iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 13 **Issue:** III **Month of publication:** March 2025

DOI: https://doi.org/10.22214/ijraset.2025.67958

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 13 Issue III Mar 2025- Available at www.ijraset.com

Non-Linear Programming Problem using Pentagonal Fuzzy Numbers

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Abstract: In this study, pentagonal fuzzy numbers are used as the decision parameters. To solve these fuzzy nonlinear programming problems, we first convert all of the pentagonal fuzzy numbers into crisp values using robust ranking method. Next, we obtain a crisp nonlinear programming problem. Finally, we apply the Kuhn-Tucker conditions to obtain the best solution.
Keywords: Fuzzy Non-Linear programming problem, Robust Ranking Method, PentagonalFuzzy Number.

I. INTRODUCTION

Traditional optimization techniques have been used successfully for many years. The decision maker's decision is based on the information available from the mathematical modelling environment. However, we have a lot of trouble formulating and addressing real-world problems because of the ambiguities and inexactness of the decision parameters. These inaccuracies are caused by subjectivity and human opinion, lack of information, estimation error, and imperfect representation of information, among other things. As a result, conventional mathematical techniques are unable to formulate and successfully solve real-world problems. Thus, fuzzy optimization techniques offer a useful and powerful tool for optimization under fuzzy environment.

In 1965, Zadeh introduced Fuzzy set [19]. It plays a crucial role in solving the real-world problems. Thereafter, Bellman and Zadeh have applied the decision-making concept in fuzzy nature [3]. In 1991, Fuzzy set theory and its applications was discussed by Zimmermann [20]. Arun Pratap Singh [2] presented Allocation of subjects in an educational institution by Robust ranking method [8]. MonalishaPattnaik [11] applied Robust ranking for two phase fuzzy Optimization. S. Yahya Mohamed [18] Solving Fuzzy Travelling Salesman Problem Using Octagon Fuzzy Numbers with alpha-Cut. P. Umamaheswari et al [15] presented fuzzy problem converted into parametric form. Then applying by Kuhn Tucker conditions, the optimal solution of the problem is obtained.

DaruneeHunwisai et al [7], M. S. Annie Christi [1] applied Robust ranking for fuzzy transportation problems [14]. Sudha et al [13] suggested a new approach for fuzzyneutrosophicquadratic problem. Kokila et al [9] used Robust ranking method forsolving Fuzzy Octagonal number.S. U. Malini et al [10] applied Modi method Fuzzy Octagonal number.Thangaraj Beaula and Vijaya [4-6] have adopted different fuzzy numbers and ranking methods to solve critical path problems. Vijaya et al [16,17] have used Pythagorean Fuzzy numbers and Neutrosophic Fuzzy numbers to find the solution of Decision making problem and critical path problems. Vijaya et al have used complex FermateanNeutrosophic fuzzy sets to solve decision making problem.

Vijaya, V., and D. Rajalaxmi. "Decision making in fuzzy environment using Pythagorean fuzzy numbers." Mathematical Statistician and Engineering Applications 71.4 (2022): 846-854.Vijaya, V., D. Rajalaxmi, and H. Manikandan. "Finding critical path in a fuzzy project network using neutrosophic fuzzy number." Advances and Applications in Mathematical Sciences 21.10 (2022): 5743-5753. Vijaya V, Said Broumi, & Manikandan H. (2025). Complex FermateanNeutrosophic Sets and their Applications in Decision Making. Neutrosophic Sets and Systems, 81, 827-839.

In this paper, we solve fuzzy nonlinear programming problems in which all of the problem's decision parameters are pentagonal fuzzy numbers. First, we use the robust ranking approach to turn all of the pentagonal fuzzy numbers into crisp values, and then we get a crisp nonlinear programming problem, which we solve using the Kuhn Tucker conditions.

II. PRELIMINARIES

Definition:2.1

If X is a universal set and xϵX, then a fuzzy set $\tilde{A}$ defined as a collection of ordered pairs,$\tilde{A}$ = {(x,$\mu_{\tilde{A}}(x)$), xϵX}
where $\mu_{\tilde{A}}(x)$ is called the membership function that maps X to the membership space M.

