Moreover, Grünbaum and Shephard (1986) explained in their book that in the past there have been many attempts that try to describe and systematize the notation of tessellation. For instance "Bourgoin (1873, 1880, 1883, 1901), Day (1903), Dresser (1862), Edwards (1932), Meyer (1888)(figure 1).

Schauermann (1892), Wersin (1953) is noteworthy mainly for the extraordinary extent, which it plagiarizes Bourgouin (1883)[2].
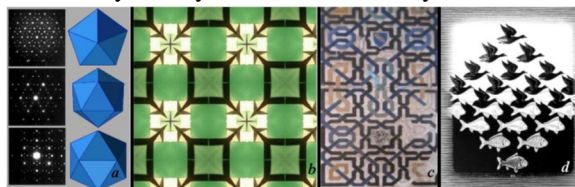

Figure 1 Tessellations in art, architecture and science a) Relationship between tessellations and x-ray crystallography, b) Origami tessellation, c)Tessellation surface, d) Echer "sky" (Source: Thinkquest, ISU, 5cense,Flickr, 2009)

To cover a lane by tessellation technique geometry is very important, to analyze which shape tessellate the planar surface?, what kind of iteration type exist and how polygons are combines. In this platform used in kinetic planar surface the tessellation is classified in two parts . firstly by iteration way and secondly based on shapes. The way of iteration combines 2 type. Periodic if finite section of the tilling that can translated in two parallel directions to recreate the entire tiling(Kinsey and Moore 2002) (figure 2). secondly, non periodic where the pattern don't repeat in a linear direction (figure 3)
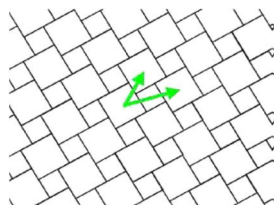

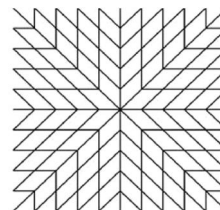Figure 2 Periodic tilling

Figure 3 Non periodic

Based on shape and polygon its been understood that all the polygons shapes can not be combined to fit in a planar surface without any gaps or overlaps. After the study its been seen that triangle is the simplest polygon and it can tessellate any kind of plane,(Seymour and britton,1989)(figure1.3) so according to Seymour and Britton only triangle and quadrilateral polygon can tessellate the plane themselves, as interior angle of triangle is 180⁰,and that of quadrilateral is 360⁰.

Jaspreet kharia displays five polygons and their properties to tessellate a plane [3](figure 4)

Triangle-The three exterior angles must add to make 360 °. 60 is a factor of 360 and so the equilateral will

Square- The four exterior angles must add to make 360 °. Therefore each one must be (360/4) 90 °.90is a factor of 360 and so a square will tessellate.

Pentagon- The five exterior angles must add to make 360 °. The interior angles are (180-72) 108 °.60 is a factor of 360 and so the equilateral will tessellate.

Hexagon- The six exterior angles must add to make 360 °. 120 is a factor of 360 and so a hexagon will tessellate.
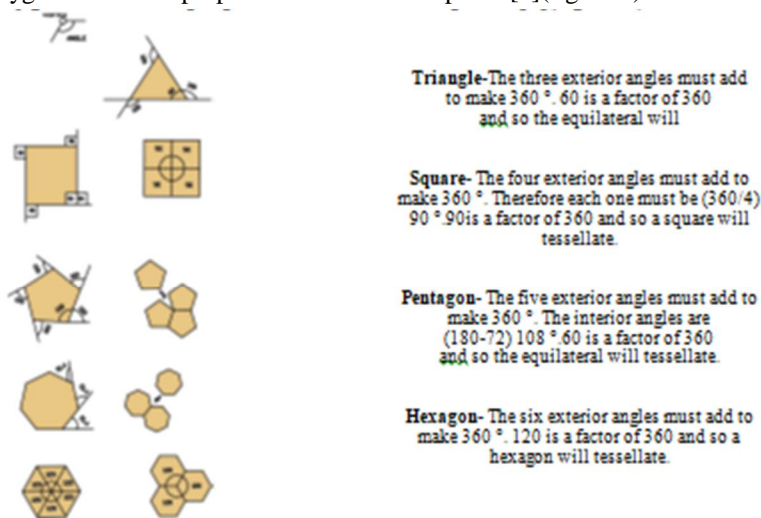
Figure 4
(source: Khaira, 2009)

