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Neutrosophic Cubic &-Fuzzy Ideals with Ternion Semigroups

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Abstract: The aim of this paper is to define neutrosophic cubic $\$ -ternion subsemigroup, neutrosophic(neut.) cubic $\$ -left ideal(LI), (resp. lateral ideal(LI), right ideal(RI)) of ternion semigroup(SG) with suitable example, to define characteristic neut. cubic $\$ -structure($\$ -S) of ternion SG. Additionally intersection of two neut. cubic $\$ -LI(resp. LI, RI) is also a neut. cubic $\$ -LI(resp. LI, RI). We find intersection between two neut. cubic $\$ -LI(resp. LI, RI) in ternion SG is neut. cubic $\$ -LI(resp. LI, RI) in ternion SG. Further if we have an neut. cubic $\$ -LI(resp. LI, RI) then its pre-image is also neut. cubic $\$ -LI(resp. LI, RI) of ternion SG. In this study a new algebraic approach has been developed in neut. cubic $\$ -ideals in ternion SG. In future this neut. cubic $\$ -future this neut.

Keywords: Neutrosophic fuzzy set, Interval Valued Neutrosophic cubic &-fuzzy ideals, Ternion Semigroup, Direct product of cubic ternion semigroup, Homomorphism of cubic ternion semigroup.

I. INTRODUCTION

R.Chinram et al.^[4] in 2023 studied a new notion on covered left ideals of ternary SG. A. Nongmanee and S. Leeratanavalee^[13] studied about quaternary rectangular bands and representations of ternary SG in 2022. L. A. Zadeh ^[23] introduced the notion of interval valued (in short i-v) fuzzy subset where the values of the membership functions are closed interval number instead of single value in 1975.

Li. Chunnu et al.^[5] discussed a new characterization of fuzzy ideals of SG in 2021. K. T. Atanassove ^[1] introduced the notion of an intuitionistic fuzzy set. F. Smarandache^[20] introduced the notion of neutrosophic sets (in short neut.) which is useful mathematical tool for dealing with membership, non membership and indeterminacy function. D. H. Lehmer^[9] introduced ternary analogue of abelian group in 1932. Madad Khan et al. ^[11] introduced the notion of neut. \mathcal{N} -subsemigroup(SSG) in SG and investigated several properties. K. Lenin Muthu Kumaran and S. Selvaraj^[10] discussed about interval valued neut. \aleph -fuzzy ideals in SG in 2023. S. Amalanila and S. Jayalakshmi^[2] discussed a cubic (1,2) ideals of cubic near-rings. V. Chinnadurai and K. Bharathivelan^[3] studied a new notion of cubic lateral ideals in ternary near-rings. Muhammad Gulistan^[12] introduced the notion of neutrosophic cubic (α, β)-ideals in semigroup with application. The above ideas motivate us to define the notion of neut. cubic \aleph -fuzzy ideals in ternion SG.

In this paper, the notion of neut. cubic \aleph -ideals in ternion (means ternary) SG is introduced and several properties are investigated such as LI, \pounds I, RI etc. Further, conditions for neut. cubic \aleph -S to be neut. cubic \aleph -ideals in ternion SG are provided. Furthermore, we explore the ideas of characteristic function, level sets of neut. cubic \aleph -S of ternion SG, direct product, intersection property of neut. cubic \aleph -S of fuzzy ideal in ternion SG and homomorphism of neut. cubic \aleph -ideals in ternion SG and its related properties.

II. METHODOLOGY

In this research work the results of neut. cubic \aleph -ideals in ternion SG are used such as neut. cubic \aleph -ternion SSG, neut. cubic \aleph -ternion SG, neut. cubic \aleph -structure in ternion SG.

III. RESULT AND DISCUSSION

In this section, we define the idea of neutrosophic \aleph -cubic ideals in ternion semigroups and investigate using this concept. Definition 3.1

Consider S_c as a neut. cubic \aleph -S defined over the set S. In such a context, S_c is termed as a neut. cubic \aleph -ternion SSG of S if it meets the following criteria:

 $T_{\overline{N}}([uvw]) \leq rmax\{T_{\overline{N}}(u), T_{\overline{N}}(v), T_{\overline{N}}(w)\},\$



$$\begin{split} I_{\overline{N}}([uvw]) &\geq rmin\{I_{\overline{N}}(u), I_{\overline{N}}(v), I_{\overline{N}}(w)\},\\ F_{\overline{N}}([uvw]) &\geq rmin\{F_{\overline{N}}(u), F_{\overline{N}}(v), F_{\overline{N}}(w)\}, \text{ and }\\ T_{N}([uvw]) &\geq min\{T_{N}(u), T_{N}(v), T_{N}(w)\},\\ I_{N}([uvw]) &\leq max\{I_{N}(u), I_{N}(v), I_{N}(w)\},\\ F_{N}([uvw]) &\leq max\{F_{N}(u), F_{N}(v), F_{N}(w)\} \text{ for all } u, v, w \in S. \end{split}$$

Definition 3.2

An neut. cubic \aleph -S S_c over set S is considered as an neut. cubic $\aleph - LI$ (resp. $\aleph - LI$, $\aleph - RI$) of S if it adheres to the following criteria:

$$\begin{split} T_{\overline{N}}([uvw]) &\leq T_{\overline{N}}(w) \left(resp. T_{\overline{N}}([uvw]] \right) \leq T_{\overline{N}}(v), T_{\overline{N}}([uvw]] \right) \leq T_{\overline{N}}(u) \right) \\ I_{\overline{N}}([uvw]) &\geq I_{\overline{N}}(w) \left(resp. I_{\overline{N}}([uvw]] \right) \geq I_{\overline{N}}(v), I_{\overline{N}}([uvw]] \right) \geq I_{\overline{N}}(u) \right) \\ F_{\overline{N}}([uvw]]) &\geq F_{\overline{N}}(w) \left(resp. F_{\overline{N}}([uvw]] \right) \geq F_{\overline{N}}(v), F_{\overline{N}}([uvw]] \right) \geq F_{\overline{N}}(u) \right), \text{ and} \\ T_{\overline{N}}([uvw]]) &\geq T_{\overline{N}}(w) \left(resp. T_{\overline{N}}([uvw]] \right) \geq T_{\overline{N}}(v), T_{\overline{N}}([uvw]] \geq T_{\overline{N}}(u) \right) \\ I_{\overline{N}}([uvw]]) &\leq I_{\overline{N}}(w) \left(resp. I_{\overline{N}}([uvw]] \right) \leq I_{\overline{N}}(v), I_{\overline{N}}([uvw]] \leq I_{\overline{N}}(u) \right) \\ F_{\overline{N}}([uvw]]) &\leq F_{\overline{N}}(w) \left(reps. F_{\overline{N}}([uvw]] \right) \leq F_{\overline{N}}(v), F_{\overline{N}}([uvw]] \leq F_{\overline{N}}(u) \right) \text{ for all } u, v, w \in S. \end{split}$$

If S_c is a neut. cubic \aleph -LI, \aleph - \mathcal{L} I, \aleph - RI of S, then S_c is said to be neut. cubic \aleph -ideal of S.