Definition:2.2

A fuzzy number A = $(a_1, a_2, a_3, a_4, a_5)$ is called a pentagonal fuzzy number. If its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & for\ x < a_1, a_2 \le x \\ \frac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ \frac{x-a_2}{a_3-a_2} & for\ a_2 \le x \le a_3 \\ 1 & for\ x = a_3 \\ \frac{a_4-x}{a_4-a_3} & for\ a_3 \le x \le a_4 \\ \frac{a_5-x}{a_5-a_4} & for\ a_5 \le x \le a_4 \end{cases}$$

Herer for PFN A = $(a_1, a_2, a_3, a_4, a_5)$, a_3 is the middle point and (a_1, a_2) and (a_4, a_5) are the left and right side points of a_3 respectively. The middle point $a_3$has the grade of membership 1 and w_1, w_2 are the grades of points a_2, a_4.

Note that, every PFN is associated with two weights $w_1\ and\ w_2$.

Definition:2.3

A fuzzy number A= $(a_1, a_2, a_3, a_4, a_5)$ is a pentagonal fuzzy number (PFN) having membership function

$$\mu_{(x,w_1,w_2)}=\begin{cases} w_1\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ 1-(1-w_1)\left(\frac{x-a_2}{a_3-a_2}\right) & for\ a_2 \le x \le a_3 \\ 1 & for\ x = a_3 \\ 1-(1-w_2)\left(\frac{x-a_3}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ w_2\left(\frac{x-a_5}{a_4-a_5}\right) & for\ a_4 < x < a_5 \\ 0 & for\ x > a_5 \end{cases}$$

Case(i):

When $w_1 = w_2 = 0$, then the pentagonal fuzzy number is reduced to a triangular fuzzy number. That is, A= $(a_1, a_2, a_3, a_4, a_5) \approx (a_2, a_3, a_4)$ where membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & for\ x \le a_2 \\ 1-\left(\frac{x-a_2}{a_3-a_2}\right) & for\ a_2 \le x \le a_3 \\ 1 & for\ x = a_3 \\ 1-\left(\frac{x-a_3}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ 0 & for\ x \ge a_4 \end{cases}$$

Case (ii):

When $w_1 = w_2 = 1$, then the pentagonal fuzzy number becomes a trapezoidal fuzzy number. That is, the membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & for\ x \le a_1 \\ \left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_5 \\ \left(\frac{a_4-x}{a_5-a_4}\right) & for\ a_4 \le x \le a_5 \\ 0 & for\ x > a_5 \end{cases}$$

Definition:2.4

A fuzzy set $\tilde{A}$ defined on X is called a normal fuzzy set if there exist at least one xϵX such that $\mu_{\tilde{A}}(x) = 1$.

Definition:2.5

If $\tilde{a}$ is a fuzzy number, the Robust ranking index is defined by

R($\tilde{a}$) = 0.5 $\int_0^1 (a_\alpha^L, a_\alpha^U)\, d\alpha$

Where (a_α^L, a_α^U) = {$2\alpha(a_2 - a_1) + a_1$,- $2\alpha(a_5 - a_4) + a_5$), $2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) - a_4 + 2a_3$} is the α-cut of the fuzzy number $\tilde{a}$.

Definition:2.6

Given a fuzzy set '$\tilde{A}$' defined on 'X' and any $\alpha\epsilon[0,1]$, the α-cut is denoted by $\tilde{A}(\alpha)$ and is defined as, $\tilde{A}(\alpha)$ = {x:μ(x)≥ α, $\alpha\epsilon[0,1]$}.

III. FUZZY NON-LINEAR PROGRAMMING PROBLEMS

The fuzzy nonlinear programming problem (FNPP)defined as

Maximize (or) Minimize $\tilde{f}(x)= \tilde{f}(x_1, x_2, \ldots, x_n)$

Subject to

$\tilde{g}_j(\tilde{x}_j) \le or \approx or \ge b_i$; i=1, 2, . . ., m

$\tilde{x}_j \ge 0$ for all j= 1, 2, . . ., n

The fuzzy version Kuhn-Tucker conditions given below,

Kuhn-Tucker condition 1: $\nabla f(x)$-$\sum_{j=1}^{n} \mu_j \nabla g_j(x)$ = 0 (1)