From the point of view, guiruband Shepherd inquired the possibilities of edge-to-edge tilings on the plane. They say that "the interior angle at each corner of a regular n{n} is (n-2) /n radians (or 180
(n-2)/n degrees) so that if an n1-{n1}, an n2- {n2},…., an nr –{nr}, meet
at a vertex of tiling then" [4]

$$\frac{n1-2}{n1} + \ldots\ldots\ldots + \frac{nr-2}{nr} = 2$$

By this study there exist three regular and eight semi-irregular tessellation and we develop a method with three regular tessellation (square, triangular, hexagonal).(figure 5 and 6)
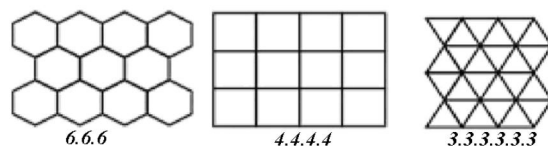
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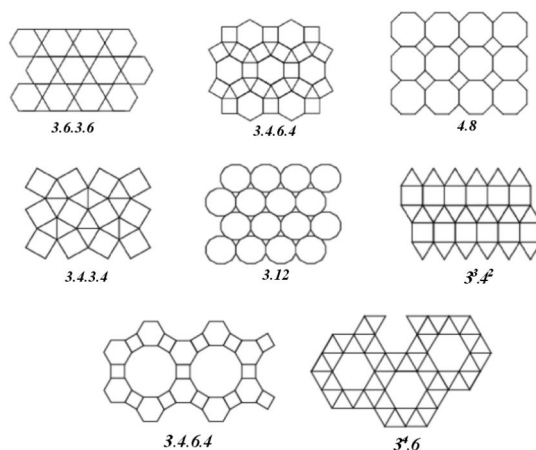*Volume 11 Issue VII Jul 2023- Available at www.ijraset.com*

Figure 5

Figure 6

II. PLANAR MECHANISM

After the analysis of tessellation techniques the study of planar mechanism is important to understand the motion on the planar surfaces, its properties of linkage and joint. the discipline of the mechanism science can be divided in two parts as static and dynamic. Static analyze the stationary system where time is not a factor but there dynamic deals with system that is in motion throughout the time. Dynamic is also slated into kinetics and kinematics. As the mechanism is categorized with the respect to the relative motion of the rigid bodies such as planar spherical and spatial mechanism.

III.DESIGN OF MECHANISM

As per the research "Robert L. Norton(2004) defines the process of applying the various techniques and scientific principles for the purpose of defining a device a process of a system in insufficient detail to permit its realization"[5]. Yan(1998) show the process in a simple flow chart(figure 7).
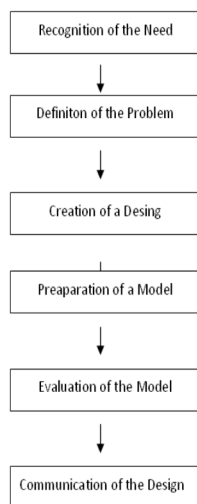

Figure 7 engineering design process
(Source yan,1998)

It's understood that the mechanism is constituted with the structural synthesis and henceforth structural synthesis is constitutes with the type and number synthesis as it is the fundamental part of the mechanism design process(figure 8). In the process various things are determined such as type of mechanism, form and size of platform and links. Therefore this the crucial point for the study because of the process of structural synthesis is the main problem in designing variable building parts.
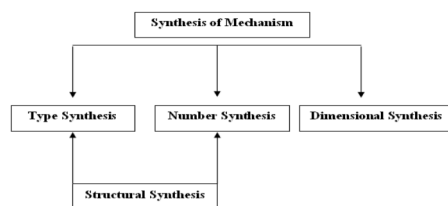
Figure 8 synthesis of mechanism

IV. DESIGN OF KINETIC PLANAR SURFACE

The method developed with the architectural and mechanical interdisciplinary approach consists of two parts. One that deals with regular planar surface and second that deals with irregular planar surface. Both the methodology try resolve the question "which form or size of the platform and linkage should be chosen and how they should be joint to reach the kinetic planar surface. This research is limited to regular tessellation.