Note 3.3

Every neut. cubic \aleph -*LI* (*resp. LI*, *RI*) within a ternion SG qualifies as an neut. cubic \aleph -ternion SSG, the reverse may not hold true, (ie) neut. cubic \aleph -ternion SSG is not necessarily an neut. cubic \aleph -LI, nor is it required to be an neut. cubic \aleph -*L*I or an neut. cubic \aleph -RI, as demonstrated in the subsequent example.

Example 3.4

Let $S = \{0, 1, 2, 3\}$ and define the ternion operation [] on S as follows:

F1	Δ	1	2	2	1	F 1	Δ	1	2	2		F1	Δ	1	2	2		F1	Δ	1	2	2
L	0	1	Z	3		L	0	1	2	3		IJ	U	1	2	3		L	0	1	Z	3
00	0	1	2	3		10	1	1	1	3		20	0	1	2	3		30	3	3	3	3
01	0	1	2	3		11	1	1	1	3		21	0	1	2	3		31	3	3	3	3
02	0	1	2	3		12	1	1	1	3		22	0	1	2	3		32	3	3	3	3
03	3	3	3	3		13	3	3	3	3		23	3	3	3	3		33	3	3	3	3
Table 1				Table 2						Table 3							Table 4					

Then, (S, []) is a ternion SG [16]. Define an neut. cubic \aleph -S, S_C over S as follows:

$T_{\overline{N}}(0) = [-0.7, -0.5],$	$I_{\overline{N}}(0) = [-0.5, -0.1],$
$T_{\overline{N}}(1) = [-0.7, -0.5],$	$I_{\overline{N}}(1) = [-0.5, -0.1],$
$T_{\overline{N}}(2) = [-0.6, -0.4],$	$I_{\overline{N}}(2) = [-0.6, -0.3],$
$T_{\overline{N}}(3) = [-0.4, -0.1],$	$I_{\overline{N}}(3) = [-0.7, -0.5],$
$T_N(0) = -0.1, I_N(0) = -0.5,$	$I_N(0) = -0.8$
$T_N(1) = -0.4, I_N(1) = -0.3,$	$I_N(1) = -0.4$
$T_N(2) = -0.5, I_N(2) = -0.1,$	$I_N(2) = -0.1$,
$T_N(3) = -0.5, I_N(3) = -0.1,$	$I_N(3) = -0.1.$

By routine calculation, $S_C = T_{\bar{N}}$, $I_{\bar{N}}$, $F_{\bar{N}}$, T_{N} , I_N , F_N is an neut. cubic ternion \aleph -SSG of *S*, but it is not an neut. cubic \aleph -LI, because $T_{\bar{N}}[130] = [-0.4, -0.1] \leq [-0.7, -0.5] = T_{\bar{N}}(0),$ $I_{\bar{N}}[130] = [-0.7, -0.5] \geq [-0.5, -0.1] = I_{\bar{N}}(0),$ $F_{\bar{N}}[130] = [-0.9, -0.8] \geq [-0.6, -0.1] = F_{\bar{N}}(0).$

 $F_{\overline{N}}(0) = [-0.6, -0.1],$ $F_{\overline{N}}(1) = [-0.6, -0.1],$ $F_{\overline{N}}(2) = [-0.7, -0.4],$ $F_{\overline{N}}(3) = [-0.9, -0.8].$



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Example 3.5 Let $S = \{0, 1, 2, 3\}$ and define the ternion operation [] on *S* as follows:

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[] 0	1	2	3		[]	0	1	2	3		[]	0	1	2	3		[]	0	1	2	3	
0	0 0	1	0	3		10	0	1	0	3		20	0	1	0	3		30	3	3	3	3	
0	01 0	1	0	3		11	0	1	0	3		21	0	1	0	3		31	3	3	3	3	
0	02 0	1	0	3		12	0	1	0	3		22	0	1	0	3		32	3	3	3	3	
0	3 3	3	3	3		13	3	3	3	3		23	3	3	3	3		33	3	3	3	3	
	Table 5 Table 6 Table 7 Table 8																						
Then, $(S, [])$ is a ternion SG [16]. Define an neut. cubic \aleph -S, S_N over S as follows:																							
$T_{\bar{N}}(0) = [-0.9, -0.8], \qquad I_{\bar{N}}(0) = [-0.5, -0.2], \qquad F_{\bar{N}}(0) = [-0.4, -0.1],$																							
	$T_{\bar{N}}(1) = [-0.8, -0.6],$ $I_{\bar{N}}(1) = [-0.6, -0.4],$ $F_{\bar{N}}(1) = [-0.5, -0.3],$																						
	$T_{\bar{N}}(2) = [-0.5, -0.2],$ $I_{\bar{N}}(2) = [-0.8, -0.7],$ $F_{\bar{N}}(2) = [-0.7, -0.6],$																						
$T_{\overline{N}}(3) = [-0.9, -0.8],$ $I_{\overline{N}}(3) = [-0.5, -0.2],$ $F_{\overline{N}}(3) = [-0.4, -0.1].$																							
$T_N(0) = -0.8, I_N(0) = -0.2, I_N(0) = -0.1,$																							
$T_N(1) = -0.2, I_N(1) = -0.7, I_N(1) = -0.6,$																							
$T_N(2) = -0.6, I_N(2) = -0.4, I_N(2) = -0.3,$																							
$T_N(3)$	$T_N(3) = -0.8, I_N(3) = -0.2, I_N(3) = -0.1.$																						
By routine calculation, $S_C = T_{\overline{N}_1} I_{\overline{N}_1} F_{\overline{N}_1} T_{N_1} I_{N_1} F_N$ is a neut. cubic ternion $\$$ -SSG of S, but it is not a neut. cubic $\$$ - $\mathcal{L}I$, because																							
								14 -		-	-	-	-	-			-	$= T_{\overline{N}}($	• • •				
								14 -			-	·	-			•	-	$=I_{\overline{N}}($	• • •				
								$F_{\overline{N}}[$	101] =	[-0	5,-0).3]	≱ [-	-0.4	1, — (0.1]	$= F_{\overline{N}}$	(0).				
By addi	tion,	S _C is	also	not	a ne	ut. cu	bic	R-א	L, b	ecau	se												
								1.4 -		-	-	-	-	-			-	$= T_{\overline{N}}($	• • •				
								1			-	•	-			•	-	$= I_{\overline{N}}($					
								$F_{\overline{N}}[$	021] =	[-0	5,-0).3]	≱ [-	-0.4	ŧ, − (0.1]	$= F_{\overline{N}}$	(0).				
Definiti																							
Let $S_A = \langle u, T_{\bar{N}}, I_{\bar{N}}, F_{\bar{N}}, T_N, I_N, F_N \rangle$, $S_B = \langle u, T_{\bar{P}}, I_{\bar{P}}, F_{\bar{P}}, T_P, I_P, F_P \rangle$ and $S_C = \langle u, T_{\bar{Q}}, I_{\bar{Q}}, F_{\bar{Q}}, I_Q, F_Q \rangle$ a neut. cubic \aleph -S																							
over S. The neut. cubic \aleph -product of $S_{\overline{N}}$, $S_{\overline{P}}$ and $S_{\overline{Q}}$ is defined by																							
$S_A \odot S_B \odot S_C = \langle u, (T_{\overline{N} \circ \overline{P} \circ \overline{Q}}, I_{\overline{N} \circ \overline{P} \circ \overline{Q}}, F_{\overline{N} \circ \overline{P} \circ \overline{Q}}, I_{N \circ P \circ Q}, I_{N \circ P \circ Q}, F_{N \circ P \circ Q})(u) \rangle$																							
Where,																							
$T_{N} = \{\lambda_{u} = [max\{T_{\bar{N}}(p), T_{\bar{P}}(q), T_{\bar{Q}}(r)\}] = T_{N} = \{\lambda_{u}\} = \{\forall_{u} = [max[T_{N}(p), T_{P}(q), T_{Q}(r)\}]$																							
<i>ĨÑ∘₽</i> ∘ <i>Q</i> ($T_{\overline{N} \circ \overline{P} \circ \overline{Q}}(u) = \begin{cases} \Lambda_{u=[pqr]}[max\{T_{\overline{N}}(p), T_{\overline{P}}(q), T_{\overline{Q}}(r)\}] \\ \overline{0} & \text{otherwise} \end{cases}, T_{N \circ P \circ Q}(u) = \begin{cases} \bigvee_{u=[pqr]}[min\{T_{N}(p), T_{P}(q), T_{Q}(r)\}] \\ 0 & \text{otherwise} \end{cases}$																						
$(\bigvee_{u=lngv}[min\{I_{\bar{N}}(p), I_{\bar{D}}(q), I_{\bar{D}}(r)\}] \qquad (\bigwedge_{u=lngv}[max\{I_{N}(p), I_{D}(q), I_{D}(r)\}]$																							

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$$\begin{split} I_{\overline{N}\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigvee_{u=[pqr]} \left[\min\{I_{\overline{N}}(p), I_{\overline{P}}(q), I_{\overline{Q}}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad I_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{I_{N}(p), I_{P}(q), I_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{I_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigwedge_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigcap_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N\circ\overline{P}\circ\overline{Q}}(u) &= \begin{cases} \bigcap_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N}(p) &= \begin{cases} \bigcap_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N}(p) &= \begin{cases} \bigcap_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N}(p) &= \begin{cases} \bigcap_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N}(p) &= \begin{cases} \bigcap_{u=[pqr]} \left[\max\{F_{N}(p), F_{P}(q), F_{Q}(r)\} \right] \\ -\overline{1} & \text{otherwise} \end{cases}, \quad F_{N}(p) &= \end{cases} \end{cases}$$

Theorem 3.7

Consider a ternion SG denoted as S. If we have two neut. cubic &-LI (resp. LI, RI)'s of S, the intersection of these ideals also qualifies as a neut. cubic &-LI (resp. LI, RI) of S. Proof.

Let
$$S_A = \langle u, (T_{\overline{N}}, I_{\overline{N}}, F_{\overline{N}}, T_N, I_N, F_N)(u) \rangle$$
 and $S_B = \langle u, (T_{\overline{M}}, I_{\overline{M}}, F_{\overline{M}}, T_M, I_N, F_M)(u) \rangle$ be neut. cubic
 $\aleph - LI (resp. \mathcal{L}I, RI)$ of S. Then for any $u, v, w \in S$, we have
 $T_{\overline{N} \cap \overline{M}}([uvw]]) = max\{T_{\overline{N}}([uvw]]), T_{\overline{M}}([uvw]])\}$
 $\leq max\{T_{\overline{N}}(w), T_{\overline{M}}(w)\} = T_{\overline{N} \cap \overline{M}}(w),$
 $I_{\overline{N} \cap \overline{M}}([uvw]]) = min\{I_{\overline{N}}([uvw]]), I_{\overline{M}}([uvw]])\}$



$$\geq \min\{I_{\overline{N}}(w), I_{\overline{M}}(w)\} = I_{\overline{N}\cap\overline{M}}(w), \\ F_{\overline{N}\cap\overline{M}}([uvw]) = \min\{F_{\overline{N}}([uvw]), F_{\overline{M}}([uvw])\} \\ \geq \min\{F_{\overline{N}}(w), F_{\overline{M}}(w)\} = F_{\overline{N}\cap\overline{M}}(w), \\ T_{N\cap M}([uvw]) = \min\{T_{N}([uvw]), T_{N}([uvw])\} \\ \geq \min\{T_{N}(w), T_{M}(w)\} = T_{N\cap M}(w), \\ I_{N\cap M}([uvw]) = \max\{I_{N}([uvw]), I_{M}([uvw])\} \\ \leq \max\{I_{N}(w), I_{M}(w)\} = I_{N\cap M}(w), \\ F_{N\cap M}([uvw]) = \max\{F_{N}([uvw]), F_{M}([uvw])\} \\ \leq \max\{F_{N}(w), F_{M}(w)\} = I_{N\cap M}(w). \\ \text{Therefore } S_{A\cap B} \text{ is a neut. cubic } \aleph - LI (resp. \mathcal{L}I, RI) \text{ of } S.$$

Corollary 3.8

Consider a ternion SG denoted as S. If $\{S_{C_i} | i \in \Lambda\}$ forms a family of neut. cubic $\aleph - LI$ (resp. LI, RI)'s of S, then the set $\bigcap S_{C_i}$ also qualifies as an neut. cubic $\aleph - LI$ (resp. LI, RI) of S.

