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# New Characterizations of Topological Spaces

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***Abstract:*** ***In this paper, we introduce and investigate topological spaces called sgw-compactness Spaces and sgw-connectedness space and we get several characterizations and some of their properties. Also we investigate its relationship with other types of functions.***
***Mathematics Subject Classification: 54D05, 54D30.***
***Keywords: sgw-open set, sgw-closed sets, sgw-compact spaces, sgw-connectedness***

## I. INTRODUCTION

The notions of compactness and connectedness are useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness and connectedness. The productivity and fruitfulness of these notions of compactness and connectedness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness and connectedness have been introduced and investigated. D. Andrijevic [1] introduced a new class of generalized open sets in a topological space called b-open sets. The class of b-open sets generates the same topology as the class of b-open sets. Since the advent of this notion, several research paper with interesting results in different respects came into existence . M. Ganster and M.Steiner [5] introduced and studied the properties of gb-closed sets in topological spaces. The aim of this paper is to introduce the concept of sgw-compactness and sgw-connectedness in topological spaces and is to give some characterizations of sgw-compact spaces in terms of nets and filter bases. S.S. Benchalli, T.D. Rayanagoudar and P.G. Patil introduced the concept of g*-closed sets and S.S. Benchalli, T.D. Rayanagoudar and P.G. Patil and Shik John studied the concept of g*- preregular, g*- pre normal and obtained their properties by utilizing g*-closed sets. The notation of closed set is fundamental in the study of topological spaces. In 1970, Levine introduced the concept of generalized closed sets in the topological space by comparing the closure of subset with its open supersets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, covering properties and generalization of continuity. T. Kong, R. Kopperman and P. Meyer shown some of the properties of generalized closed set have been found to be useful in computer science and digital topology. Caw, Ganster and Reilly and has shown that generalization of closed set is also useful to characterize certain classes of topological spaces and there variations, for example the class of extremely disconnected spaces and the class of submaximal spaces. In 1990, S.P. Arya and T.M. Nour define generalized semi-open sets, generalized semi closed sets and use them to obtain some cauterization of *s*-normal spaces.

In 1993,N. PalaniInappan and K. Chandrasekhara Rao introduced regular generalized closed (briefly *rg*-closed) sets and study there properties relative to union, intersection and subspaces. In 2000, A. Pushpalatha introduce new class of closed set called weakly closed (briefly *w*-closed) sets and study there properties.In 2007, S.S. Benchalli and R.S. Wali introduced the new class of the set called regular *w*-closed (briefly *rw*-closed) sets in topological spaces. In this this paper is to introduce and study two new classes of spaces,namely Semi weakly generalized-normal and Semi weakly generalized- regular spaces and obtained their properties by utilizing Semi weakly generalized-closed sets.

## II. PRELIMINARY NOTES

Throughout this paper $(X, \tau)$, $(Y, \sigma)$ are topological spaces with no separation axioms assumed unless otherwise stated. Let $A \subseteq X$. The closure of A and the interior of A will be denoted by Cl(A) and Int(A) respectively.

1) *Definition 2.1:* A subset A of X is said to be b-open [1] if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$. The complement of b-open set is said to be b-closed. The family of all b-open sets (respectively b-closed sets) of $(X, \tau)$ is denoted by $bO(X, \tau)$[respectively $bCL(X, \tau)$].
2) *Definition 2.2:* Let A be a subset of X. Then

(i) b-interior [1] of A is the union of all b-open sets contained in A.
(ii) b-closure [1] of A is the intersection of all b-closed sets containing A.
The b-interior [respectively b-closure] of A is denoted by b-Int(A) [respectivelyb-Cl(A)].

3) *Definition 2.3:* Let A be a subset of X. Then A is said to be sgw-closed [12]

If cl (int(A) ⊆U whenever A ⊆U and U∈regular-semi open in (X, τ). The complement of sgw-closed [12] set is called sgw-open. The family of all sgw-open [respectively sgw-closed] sets of (X, τ) is denoted by sgwO(X, τ) [respectively,sgw-CL(X, τ)].

4) *Definition 2.4:*The sgw-closure [14] of a set A, denoted by sgw-Cl(A), is the Intersection of all sgw-closed sets containing A.
5) Definition 2.5: The sgw-interior [14] of a set A, denoted by sgw-Int (A), is the Union of all sgw-open sets contained in A.
6) *Remark 2.6:* Every pre-closed set is sgw-closed.