V. METHOD FOR KINETIC REGULAR TESSELLATION

Tessellation with regarding to shapes is known as regular and semi-regular tessellation. The most important point in this case is that the planar mechanism reaches the regular tessellation on the first point of the motion and on the last point of the motion . the method is inspired from the duality of tessellation. Geometric forms transform to unique form and again transform to first form. For example "hexagonal can transform triangle and vise versa", similar to motion of kinetic structure. To examine the kinetic tessellation the method have following three steps.

1) To determine the platform which compose regular tessellation
2) To determine the link by the help of duality
3) To determine the combination of order of the platform and the link.

VI. KINETIC SQUARE TESSELLATION
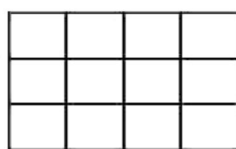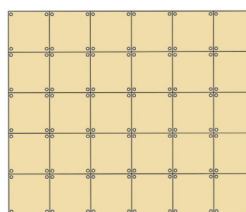
It's a form of regular tessellation ({4,4}figure 9-10).

{4, 4}

Figure 9
closed form of square tessellation

Figure 10
Square tessellation

The first step of methodology is to find the form of the first link by regular tessellation (figure 11)
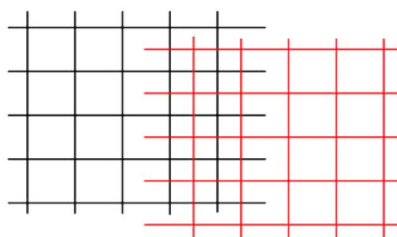
Figure 11 dual tessellation of square

Black line shows the real square tessellation while the red shows the dual of the square tessellation. This give the desired information about the form of the platform and the link. Now the determination of the vertices is important. Thus , the link length will be determined by the pointing one vertex to another(figure 12)
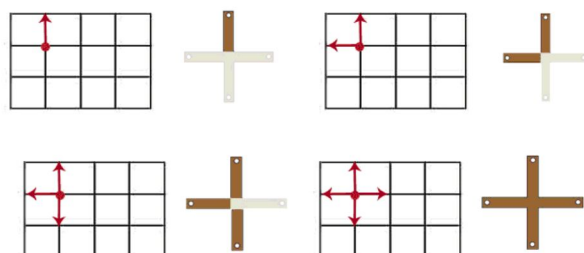

Figure 12 Process of achieving the form of the first link.

The second step comprises of dual of the tessellation providing the form of the platform of mechanism (figure 13).
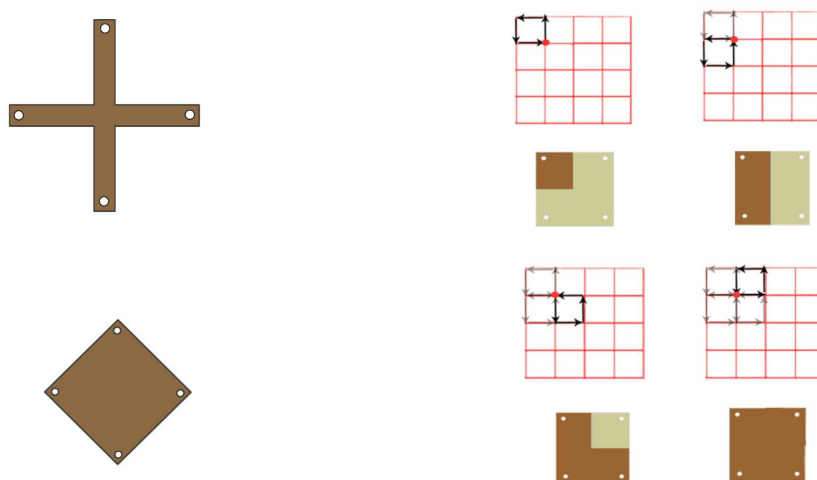
Figure 13 Process of obtaining the form of the platform and form of square tessellation platform and link