Theorem 3.9

Assume that S_A , S_B and S_N be a neut. cubic \aleph -S's over S. If S_C is a neut. cubic \aleph -RI of S, then $S_A \odot S_B \odot S_N$ is also a neut. cubic \aleph -RI of S.

Proof.

Assume that S_c is a neut. cubic \aleph -RI of S. If there exist $p, q, r \in S$ such that u = [pqr], then [uvw] = [[pqr]vw] = [pq[rvw]] for all $u, v, w \in S$. Then,

$$\begin{split} T_{\bar{A}\circ B\circ N}(u) &= \bigwedge_{u=[pqr]} \{ max\{T_{\bar{A}}(p), T_{\bar{B}}(q), T_{\bar{N}}(r)\} \} \\ &= \bigwedge_{[uvw]=[pqn]} \{ max\{T_{\bar{A}}(p), T_{\bar{B}}(q), T_{\bar{N}}([rvw])\} \} \\ &= \bigwedge_{[uvw]=[pqn]} \{ max\{T_{\bar{A}}(p), T_{\bar{B}}(q), T_{\bar{N}}(h)\} \} \\ &\geq T_{\bar{A}\circ\bar{B}\circ\bar{N}}(u) = \bigvee_{u=[pqr]} \{ min\{I_{\bar{A}}(p), I_{\bar{B}}(q), I_{\bar{N}}(r)\} \} \\ &= \bigvee_{[uvw]=[pqr]} \{ min\{I_{\bar{A}}(p), I_{\bar{B}}(q), I_{\bar{N}}(h)\} \} \\ &\leq I_{\bar{A}\circ\bar{B}\circ\bar{N}}(u) = \bigvee_{u=[pqr]} \{ min\{F_{\bar{A}}(p), F_{\bar{B}}(q), F_{\bar{N}}(r)\} \} \\ &= \bigvee_{[uvw]=[pqh]} \{ min\{F_{\bar{A}}(p), F_{\bar{B}}(q), F_{\bar{N}}(r)\} \} \\ &= \bigvee_{[uvw]=[pqr]} \{ min\{F_{\bar{A}}(p), F_{\bar{B}}(q), F_{\bar{N}}(frvw]) \} \} \\ &= \bigvee_{[uvw]=[pqr]} \{ min\{F_{\bar{A}}(p), F_{\bar{B}}(q), F_{\bar{N}}(frvw]) \} \} \\ &= \bigvee_{[uvw]=[pqn]} \{ min\{F_{\bar{A}}(p), F_{\bar{B}}(q), F_{\bar{N}}(h) \} \} \\ &\leq F_{\bar{A}\circ\bar{B}\circ\bar{N}}(u) = \bigvee_{u=[pqr]} \{ min\{T_{A}(p), T_{B}(q), T_{N}(r) \} \} \\ &= \bigvee_{[uvw]=[pqr]} \{ min\{T_{A}(p), T_{B}(q), T_{N}(h) \} \} \\ &\leq T_{A\circ\bar{B}\circ\bar{N}}([uvw]) . \end{cases} \\ T_{A\circ\bar{B}\circ\bar{N}}(u) = \bigvee_{u=[pqr]} \{ max\{I_{A}(p), I_{B}(q), I_{N}(r) \} \} \\ &= \bigwedge_{[uvw]=[pqn]} \{ max\{I_{A}(p), I_{B}(q), I_{N}(h) \} \} \\ &\leq T_{A\circ\bar{B}\circ\bar{N}}([uvw]) . \end{cases} \\ I_{A\circ\bar{B}\circ\bar{N}}(u) = \bigwedge_{u=[pqr]} \{ max\{I_{A}(p), I_{B}(q), I_{N}(r) \} \} \\ &= \bigwedge_{[uvw]=[pqn]} \{ max\{I_{A}(p), I_{B}(q), I_{N}(h) \} \} \\ &\geq I_{A\circ\bar{B}\circ\bar{N}}([uvw]) . \end{cases} \\ F_{A\circ\bar{B}\circ\bar{N}}(u) = \bigwedge_{u=[pqr]} \{ max\{F_{A}(p), F_{B}(q), F_{N}(r) \} \} \\ &= \bigwedge_{[uvw]=[pqn]} \{ max\{F_{A}(p), F_{B}(q), F_{N}(r) \} \} \\ &= \bigwedge_{[uvw]=[pqn]} \{ max\{F_{A}(p), F_{B}(q), F_{N}(r) \} \} \\ &= \bigwedge_{[uvw]=[pqn]} \{ max\{F_{A}(p), F_{B}(q), F_{N}(h) \} \} \\ &\geq F_{A\circ\bar{B}\circ\bar{N}}([uvw]) . \end{cases}$$

Therefore, $S_A \odot S_B \odot S_N$ is a neut. cubic \aleph -RI of S.



Theorem 3.10

Assume that S_A , S_B and S_N be a neut. cubic \aleph -S's over S. If S_C is a neut. cubic \aleph -LI of S, then $S_A \odot S_B \odot S_N$ is also a neut. cubic \aleph -LI of S.

Proof.