## III. SGW-COMPACTNESS.

1) *Definition 3.1:* A collection $\{A_i: i \in \wedge\}$ of sgw-open sets in a topological space X is called a sgw-open cover of a subset B of X if B ⊂ {Ai: i ∈∧} holds.
2) *Definition 3.2:* A topological space X is sgw-compact if every sgw-open cover of X has a finite sub-cover.
3) *Definition 3.3:*A subset B of a topological space X is said to be sgw-compact relative to X if, for every collection $\{A_i: i \in \wedge\}$ of sgw-open subsets of X such that $B \subset U \{A_i : i \in \wedge\}$ there exists a finite subset $\wedge_0$of ∧ such that $B \subseteq U \{A_i: i \in \wedge_0\}$.
4) *Definition 3.4:* A subset B of a topological space X is said to be sgw-compact If B is sgw-compact as a subspace of X.
5) *Theorem 3.5:* Every sgw-closed subset of a sgw-compact space is sgw-compact Relative to X.

Proof: Let A be sgw-closed subset of sgw-compact space X. Then $A^C$is sgw-open

in X. Let M = $\{G_\alpha:\alpha \in \wedge\}$ be a cover of A by sgw-open sets in X. Then $M{*}= M\cup A^c$ is a sgw-open cover of X. Since X is sgw-compact M∗is reducible to a finite subcover of X, say $X = G\alpha_1 \cup G\alpha_2 \cup \cdot\cdot\cdot \cup G\alpha_m \cup A^C$, $G\alpha_k \in M$. But A and $A^C$ are disjoint hence A $\subset G\alpha_1 \cup \cdot\cdot\cdot \cup G\alpha_m$, $G\alpha_k \in M$, which implies that any sgw-open cover M of A contains a finite sub-cover.

Therefore A is sgw-compact relative to X. Thus every sgw-closed subset of a sgw-compact space X is sgw-compact.

6) *Definition 3.6:* A function f : X → Y is said to be sgw-continuous [5] if $f^{-1}(V)$ is sgw-closed in X for every closed set V of Y .
7) *Definition 3.7:* A function f : X → Y is said to be sgw-irresolute [5] if $f^{-1}(V)$ is sgw-closed in X for every sgw-closed set V of Y.

8) *Theorem 3.8:* A sgw-continuous image of a sgw-compact space is compact

Proof. Let f : X → Y be a sgw-continuous map from a sgw-compact space X onto a topological space Y. Let $\{A_i: i \in \wedge\}$ be an open cover of Y . Then $\{f^{-1}(A_i) : i \in \wedge\}$ is a sgw-open cover of X. Since X is sgw-compact it has a finite sub-cover say $\{f^{-1}(A_1), f^{-1}(A_2), \cdot\cdot\cdot, f^{-1}(A_n)\}$. Since f is onto $\{A_1, \cdot\cdot\cdot, An\}$ is a cover of Y , which is finite. Therefore Y is compact.

9) *Theorem 3.9:* If a map f : X → Y is sgw-irresolute and a subset B of X is

sgw-compact relative to X, then the image f(B) is sgw-compact relative to Y .

Proof: Let $\{A_\alpha: \alpha \in \wedge\}$ be any collection of sgw-open subsets of Y such that $f(B) \subset \cup\{A_\alpha: \alpha \in \wedge\}$. Then $B \subset \cup\{f^{-1}(A_\alpha) : \alpha \in \wedge\}$ holds. Since by hypothesis B is sgw-compact relative to X there exists a finite subset $\wedge_0$of ∧ such that $B \subset U \{f^{-1}(A_\alpha) : \alpha \in \wedge_0\}$ Therefore we have $f(B) \subset \cup\{A_\alpha: \alpha \in \wedge_0\}$, which shows that f(B) is sgw compact relative to Y.

## IV. SGW-CONNECTEDNESS

1) *Definition 4.1:* A topological space X is said to be sgw-connected if X cannot be expressed as a disjoint union of two non-empty sgw-open sets. A subset of X is sgw-connected if it is sgw-connected as a subspace.
2) *Example 4.2:* Let X = {a, b} and let τ = {X, φ, {a}}. Then it is sgw-connected.
3) *Remark 4.3:* Every sgw-connected space is connected but the converse need not be true in general, which follows from the following example.
4) *Example 4.4:* Let X = {a, b} and let τ = {X, φ}. Clearly (X, τ) is connected. The sgw-open sets of X are {X, φ, {a} , {b}}. Therefore (X, τ) is not a sgw-connected space, because X = {a} ∪ {b} where {a} and {b} are non-empty sgw-open sets.
5) *Theorem 4.5:* For a topological space X the following are equivalent.