The third step is the determination of the placement of combination of the link and platform. Firstly the platform of the mechanism will be placed where the square tessellation is pointed and its dual intersect, and secondly the link of the mechanism will be added to the platform(figure 14-16)
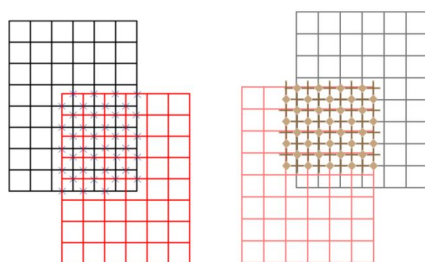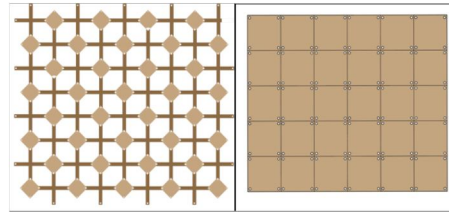

Figure 14 placement of the kinetic square tessellation

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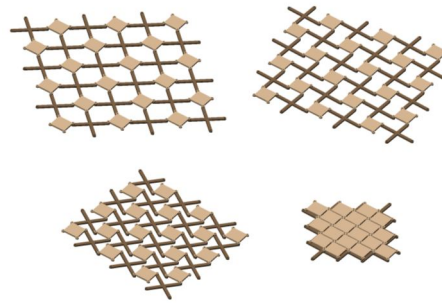Figure 15 Open and closed form of kinetic regular tessellation

Figure 16 Kinetic square tessellation

VII. KINETIC HEXAGONAL TESSELLATION
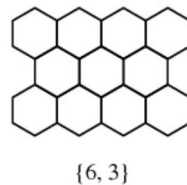
Hexagonal tesellationn {6,3}(figure 17-18).

$\{6, 3\}$

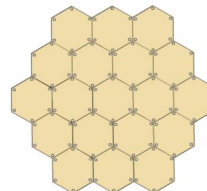| Figure 17	Figure 18
Hexagonal tessellation	closed form of kinetic
hexagonal tesellation	

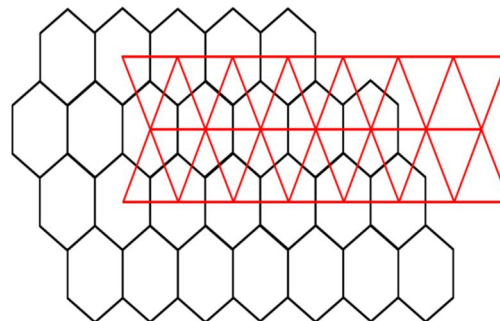The fist step is to find the first link of the form (figure 19-20)

Figure 19
Dual tessellation of the hexagonal tesellation

The black line shows the eal part and red line depicts the dual of hexagonal tessellation. the length of the linkis determined bythe pointing of one vertex to the other around.
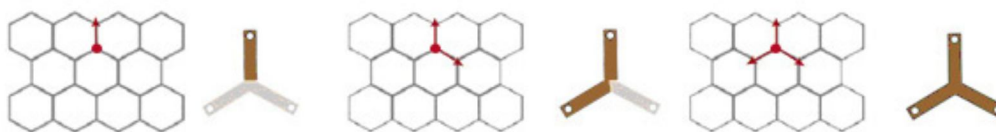
Figure 20 Process of obtaining the form of first link

the second step is to dual the tessellation that provide the form of the platform. the dual of heagonal tesellation is triangulaer to the ceters of adjacent. the successively line drawn is started from the vertex point until the same vertex is reached(figure 21).
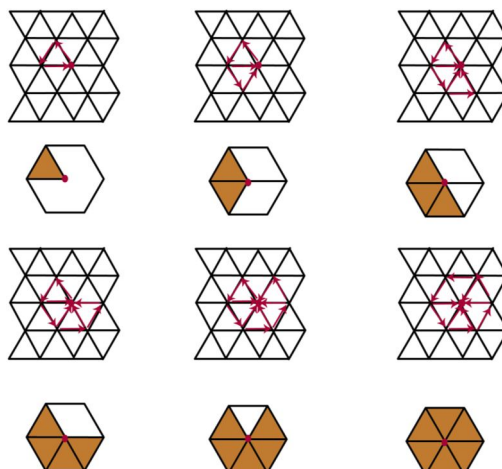