It follows similarly from the theorem 3.12

f

Definition 3.11

Let $f: S \to S'$ be a function of sets. If $S'_D = \langle u, T_{\bar{B}}, I_{\bar{B}}, F_{\bar{B}}, T_B, I_B, F_B \rangle$ is a neut. cubic \aleph -S over ', the preimage of S'_D under f is defined to be a neut. cubic \aleph -S's.

$${}^{-1}(S'_{D})(u) = (f^{-1}(T_{\bar{B}}), f^{-1}(I_{\bar{B}}), f^{-1}(F_{\bar{B}}), f^{-1}(T_{B}), f^{-1}(I_{B}), f^{-1}(F_{B}))(u),$$

For every $u \in S$, the inverse function f^{-1} satisfies the equations $f^{-1}(T_{\bar{B}})(u) = T_{\bar{B}}(f(u))$, $f^{-1}(I_{\bar{B}})(u) = I_{\bar{B}}(f(u))$, $f^{-1}(F_{\bar{B}})(u) = F_{\bar{B}}(f(u))$, $f^{-1}(T_{B})(u) = T_{B}(f(u))$, $f^{-1}(I_{B})(u) = I_{B}(f(u))$, $f^{-1}(F_{B})(u) = F_{B}(f(u))$, $f^{-1}(I_{B})(u) = I_{B}(f(u))$, $f^{-1}(I_{B})(u$

Theorem 3.12

Assume that $f: S \to S'$ be a homomorphism of ternion SG's. If $S'_D = (T_{\bar{B}}, I_{\bar{B}}, F_{\bar{B}}, I_B, F_B)$ is an neut. cubic $\aleph - LI$ (resp. LI, RI) of S', then the preimage of S'_D under f is an neut. cubic $\aleph - LI$ (resp. LI, RI) of S. Proof.

Let
$$f^{-1}(S'_D) = (f^{-1}(T_B), f^{-1}(I_B), f^{-1}(T_B), f^{-1}(I_B), f^{-1}(F_B))$$
 is the preimage of S'_D under f . Let $u, v, w \in S$, then,
 $f^{-1}(T_B)([uvw]) = T_B(f([uvw])) = T_B([f(u)f(v)f(w)])$
 $\leq T_B(f(w)) = f^{-1}(T_B)(w),$
 $f^{-1}(I_B)([uvw]) = I_B(f([uvw])) = I_B([f(u)f(v)f(w)])$
 $\geq I_B(f(w)) = f^{-1}(I_B)(w),$
 $f^{-1}(F_B)([uvw]) = F_B(f([uvw])) = F_B([f(u)f(v)f(w)])$
 $\geq F_B(f(w)) = f^{-1}(F_B)(w),$
 $f^{-1}(T_B)([uvw]) = T_B(f([uvw])) = I_B([f(u)f(v)f(w)])$
 $\geq T_B(f(w)) = f^{-1}(T_B)(w),$
 $f^{-1}(I_B)([uvw]) = I_B(f([uvw])) = I_B([f(u)f(v)f(w)])$
 $\leq I_B(f(w)) = f^{-1}(I_B)(w),$
 $f^{-1}(F_B)([uvw]) = I_B(f([uvw])) = F_B([f(u)f(v)f(w)])$
 $\leq I_B(f(w)) = f^{-1}(I_B)(w),$
 $f^{-1}(F_B)([uvw]) = F_B(f([uvw])) = F_B([f(u)f(v)f(w)])$
 $\leq I_B(f(w)) = f^{-1}(I_B)(w),$
 $f^{-1}(F_B)([uvw]) = F_B(f([uvw])) = F_B([f(u)f(v)f(w)])$

Hence, $f^{-1}(S'_D)$ is a neut. cubic \aleph -LI of *S*.

Definition 3.13

For a subset A of a nonempty S, consider the neut. cubic &-S over S.

$$\chi_{\bar{A}}(S_{\bar{N}}) = \langle u_{,} (\chi_{\bar{A}}(T_{\bar{N}}), \chi_{\bar{A}}(I_{\bar{N}}), \chi_{\bar{A}}(F_{\bar{N}}), \chi_{A}(T_{N}), \chi_{A}(I_{N}), \chi_{A}(F_{N})) \rangle,$$

where,

$\chi_{\bar{A}}(T_{\bar{N}}): S \to D[-1,0], u \to \begin{cases} -\overline{1} \\ \overline{0} \end{cases}$	if $u \in A$ otherwise,	$\chi_A(T_N): S \to [-1,0], u \to \begin{cases} 0\\ -1 \end{cases}$	if $u \in A$ otherwise,
$\chi_{\bar{A}}(I_{\bar{N}}):S \to D[-1,0], u \to \begin{cases} \bar{0} \\ -\bar{1} \end{cases}$	if $u \in A$ otherwise,'	$\chi_A(I_N): S \to [-1,0], u \to \begin{cases} -1 \\ 0 \end{cases}$	if $u \in A$ otherwise,
$\chi_{\bar{A}}(F_{\bar{N}}): S \to D[-1,0], u \to \begin{cases} \overline{0} \\ -\overline{1} \end{cases}$	if $u \in A$ otherwise,'	$\chi_A(F_N): S \to [-1,0], u \to \begin{cases} -1\\ 0 \end{cases}$	if $u \in A$ otherwise,

is said to be a characteristic neut. cubic \aleph -S of S.



Theorem 3.14

Let $\chi_A(S_C), \chi_B(S_C)$ and $\chi_C(S_C)$ be an *i*-*v* characteristic neut. cubic \aleph -S's over S for any subsets A, B and C of S. Then the following condition holds:

$$\chi_A(S_C) \odot \chi_B(S_C) \odot \chi_C(S_C) = \chi_{[ABC]}(S_C)$$

Proof.