(i) X is sgw-connected.

(ii) X and φ are the only subsets of X which are both sgw-open and sgw-closed.

(iii) Each sgw-continuous map of X into a discrete space Y with at least two

Points are a constant map.

Proof:

(i) ⇒(ii) :Let O be any sgw-open and sgw-closed subset of X. Then $O^C$ is both sgw-open and sgw-closed. Since X is disjoint union of the sgw-open sets O and $O^C$ implies from the hypothesis of (i) that either O = φ or O = X.

(ii) ⇒(i) :Suppose that X = A ∪B where A and B are disjoint non-empty sgw-open subsets of X. Then A is both sgw-open and sgw-closed. By assumption A = φ or X. Therefore X is sgw-connected.

(ii) ⇒(iii) :Let f : X → Y be a sgw-continuous map. Then X is covered by sgw-open and sgw-closed covering {$f^{-1}$(Y ) : y ∈(Y )}. By assumption $f^{-1}$(y) = φ or X for each y ∈Y. If f−1(y) = φ for ally ∈Y , then f fails to be a map. Then there exists only one point y ∈Y such that $f^{-1}$(y) ≠φ and hence $f^{-1}$(y) = X. This shows that f is a constant map.

(iii) ⇒(ii) :Let O be both sgw-open and sgw-closed in X. Suppose O ≠φ. Let f : X → Y be a sgw-continuous map defined by f(O) = y and f($O^C$) = {w} for some distinct points y and w in Y .By assumption f is constant. Therefore we have O = X.

6) *Theorem 4.6:* If f : X → Y is a sgw-continuous and X is sgw-connected, then Y is connected.

Proof: Suppose that Y is not connected. Let Y = A∪B where A and B are disjoint non-empty open set in Y . Since f is sgw-continuous and onto, X = $f^{-1}$(A) ∪$f^{-1}$(B) where $f^{-1}$(A) and
$f^{-1}$(B) are disjoint non-empty sgw-open sets in X. This contradicts the fact that X is sgw-connected. Hence Y is connected.

7) *Theorem 4.7:*If f : X → Y is a sgw-irresolute surjection and X is sgw-connected,
then Y is sgw-connected.

Proof: Suppose that Y is not sgw-connected. Let Y =A∪B where A and B are disjoint non-empty sgw-open set in Y. Since f is sgw-irresolute and onto, X = $f^{-1}$(A) ∪$f^{-1}$(B) where $f^{-1}$(A) and$f^{-1}$(B) are disjoint non-empty sgw-open sets in X. This contradicts the fact that X is sgw-connected. Hence Y is connected.

8) *Theorem 4.8:*In a topological space (X, τ) with at least two points, if sgw-O(X, τ) = sgw-CL(X, τ) then X is not sgw-connected.

Proof: By hypothesis we have sgw-O(X, τ) = sgw-CL(X, τ) and by Remark 2.6 we have every pre-closed set is sgw-closed, there exists some non-empty proper subset of X which is both sgw-open and sgw-closed in X. So by last Theorem 4.5 we have X is not sgw-connected.

9) *Definition 4.9:* A topological space X is said to be $T_{sgw}$-space if every sgw-closed
subset of X is closed subset of X.

10) *Theorem 4.10:* Suppose that X is a $T_{sgw}$-space then X is connected if and only if it is sgw-connected.

Proof: Suppose that X is connected. Then X cannot be expressed as disjoint union of two non-empty proper subsets of X. Suppose X is not a sgw-connected space. Let A and B be any two sgw-open subsets of X such that X = A∪B, where A ∩ B = φ and A ⊂X, B ⊂X. Since X is $T_{sgw}$-space and A,B are sgw-open,A,B are open subsets of X, which contradicts that X is connected.Therefore X is sgw-connected. Conversely, every open set is sgw-open. Therefore every sgw-connected space is connected.

11) *Theorem 4.11:* If the sgw-open sets C and D form a separation of X and if Y is sgw-connected subspace of X, then Y lies entirely within C or D.

Proof: Since C and D are both sgw-open in X the sets C∩Y and D∩Y are sgw-open in Y these two sets are disjoint and their union is Y. If they wereboth non-empty, they would constitute a separation of Y. Therefore, one ofthem is empty. Hence Y must lie entirely in C or in D.