Kuhn-Tucker condition 2: $\mu_j \nabla g_j(x)$ = 0, ∀ j = 1,2,3,…,n (2)

Kuhn-Tucker condition 3: $g_j(x)$ - $b_i \le 0$, j = 1,2,3,…,n (3)

IV. NUMERICAL EXAMPLE

Consider a fuzzy nonlinear programming problem

Max $\widetilde{w}$ = (1,1,2,3,4)$\tilde{c}_1^2$ + (1,1,2,2,3)$\tilde{c}_2^2$

Subject to

(0,1,2,2,3)$\widetilde{c_1}$+ (2,3,3,4,5)$\widetilde{c_2}$ ≤(3,4,5,5,6)

(1,2,3,3,4)$\widetilde{c_1}$- (0,1,1,2,2)$\widetilde{c_2}$ ≤(1,2,3,4,5) and

$\widetilde{c_1}$,$\widetilde{c_2}$ ≥ 0

Now convert the given fuzzy problem into crisp form using the Robust Ranking method

Max $\widetilde{w}$ = R(1,1,2,3,4)$\tilde{c}_1^2$ + R(1,1,2,2,3)$\tilde{c}_2^2$

Subject to

R(0,1,2,2,3)$\widetilde{c_1}$+ R(2,3,3,4,5)$\widetilde{c_2}$ ≤ R(3,4,5,5,6)

R(1,2,3,3,4)$\widetilde{c_1}$- R(0,1,1,2,2)$\widetilde{c_2}$ ≤ R(1,2,3,4,5) and

R ($\tilde{a}$) = 0.5 $\int_0^1 (a_\alpha^L, a_\alpha^U)$ dα

Where (a_α^L, a_α^U) = {$2\alpha(a_2-a_1)+a_1$,- $2\alpha(a_5-a_4)+a_5$), $2\alpha(a_3-a_2)$ $+2a_2 - a_3, 2\alpha(a_4-a_3) - a_4 + 2a_3$}

Therefore

R(1,1,2,3,4) = 0.5$\int_0^1\{2\alpha(0) + 1 - 2\alpha(1) + 4 + 2\alpha(1) + 2 - 2 + 2\alpha(1) - 3 + 2(2)\}$ dα

= 0.5$\int_0^1\{1 - 2\alpha + 4 + 2\alpha + 2 - 2 + 2\alpha - 3 + 4\}$ dα

= 0.5$\int_0^1\{6 + 2\alpha\}$ dα

= 3.0

Similarly, we get

R(1,1,2,2,3) = 3.0 , R(0,1,2,2,3) = 2.5 , R(2,3,3,4,5) = 6.0,R(3,4,5,5,6) = 8.5

R(1,2,3,3,4) = 4.5 , R(0,1,1,2,2) = 3.0 , R(1,2,3,4,5) = 9.0

We obtain crisp nonlinear programming problem is

Max $\widetilde{w}$ =3.0$\tilde{c}_1^2$ + 3.0$\tilde{c}_2^2$

Subject to

2.5$\widetilde{c_1}$+ 6.0$\widetilde{c_2}$ ≤ 8.5

4.5$\widetilde{c_1}$-3.0$\widetilde{c_2}$ ≤ 9.0

$\widetilde{c_1}$,$\widetilde{c_2}$ ≥ 0

By applying KT condition (1) we get

6.0$\widetilde{c_1}$-2.5ϕ_1 - 4.5ϕ_2 = 0

6.0$\widetilde{c_1}$- 6.0ϕ_1–3.0ϕ_2 = 0

By applying KT condition (2) we get

ϕ_1 $[2.5\tilde{c_1} + 6.0\tilde{c_2} - 8.5] = 0$
ϕ_2 $[4.5\tilde{c_1} - 3.0\tilde{c_2} - 9.0] = 0$
By applying KT condition (3) we get
$2.5\tilde{c_1} + 6.0\tilde{c_2} - 8.5 \leq 0$
$4.5\tilde{c_1} - 3.0\tilde{c_2} - 9.0 \leq 0$
Solving the above equations, we get the optimal solution is
$\tilde{c_1} = 2.304$ and $\tilde{c_1} = 0.456$
Max $\tilde{w} = 3.0\tilde{c}_1^2 + 3.0\tilde{c}_2^2$
Max $\tilde{w} = 3.0(2.304)^2 + 3.0(0.456)^2$
$= 15.925 + 0.623$
Max $\tilde{w}$= 16.548

V. CONCLUSION

We used the proposed method in this research to achieve optimal solution to non-linear programming problem with linear constraints, and we confirmed the accuracy of the proposed method using numerical examples.

