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New Trigonometrically Method for Solving Non-Linear Transcendental Equations

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Abstract: The transcendental equations' non-zero positive real roots can be found using a new approach presented in this research. The proposed approach is based on the union of the Newton-Raphson method and the inverse tan(x) function. To ensure the method's applicability, it is implemented in MATLAB and applied to various issues. The suggested approach is evaluated on a variety of numerical instances, the results show that our approaches are superior and more efficient than widely used methods. For both the new proposed method and the currently accessible existing methods, error calculations have been made. When compared to well-known procedures, the mistakes were quickly minimised, and the genuine root was discovered in fewer repetitions. The proposed method's convergence is studied, and it is demonstrated that it reduces to the quadratic convergent Newton-Raphson method. This method will also assist in the production of a non-zero real root of a specified nonlinear equation in the commercial software (transcendental, algebraic, and exponential).
Keywords: Spherical fuzzy, Graph Theory, Fuzzy Graph, k-Vertex Coloring, Chromatic Number, Vertices, Traffic, Membership Values, Edge, Strong

I. INTRODUCTION

Numerous engineering design fields, including circuit analysis, analysis of state equations for a real gas, mechanical motions/oscillations, weather forecasting, optimization, and tracking of sustainable photovoltaic energy generation, are heavily dependent on root finding methods. Other examples include computing gradient retention times in liquid chromatography, tracking of photovoltaic energy generation, tracking of mechanical motions/oscillations, and forecasting weather. In discrete stochastic arithmetic (DSA), root finding techniques can also be used to test the class of multi-step iterative approaches and identify the best numerical solution to non-linear equations.

In order to accomplish effective iterative approaches, Gemechu used derivative estimations up to the third order (in root discovery, some new initiatives) in Taylor's approximation of a non-linear function/equation. Investigated are effective approaches of higher order for resolving simple roots of nonlinear equations that enhance the convergence of several fundamentally established methods. In [1], the library for Control of Accuracy and Debugging for Numerical Applications (CADNA) is used. Using this method, the ideal solution with the ideal precision and the ideal amount of iterations are discovered. In this instance, a new criterion that is independent of the provided tolerance is used to substitute the iterative procedure's customary halting termination, allowing the computation of the best results. In [2], Kwasinski and Chun investigated the use of traditional mathematical root-finding optimization techniques for photovoltaic (PV) systems' maximum power point tracking (MPPT). Due to the fact that in this study these approaches are digitally implemented, difficulties in digitally implementing a method that was originally based on a continuous domain are also explored. This study specifically addresses inherent digital process defects, such as algorithm numerical stability, quantization error, and discretization error analysis, which have received less attention in earlier MPPT papers.

Using root-finding techniques, Lopez-Ure'na et al. [6] improved the computation of gradient retention durations in liquid chromatography. The fundamental equation of gradient elution, an integral equation that can only be solved analytically for specific combinations of the retention model and gradient programme, was solved by the authors using this method. Numerical integration is a universal method that can be used to get around this restriction, but it comes at a cost of higher computation times. The author of [10] suggests a straightforward methodology to create Newton iteration formulae of arbitrary order, starting from the widely used quadratically convergent Newton-Raphson method and the conventional linearly convergent fixed point iteration method. It is also demonstrated that the general case can be used to reproduce well-known versions like Halley's method or Haouseholder's high order methods.

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The majority of engineering and scientific issues are formulated as non-linear transcendental equations, whose root evaluation is more challenging. Such non-linear equations are involved in a number of physical issues, including the van der Waal equation, the decay equation, the earthquake Richter scale, and the surface wave formula. Mahesh et al. presented a quadratic convergent iterative method in [7] that is highly effective at finding the roots of non-linear equations and rapidly reduces error. For the purpose of locating the roots of non-linear equations, Noor [9] presented a two-step iterative method. This method outperforms Newton method and other one-step iterative methods. A new family of iterative methods for solving non-linear equations employing a system of coupled equations and the decomposition approach [8] was also proposed and examined by him.

II. PROPOSED METHOD

The alternative iterative trigonometric equation is proposed as

$$x_{n+1} = x_n\left[1 + 2tan^{-1}\left(\frac{\left(\frac{(f'x_n)}{x_n f'(x_n)}\right)}{1+\sqrt{1-\left(\frac{-f(x_n)}{x_n f'(x_n)}\right)^2}}\right)\right] \qquad n = 0,1,2,\ldots \qquad (2.1)$$

The classic Newton-Raphson approach can be achieved by extending the aforementioned iterative formula, as in the first two terms, and many methods based on series truncation can also be obtained. In actuality:

Theorem 2.1. Suppose α ≠0 is a real exact root of the algebraic/trancendental equation f(x) and h is a very small neighborhood of α. Let f''(x) exists and f'(x) ≠ 0 in h. Then the proposed method given in (2.1) produces a sequence of terms {x_n : n = 0, 1, 2, · · · } with quadratically convergent.