Figure 21
Process of obtaining the form of the platform

the final step is determined for the placementt of the combiination of link and platform as the intersection of the hexangonal tessellation and its dual that is the triangular tessellation(figure 22).
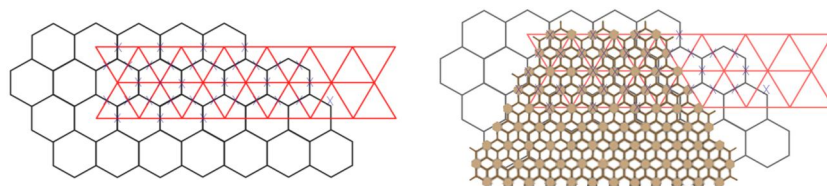
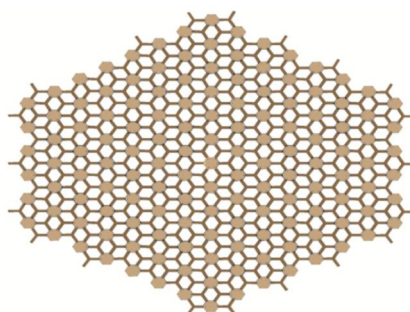

Figure 22
Placement of the kinetic hexagonal tessellation

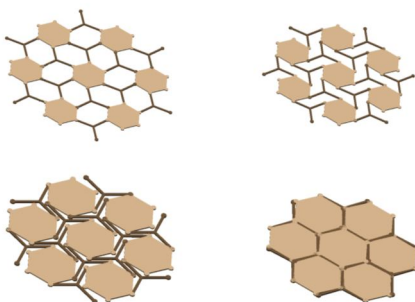

Figure 23
Open form of the kinetic hexagonal tessellation

Figure 24
Kinetic hexagonal tessellation.

VIII. MOBILITY OF THE KNETIC REGULAR TESSELLATION

the visual Nastran 4d is used to analyze the performance and motion of regular kinetic tessellation and mobility are calculated by the alizade formulation

where f_i is the degrees of freedom of ith kinematic pair

λ_k is the space or subspace number of the kth

independent loops

L is the total number of independent loops,

q is the number of excessive link

p_j is the number of passive joints.

$$\sum f_i - \sum_{k=1}^{L} \lambda_k + q - j_p = M$$

Now Acoording to Grubber mobility criteria formula,

where l is the total number of links including ground link

i_P is the total number of joints with

ith degrees of freedom.

$$\lambda(l-1) - \sum_{i=1}^{\lambda-1}(\lambda-i)P_i + q - j_p$$

As the research deals in planar surface,so $(\lambda=3),$ the equation become

$$P_1 - 3L + q - j_p = M$$

$$3(l-1) - 2P_1 + q - j_p = M$$

So start the process, smallest mobile element of regular tessellation should be considered.

if the mobility of the smallest system wll be calculated as unity (M=1)then the following variabels are instered in formulation.
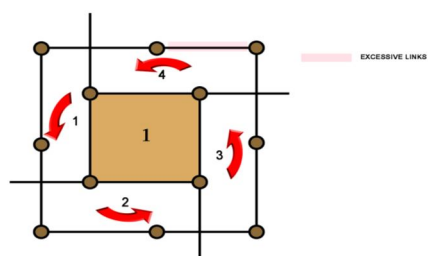

Figure 25 smallest mobile element

L	l	p_j	q	1_P
4	9	0	1	12

Table 1 variable for eqation of figure 25

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After the calculation of first module. now lets calculate two modules sharing one loop as shown variables in table 2 of figure 26