12) *Theorem 4.12:* Let A be a sgw-connected subspace of X. If A⊂B⊂sgw-Cl(A)
then B is also sgw-connected.

Proof**:** Let A be sgw-connected and let A⊂B⊂sgw-Cl(A). Suppose that B = C∪D is a separation of B by sgw-open sets. Then by Theorem 4.11 above A must lie entirely in C or in D.Suppose that A ⊂C, then sgw-Cl(A) ⊆sgw-Cl(C). Since sgw-Cl(C) and D are disjoint, B cannot intersect D. This contradicts the fact that D is non-empty subset of B. So D = φ which implies B is sgw-connected.

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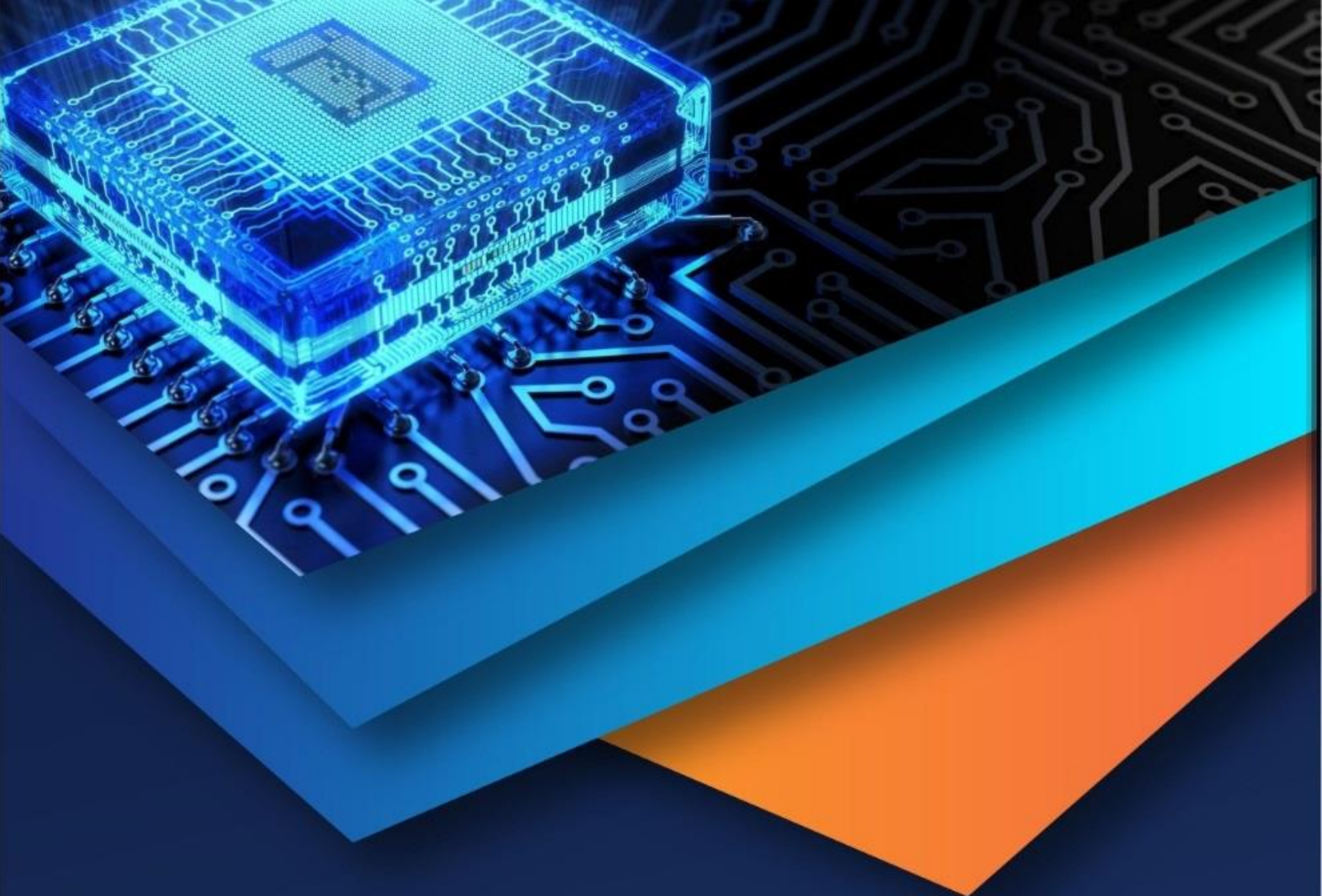


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