PROOF:

The proposed method in (2.1) can be expressed in the form as

$$x_{n+1} = x_n\left[1 + 2\left(\frac{\left(\frac{(f'x_n)}{x_n f'(x_n)}\right)}{1+\sqrt{1-\left(\frac{-f(x_n)}{x_n f'(x_n)}\right)^2}}\right) - \frac{1}{3}\left(\frac{\left(\frac{(f'x_n)}{x_n f'(x_n)}\right)}{1+\sqrt{1-\left(\frac{-f(x_n)}{x_n f'(x_n)}\right)^2}}\right) + \frac{1}{5}\left(\frac{\left(\frac{(f'x_n)}{x_n f'(x_n)}\right)}{1+\sqrt{1-\left(\frac{-f(x_n)}{x_n f'(x_n)}\right)^2}}\right) - \cdots\right]$$

Using standard expansion tan−1 (x) = x $-\frac{x^3}{3}+\frac{x^5}{5}$ − · · · , Neglecting the higher order terms, we get

$$x_{n+1} = x_n\left[1 + 2\left(\frac{\left(\frac{(f'x_n)}{x_n f'(x_n)}\right)}{1+\sqrt{1-\left(\frac{-f(x_n)}{x_n f'(x_n)}\right)^2}}\right)\right]$$

$$x_{n+1} = x_n\left[1 + 2\left(\frac{\left(\frac{(f'x_n)}{x_n f'(x_n)}\right)}{\frac{x_n f'(x_n) + \sqrt{(x_n f'x_n))^2 - (f'(x_n)^2}}{x_n f'(x_n)}}\right)\right]$$

$$= x_n\left[1 + 2\left(\frac{-f(x_n)}{x_n f'(x_n) + \sqrt{(x_n f'x_n))^2 - (f'(x_n)^2}}\right)\right]$$

as $\left(\frac{(f'x_n)}{f'(x_n)}\right)^2$ = h 2 → 0 since hisverysmall.

the above equation (2.3) reduces to

$$x_{n+1} = x_n\left[1 - 2\left(\frac{(f'x_n)}{2x_n f'(x_n)}\right)\right]$$

$$= x_n - \frac{(f'x_n)}{f'(x_n)}$$

This demonstrates how the suggested approach reduces to the Newton-Raphson approach and has quadratic convergence.

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A. *Proposed Algorithm*

1) Finding the initial approximations x 0& x 1 that cause f(x 0) f(x 1) 0 is necessary.
2) Use the suggested technique to locate the following approximation root of the given equation.
3) Continue in this manner until we obtain the desired approximation of the root. Figure 1 shows the flowchart for the suggested method.

III. NUMERICAL EXPERIMENTS

This section presents some numerical examples to explain the efficiency of the proposed method provided in the section above, and comparisons are taken into consideration to ensure that the proposed method is more efficient than other methods.

The following problems are considered to show the effectiveness of our proposed method. Here x0 is considered as the initial approximation to the root

- Example 3.1. f(x) = xe^{-x}− 0.1 with x_0= 0.1.
- Example 3.2. f(x) = 11x^{11} − 1 with x_0 = 1.
- Example 3.3. f(x) = x −e^{sinx} + 1 with x_0 = 2.
- Example 3.4. f(x) = ln x with x_0 = 0.5.

In the numerical experiments, the errors are taken as10^{-15} , and the maximum number of iterations is limited to 100, the results of examples 1 to 4 are shown in Table 1

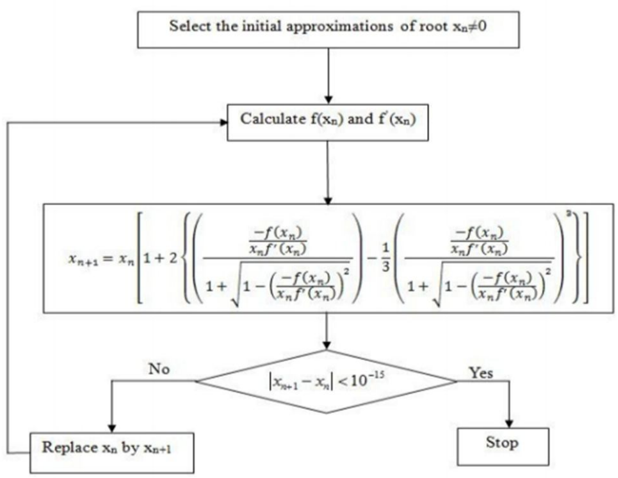

Figure 1: flow chart of the proposed method.

Table 1: The numerical comparison of examples using various existing methods.