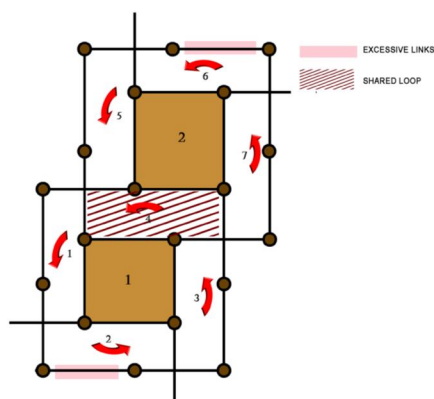Figure 26 First iterated mobile module

L	l	p j	q	1 P
7	14	0	1+1	20

Table 2 Variables for equation figure 26
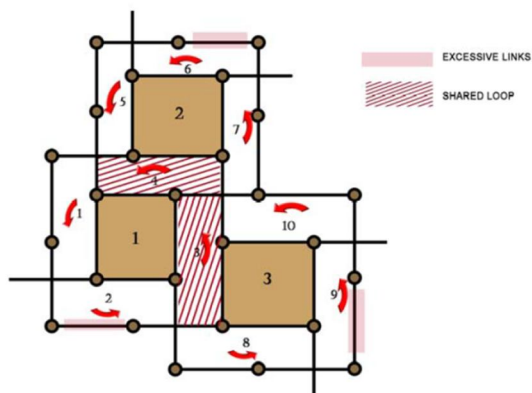
Figure 27 second iterated mobile module

L	l	$p\,j$	q	1 P
10	19	0	2+1	28

Table 3 Variables for equation figure 27
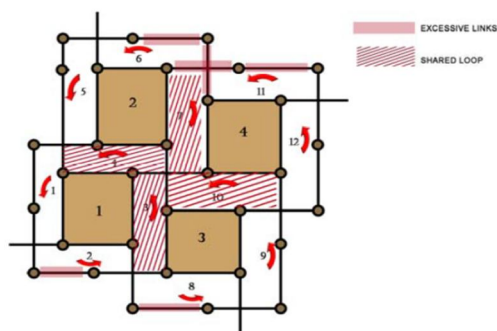
Figure 28 Third iterated mobile module

L	l	$p\,j$	q	1 P
12	20	0	4+1	32

Table 4 Variables for equation figure 28

As the result of following iterations in module of regular tessellation it is noted that the result of mobility of the system will always revelead to be as unity (m=1). in fact the last iteration where there is sharing of two loops with addition of one module will add two more exceesive link to the syatem.
 as of the study of iteration procedure, a new theorm can be introduced with mobility calculation in regular tessellation.

Theorm: *Number of redundant or excessive links is equal to the onle plus the number of loops that will be shared during the whole iteration process.*

IX. CONCLUSION

As per the above studies and properties of methedolgy following conclusions are founded,as one of the concept of kinetic architecture is to fulfil the rapid changing of life conditions, and the result of this , the kinetic building parts should be constructed easily and cheap. this methedology can be used. the regular kinetic tessellation method, form, sixze and placement of platforms and links can easily be determined and it help in designing more comples pattern with simple construction fastly.Many architects and researchers deals with kinetic structures ends up covering the kinetic structure usually by textile or flexible material.
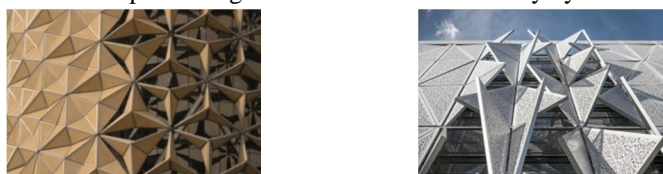

Figure 29 kinetic facade through tessellation

Kinetic architecture is an interdiciplinary controversial approach between mechanism science and architecture. the rapid changes in environmental conditions effected human sourcs and architecture traditional aspect. there are few examples in application of structure as building parts .

NextOffice   Bund Finance Center

Figure 30 kinetic building parts

Finally the relationship between mathematical knowledge, mechanism science and building parts dereves more serious considderation in kinetic architecture.

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