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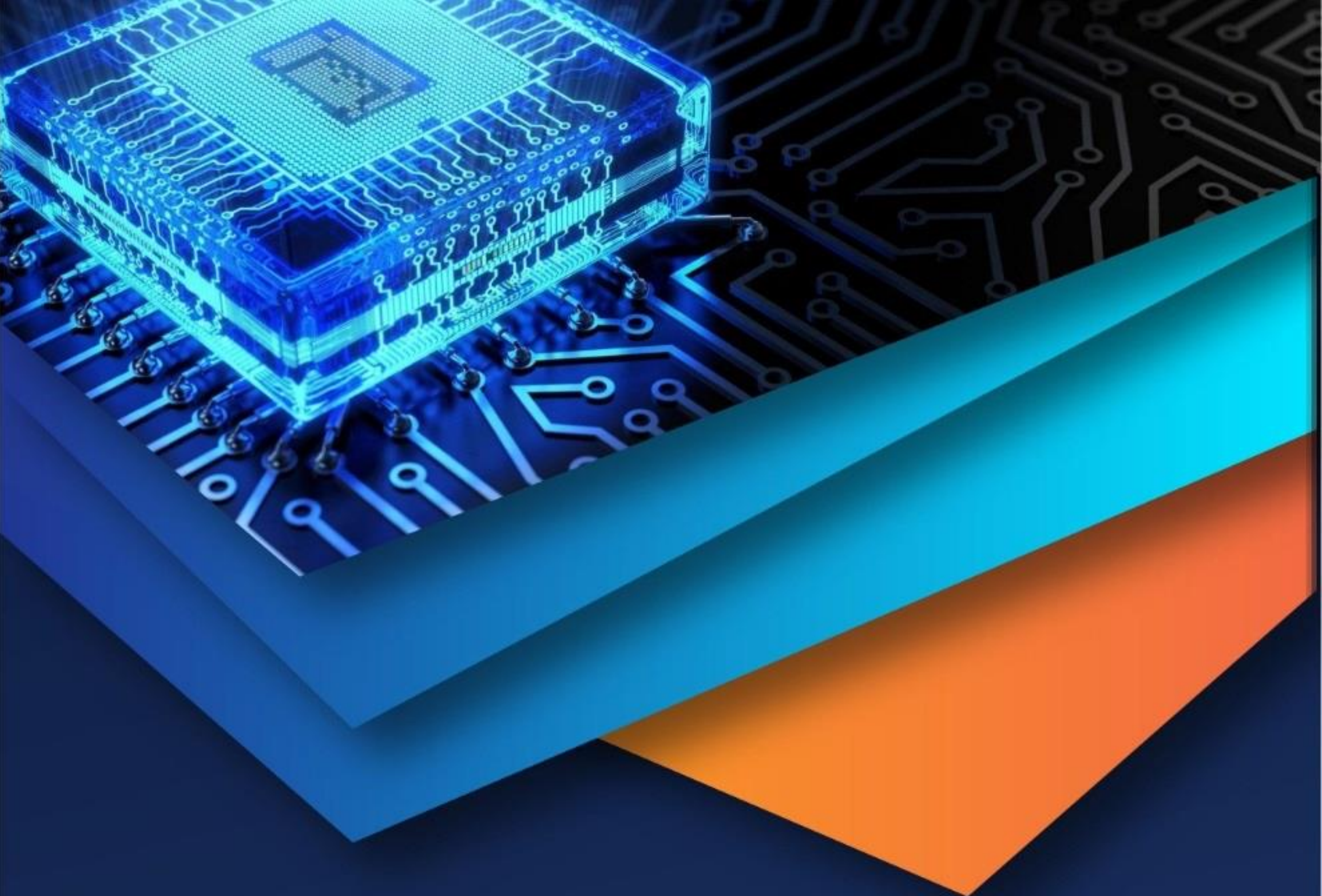

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