Ex.	Chen & Li [3]			Chen & Li [4]			Proposed method		
	n	x_n	$\|f(x_n)\|$	n	x_n	$\|f(x_n)\|$	n	x_n	$\|f(x_n)\|$
1	6	1.11833e-01	0.00000e+00	6	1.11833e-01	0.00000e+00	3	1.11833e-01	0.0000e+00
2	7	8.04133e-01	1.22124e-15	8	8.04133e-01	4.44089e-16	7	8.04133e-01	4.44089e-16
3	5	1.69681e+00	4.44089e-16	11	1.69681e+00	4.44089e-16	5	1.69681e+00	4.44089e-16
4	6	1.00000e+00	0.00000e+00	7	1.00000e+00	0.00000e+00	5	1.00000e+00	0.0000e+00

1) *Example 3.1*

Consider a transcendental equation

$$f(x) = xe^{-x} - 0.1 \qquad (3.1)$$

The following Table 2 shows the comparison between different existing methods and proposed method with initial approximations x_0= 0 and x_1 = 1. Here n represents iteration numbers and $x_n$represents the corresponding approximation root.

Table 2: The numerical comparison of Example 3.1 using different existing methods.

Bisection		Regula-Falsi method		Proposed method	
n	x_n	n	x_n	n	x_n
1	0.5	1	0.271828182845905	1	0.11171239019496900
2	0.25000000000000000	2	0.13123614957214600	2	0.11183254383445500
3	0.12500000000000000	3	0.11402370160043800	3	0.11183255915896300
4	0.06250000000000000	4	0.11207786888180100	4	0.11183255915896300
⋮	⋮	⋮	⋮		
42	0.11183255915898400	17	0.11183255915896300		

2) *Example 3.2.*

Consider a transcendental equation

$$f(x) = 11x^{11} - 1. \qquad (3.2)$$

The following Table 3 shows the comparison between different existing methods and proposed method with initial approximations $x_0 = 0$ and $x_1 = 1$. Here n represents iteration numbers and x_n represents corresponding approximation root.

Table 3: The numerical comparison of Example 3.2 using different existing methods.

Bisection		Regula-Falsi method		Proposed method	
n	x_n	n	x_n	n	x_n
1	0.5	1	0.090909090909091	1	0.917261051121725
2	0.75	2	0.173553719005368	2	0.853423490462349
3	0.875	3	0.248685195862868	3	0.816152164916169
4	0.8125	4	0.316986388027222	4	0.804979772313121
⋮	⋮	⋮	⋮		
49	0.804133097503664	108	0.804133097503664	7	0.804133097503664

3) *Examples 3.3*

Consider a transcendental equation

$$f(x) = x - e^{sinx} + 1. \qquad (3.3)$$

The following Table 4 shows the comparison between different existing methods and proposed method with initial approximations $x_0 = 1.5$ and $x_1 = 2$. Here n represents iteration numbers and xn represents corresponding approximation root.

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Table 4: The numerical comparison of Example 3.3 using different existing methods.

Bisection		Regula-Falsi method		Proposed method	
n	x_n	n	x_n	n	x_n
1	1.75	1	1.645067953924812	1	1.744811921632327
2	1.625	2	1.685074247441264	2	1.698840973066092
3	1.6875	3	1.694253896381327	3	1.696816413391276
4	1.71875	4	1.696259793878769	4	1.696812386825692
⋮	⋮	⋮	⋮	⋮	⋮
49	1.696812386809751	24	1.696812386809751	6	1.696812386809751

4) Example 3.4.

Consider a transcendental equation

$$f(x) = \ln x. \quad (3.4)$$

The following Table 5 shows the comparison between different existing methods and proposed method with initial approximations x_0= 0.5 and x_1= 1.2. Here x_n represents iteration numbers and represents corresponding approximation root.

Table 5: The numerical comparison of Example 3.4 using different existing methods.

Bisection		Regula-Falsi method		Proposed method	
n	x_n	n	x_n	n	x_n
1	0.85	1	1.000001949490732	1	0.874016607739766
2	1.025	2	1.000000543230269	2	0.992066210397659
3	0.9375	3	1.000000151372449	3	0.999968527492996
4	0.98125	4	1.000000042180307	4	0.999999999504741
⋮	⋮	⋮	⋮		
53	1.000000000000000	23	1.000000000000000	5	1.000000000000000

IV. CONCLUSION

The suggested work establishes that the bisection, regula-falsi, and secant methods are not as effective as the primacy for estimating the root of a given transcendental function. The act has been demonstrated using conventional numerical examples. The combined use of the Newton-Raphson method and the inverse tan series forms the basis of the suggested approach. The proposed method's rate of convergence is examined and determined to be quadratic. The proposed method is implemented using Matlab programming. Overall, compared to the previously used conventional approaches, the proposed method executes more faster and with greater accuracy to the exact solution. Maximum power point tracking (MPPT) of photovoltaic (PV) systems and the class of multi-step iterative methods can both be solved numerically using this suggested methodology in discrete stochastic arithmetic (DSA). This approach can also be used to identify potential flaws in digital processes such as quantization, concretization, and algorithm numerical stability issues.

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