Let $u \in S$. If $u \notin [ABC]$, then $\left(\chi_{\bar{A}}(T_{\bar{N}}) \odot \chi_{\bar{B}}(T_{\bar{N}}) \odot \chi_{\bar{C}}(T_{\bar{N}})\right)(u) = \bar{0} = \chi_{[\bar{A}\bar{B}\bar{C}]}(T_{\bar{N}})(u)$ $\left(\chi_{\bar{A}}(I_{\bar{N}}) \odot \chi_{\bar{B}}(I_{\bar{N}}) \odot \chi_{\bar{C}}(I_{\bar{N}})\right)(u) = -\overline{1} = \chi_{[\bar{A}\bar{B}\bar{C}]}(I_{\bar{N}})(u)$ $\left(\chi_{\bar{A}}(F_{\bar{N}}) \odot \chi_{\bar{B}}(F_{\bar{N}}) \odot \chi_{\bar{C}}(F_{\bar{N}})\right)(u) = -\bar{1} = \chi_{[\bar{A}\bar{B}\bar{C}]}(F_{\bar{N}})(u)$ $(\chi_A(T_N) \odot \chi_B(T_N) \odot \chi_C(T_N))(u) = -1 = \chi_{[ABC]}(T_N)(u)$ $(\chi_A(I_N) \odot \chi_B(I_N) \odot \chi_C(I_N))(u) = 0 = \chi_{[ABC]}(I_N)(u)$ $(\chi_A(F_N) \odot \chi_B(F_N) \odot \chi_C(F_N))(u) = 0 = \chi_{[ABC]}(F_N)(u).$ Thus, $\chi_A(S_C) \odot \chi_B(S_C) \odot \chi_C(S_C) = \chi_{[ABC]}(S_C)$. If $u \in [ABC]$, then u = [pqr] for some $p \in A, q \in B$ and $r \in C$. It follows that $\left(\chi_{\bar{A}}(T_{\bar{N}}) \odot \chi_{\bar{B}}(T_{\bar{N}}) \odot \chi_{\bar{C}}(T_{\bar{N}})\right)(u) = \bigwedge_{u = \lfloor lmn \rfloor} \left\{ max\{\chi_{\bar{A}}(T_{\bar{N}})(l), \chi_{\bar{B}}(T_{\bar{N}})(m), \chi_{\bar{C}}(T_{\bar{N}})(n) \} \right\}$ $\leq \max\{\chi_{\bar{A}}(T_{\bar{N}})(p), \chi_{\bar{B}}(T_{\bar{N}})(q), \chi_{\bar{C}}(T_{\bar{N}})(r)\}$ $= -\overline{1} = \chi_{[\bar{A}\bar{B}\bar{C}]}(T_{\bar{N}})(u),$ $\left(\chi_{\bar{A}}(I_{\bar{N}}) \odot \chi_{\bar{B}}(I_{\bar{N}}) \odot \chi_{\bar{C}}(I_{\bar{N}})\right)(u) = \bigvee_{u = \lfloor lmn \rfloor} \left\{ \min\{\chi_{\bar{A}}(I_{\bar{N}})(l), \chi_{\bar{B}}(I_{\bar{N}})(m), \chi_{\bar{C}}(I_{\bar{N}})(n) \} \right\}$ $\geq \min\{\chi_{\bar{A}}(I_{\bar{N}})(p), \chi_{\bar{B}}(I_{\bar{N}})(q), \chi_{\bar{C}}(I_{\bar{N}})(r)\}\}$ $= \overline{0} = \chi_{[\overline{A}\overline{B}\overline{C}]}(I_{\overline{N}})(u),$ $\left(\chi_{\bar{A}}(F_{\bar{N}}) \odot \chi_{\bar{B}}(F_{\bar{N}}) \odot \chi_{\bar{C}}(F_{\bar{N}})\right)(u) = \bigvee_{u = [lmn]} \left\{ \min\{\chi_{\bar{A}}(F_{\bar{N}})(l), \chi_{\bar{B}}(F_{\bar{N}})(m), \chi_{\bar{C}}(F_{\bar{N}})(n) \} \right\}$ $\geq \min\{\chi_{\bar{A}}(F_{\bar{N}})(p), \chi_{\bar{B}}(F_{\bar{N}})(q), \chi_{\bar{C}}(F_{\bar{N}})(r)\}\}$ $=\overline{0}=\chi_{[\bar{A}\bar{B}\bar{C}]}(F_{\bar{N}})(u),$ $\left(\chi_A(T_N) \odot \chi_B(T_N) \odot \chi_C(T_N)\right)(u) = \bigvee_{u = [lmn]} \left\{ \min\{\chi_A(T_N)(l), \chi_B(T_N)(m), \chi_C(T_N)(n)\} \right\}$ $\geq \min\{\chi_A(T_N)(p), \chi_B(T_N)(q), \chi_C(T_N)(r)\}$ $= 0 = \chi_{[ABC]}(T_N)(u),$ $\left(\chi_A(I_N) \odot \chi_B(I_N) \odot \chi_C(I_N)\right)(u) = \bigwedge_{u = [lmn]} \left\{ \max\{\chi_A(I_N)(l), \chi_B(I_N)(m), \chi_C(I_N)(n)\} \right\}$ $\leq \max\{\chi_A(I_N)(p), \chi_B(I_N)(q), \chi_C(I_N)(r)\}$ $= -1 = \chi_{[ABC]}(I_N)(u),$ $\left(\chi_A(F_N) \odot \chi_B(F_N) \odot \chi_C(F_N)\right)(u) = \bigwedge_{u = \lceil lmn \rceil} \left\{ max\{\chi_A(F_N)(l), \chi_B(F_N)(m), \chi_C(F_N)(n)\} \right\}$ $\leq \max\{\chi_A(F_N)(p), \chi_B(F_N)(q), \chi_C(F_N)(r)\}\$ $= -1 = \chi_{[ABC]}(F_N)(u),$ Therefore, $\chi_A(S_C) \odot \chi_B(S_C) \odot \chi_C(S_C) = \chi_{[ABC]}(S_C)$.

Theorem 3.15

Let $A \neq \emptyset$ subset of S. Then the following statements are equivalent:

1. A is a LI (resp. $\mathcal{L}I$, RI) of S,

2. The characteristic neut. cubic \aleph -S, $\chi_A(S_C)$ over S is an neut. cubic \aleph -LI (resp. LI, RI) of S. Proof.

(1) \Rightarrow (2) Assume that *A* is a LI of *S*. Let $u, v, w \in S$. If $w \notin A$, then

$$\begin{split} \chi_{\bar{A}}(T_{\bar{N}})([uvw]) &\leq \bar{0} = \chi_{\bar{A}}(T_{\bar{N}})(w), \\ \chi_{\bar{A}}(I_{\bar{N}})([uvw]) &\geq -\bar{1} = \chi_{\bar{A}}(I_{\bar{N}})(w), \\ \chi_{\bar{A}}(F_{\bar{N}})([uvw]) &\geq -\bar{1} = \chi_{\bar{A}}(F_{\bar{N}})(w), \\ \chi_{A}(T_{N})([uvw]) &\geq -1 = \chi_{A}(T_{N})(w), \\ \chi_{A}(I_{N})([uvw]) &\leq 0 = \chi_{A}(I_{N})(w), \\ \chi_{A}(F_{N})([uvw]) &\leq 0 = \chi_{A}(F_{N})(w). \end{split}$$



On the other hand, suppose that $w \in A$. Then, $[uvw] \in A$. It follows that

$$\begin{split} \chi_{\bar{A}}(T_{\bar{N}})([uvw]) &= -\bar{1} = \chi_{\bar{A}}(T_{\bar{N}})(w), \\ \chi_{\bar{A}}(I_{\bar{N}})([uvw]) &= \bar{0} = \chi_{\bar{A}}(I_{\bar{N}})(w), \\ \chi_{\bar{A}}(F_{\bar{N}})([uvw]) &= \bar{0} = \chi_{\bar{A}}(F_{\bar{N}})(w), \\ \chi_{A}(T_{N})([uvw]) &= 0 = \chi_{A}(T_{N})(w), \\ \chi_{A}(I_{N})([uvw]) &= -1 = \chi_{A}(I_{N})(w), \\ \chi_{A}(F_{N})([uvw]) &= -1 = \chi_{A}(F_{N})(w). \end{split}$$

Therefore, $\chi_A(S_C)$ is a neut. cubic \aleph -LI of S.

 $(2) \Rightarrow (1) \text{ Assume that } \chi_A(S_C) \text{ is a neut. cubic } \mathcal{N}\text{-LI of } S. \text{ Let } u, v \in S \text{ and } a \in A. \text{ Then} \\ \chi_{\bar{A}}(T_{\bar{N}})([uva]) \leq \chi_{\bar{A}}(T_{\bar{N}})(a) = -\bar{1}, \\ \chi_{\bar{A}}(I_{\bar{N}})([uva]) \geq \chi_{\bar{A}}(I_{\bar{N}})(a) = \bar{0}, \\ \chi_{\bar{A}}(F_{\bar{N}})([uva]) \geq \chi_{\bar{A}}(F_{\bar{N}})(a) = \bar{0}, \\ \chi_A(T_N)([uva]) \geq \chi_A(T_N)(a) = 0, \\ \chi_A(I_N)([uva]) \leq \chi_A(I_N)(a) = -1, \\ \chi_A(F_N)([uva]) \leq \chi_A(F_N)(a) = -1. \\ \text{Hence, } \chi_{\bar{A}}(T_{\bar{N}})([uva]) = -\bar{1}, \chi_{\bar{A}}(I_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = \bar{0}, \chi_A(T_N)([uva]) = -1, \\ \text{Hence, } \chi_{\bar{A}}(T_{\bar{N}})([uva]) = -\bar{1}, \chi_{\bar{A}}(I_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = \bar{0}, \chi_A(T_N)([uva]) = -1, \\ \text{Hence, } \chi_{\bar{A}}(T_{\bar{N}})([uva]) = -\bar{1}, \chi_{\bar{A}}(I_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -1, \\ \text{Hence, } \chi_{\bar{A}}(T_{\bar{N}})([uva]) = -\bar{1}, \chi_{\bar{A}}(I_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -1, \\ \text{Hence, } \chi_{\bar{A}}(T_{\bar{N}})([uva]) = -\bar{1}, \chi_{\bar{A}}(I_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -1, \\ \text{Hence, } \chi_{\bar{A}}(T_{\bar{N}})([uva]) = -\bar{1}, \chi_{\bar{A}}(I_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -1, \\ \text{Hence, } \chi_{\bar{A}}(T_{\bar{N}})([uva]) = -\bar{1}, \chi_{\bar{A}}(I_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -1, \\ \text{Hence, } \chi_{\bar{A}}(T_{\bar{N}})([uva]) = -\bar{1}, \chi_{\bar{A}}(I_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = \bar{0}, \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -1, \\ \text{Hence, } \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -\bar{1}, \\ \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -\bar{1}, \\ \chi_{\bar{A}}(F_{\bar{N}})([uva]) = \bar{1}, \\ \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -\bar{1}, \\ \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -\bar{1}, \\ \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -\bar{1}, \\ \chi_{\bar{A}}(F_{\bar{N}})([uva]) = -\bar{1}, \\ \chi_{\bar{A}}(F_{\bar{A}})([uva]) = -\bar{1}, \\$

 $\chi_A(F_N)([uva]) = -1$. This implies that $[uva] \in A$. Consequently, A is a LI of S

Theorem 3.16

Assume that S_R be a neut. cubic \aleph -S over S. Then S_R is a neut. cubic \aleph -RI of S iff $S_R \odot S_P \odot S_Q \subseteq S_R$ for every neut. cubic \aleph -S's S_P and S_Q over S.

Proof.

Let S_R is a neut. cubic \aleph -RI of S, and let S_P and S_Q an neut. cubic \aleph -S's over S. Obviously, $S_R \odot S_P \odot S_Q \subseteq S_R \forall a, b, c \in S$ such that $u \neq [abc]$.

If there are elements a, b and c in set S such that u = [abc], then we obtain

$$\begin{split} T_{\bar{R}}(u) &= T_{\bar{R}}[abc] \leq T_{\bar{R}}(a) \leq max\{T_{\bar{R}}(a), T_{\bar{P}}(b), T_{\bar{Q}}(c)\},\\ I_{\bar{R}}(u) &= I_{\bar{R}}[abc] \geq I_{\bar{R}}(a) \geq min\{I_{\bar{R}}(a), I_{\bar{P}}(b), I_{\bar{Q}}(c)\},\\ F_{\bar{R}}(u) &= F_{\bar{R}}[abc] \geq F_{\bar{R}}(a) \geq min\{F_{\bar{R}}(a), F_{\bar{P}}(b), F_{\bar{Q}}(c)\},\\ T_{R}(u) &= T_{R}[abc] \geq T_{R}(a) \geq min\{T_{R}(a), T_{P}(b), T_{Q}(c)\},\\ I_{R}(u) &= I_{R}[abc] \leq I_{R}(a) \leq max\{I_{R}(a), I_{P}(b), I_{Q}(c)\},\\ F_{R}(u) &= F_{R}[abc] \leq F_{R}(a) \leq max\{F_{R}(a), F_{P}(b), F_{Q}(c)\}. \end{split}$$

This implies that

$$\begin{split} T_{\bar{R}}(u) &= \bigwedge_{u=[abc]} max \{ T_{\bar{R}}(a), T_{\bar{P}}(b), T_{\bar{Q}}(c) \} = T_{\bar{R} \circ \bar{P} \circ \bar{Q}}(u), \\ I_{\bar{R}}(u) &= \bigvee_{u=[abc]} min \{ I_{\bar{R}}(a), I_{\bar{P}}(b), I_{\bar{Q}}(c) \} = I_{\bar{R} \circ \bar{P} \circ \bar{Q}}(u), \\ F_{\bar{R}}(u) &= \bigvee_{u=[abc]} min \{ F_{\bar{R}}(a), F_{\bar{P}}(b), F_{\bar{Q}}(c) \} = F_{\bar{R} \circ \bar{P} \circ \bar{Q}}(u), \\ T_{R}(u) &= \bigvee_{u=[abc]} min \{ T_{R}(a), T_{P}(b), T_{Q}(c) \} = T_{R \circ P \circ Q}(u), \\ I_{R}(u) &= \bigwedge_{u=[abc]} max \{ I_{R}(a), I_{P}(b), I_{Q}(c) \} = I_{R \circ P \circ Q}(u), \\ F_{R}(u) &= \bigwedge_{u=[abc]} max \{ F_{R}(a), F_{P}(b), F_{Q}(c) \} = F_{R \circ P \circ Q}(u). \end{split}$$

Therefore, $S_R \odot S_P \odot S_Q \subseteq S_R$. Conversely, assume that S_R is an neut. cubic \aleph -S over S, such that $S_R \odot S_P \odot S_Q \subseteq S_R$ for every neut. cubic \aleph -S's S_P and S_Q over S. Let $u, v, w \in S$ and a = [uvw]. Then,

$$T_{\bar{R}}([uvw]) = T_{\bar{R}}(a) \leq \left(T_{\bar{R}} \circ \chi_{S}(T_{\bar{P}}) \circ \chi_{S}(T_{\bar{Q}})\right)(a)$$

$$= \bigwedge_{a=[lmn]} max\{T_{\bar{R}}(l), \chi_{S}(T_{\bar{P}})(m), \chi_{S}(T_{\bar{Q}})(n)\}$$

$$\leq max\{T_{\bar{R}}(u), \chi_{S}(T_{\bar{P}})(v), \chi_{S}(T_{\bar{Q}})(w)\} = T_{\bar{R}}(u)$$

$$I_{\bar{R}}([uvw]) = I_{\bar{R}}(a) \geq \left(I_{\bar{R}} \circ \chi_{S}(I_{\bar{P}}) \circ \chi_{S}(I_{\bar{Q}})\right)(a)$$

$$= \bigwedge_{a=[lmn]} min\{I_{\bar{R}}(l), \chi_{S}(I_{\bar{P}})(m), \chi_{S}(I_{\bar{Q}})(n)\}$$



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$$\geq \min\{I_{\bar{R}}(u), \chi_{S}(I_{\bar{P}})(v), \chi_{S}(I_{\bar{Q}})(w)\} = I_{\bar{R}}(u),$$

$$F_{\bar{R}}([uvw]) = F_{\bar{R}}(a) \geq \left(F_{\bar{R}} \circ \chi_{S}(F_{\bar{P}}) \circ \chi_{S}(F_{\bar{Q}})\right)(a)$$

$$= \bigwedge_{a=[lmn]} \min\{F_{\bar{R}}(l), \chi_{S}(F_{\bar{P}})(m), \chi_{S}(F_{\bar{Q}})(n)\}$$

$$\geq \min\{F_{\bar{R}}(u), \chi_{S}(F_{\bar{P}})(v), \chi_{S}(F_{\bar{Q}})(w)\} = F_{\bar{R}}(u),$$

$$T_{R}([uvw]) = T_{R}(a) \geq \left(T_{R} \circ \chi_{S}(T_{P}) \circ \chi_{S}(T_{Q})\right)(a)$$

$$= \bigwedge_{a=[lmn]} \min\{T_{R}(l), \chi_{S}(T_{P})(m), \chi_{S}(T_{Q})(n)\}$$

$$\geq \min\{T_{R}(u), \chi_{S}(T_{P})(v), \chi_{S}(T_{Q})(w)\} = T_{R}(u),$$

$$I_{R}([uvw]) = I_{R}(a) \leq \left(I_{R} \circ \chi_{S}(I_{P}) \circ \chi_{S}(I_{Q})\right)(a)$$

$$= \bigwedge_{a=[lmn]} \max\{I_{R}(l), \chi_{S}(I_{P})(m), \chi_{S}(I_{Q})(n)\}$$

$$\leq \max\{I_{R}(u), \chi_{S}(I_{P})(v), \chi_{S}(I_{Q})(w)\} = I_{R}(u),$$

$$F_{R}([uvw]) = F_{R}(a) \leq \left(F_{R} \circ \chi_{S}(F_{P}) \circ \chi_{S}(F_{Q})\right)(a)$$

$$= \bigwedge_{a=[lmn]} \max\{F_{R}(l), \chi_{S}(F_{P})(m), \chi_{S}(F_{Q})(n)\}$$

$$\leq \max\{F_{R}(u), \chi_{S}(F_{P})(v), \chi_{S}(F_{Q})(w)\} = F_{R}(u).$$

Consequently, S_R is an neut. cubic \aleph -RI of S.

Theorem 3.17

Assume that S_P be a neut. cubic \aleph -S over S. Then $S_{\overline{P}}$ is a neut. cubic \aleph -LI of S iff $S_R \odot S_P \odot S_Q \subseteq S_P$ for every neut. cubic \aleph -S's S_R and S_Q over S.

Proof.

It follows similarly from the theorem 3.16

Theorem 3.18

Assume that S_Q be a neut. cubic \aleph -S over S. Then S_Q is a neut. cubic \aleph -LI of S iff $S_R \odot S_P \odot S_Q \subseteq S_Q$ for every neut. cubic \aleph -S's S_R and S_P over S.

Proof.

It follows similarly from the theorem 3.16

IV. CONCLUSION

This paper explores the notion of a SSG within the framework of neut. cubic \aleph -ternion SG, investigating its inherent properties. The study encompasses neut. cubic \aleph -fuzzy ideals within a ternion SG, scrutinizing their algebraic features. Through illustrative examples, we establish that the combination of two neut. cubic \aleph -fuzzy ideals within a ternion SG results in another neut. cubic \aleph -fuzzy ideal within the same ternion SG. Additionally, we present the notion of the direct product for neut. cubic \aleph -fuzzy ideals in a ternion SG, emphasizing its nature as a neut. cubic \aleph -fuzzy ideal. Furthermore, we can extend this neut. cubic fuzzy ternion SG to a neut. cubic fuzzy ternion semi-ring etc.

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