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Numerical Analysis of Two-Dimensional Natural Convection in an Open Vertical Tube

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Abstract: Numerical analysis of two-dimensional natural convection in an open vertical tube with volumetric heat generation involves solving the governing equations for fluid flow and heat transfer in such a configuration. This problem is typically addressed by considering the Navier-Stokes equations (for fluid flow), the energy equation (for heat transfer), and appropriate boundary conditions. The presence of volumetric heat generation adds complexity to the thermal analysis. Numerical analysis of natural convection in open ended ended two dimensional axisymmetric vertical tube has been carried out for a wide temperature range to find out the relation between non-dimensionalized number, Numerical analysis of two-dimensional natural convection in open ended vertical tube has been carried out for a wide range of heat generation rate to find out the appropriate non-dimensional numbers governing the process. Buoyancy induced flows in a tube with adiabatic wall boundary condition and uniform volumetric heat generation rate in fluids were analyzed by varying heat generation rate and the geometrical aspect ratio of the tubes with and without considering surface radiation. Curve fits have been provided for non-dimensionalized mass flow rate versus other relevant non-dimensional numbers based on heat generated and geometry of the tube. Non-dimensional correlation has been found out for natural convection flow in an open adiabatic tube with internal heat generation and neglecting surface radiation. One more non-dimensional correlation has been found out for natural convection flow in an open adiabatic tube with internal heat generation and considering surface radiation.
Keywords: Natural convection, vertical tubes, heat generation rate, variable properties, non-dimensional numbers, buoyancy flow.

I. INTRODUCTION

Analysis of high temperature natural convection systems is more complex than that of low temperature natural convection systems due to strong dependence of properties. Thus we cannot extend the results of low temperature work directly to the high temperature cases. In particular, the issue of identifying the relevant non-dimensional numbers which govern the high temperature natural combustion flow has not been addressed in the literature.

The focus of this project was to study high temperature natural convection in an open vertical tube through numerical simulation. For the results to be useful in combustion devices, the source of buoyancy is taken as the heat generated in the bulk of the gas.

A. Problem Defination

The problem that is studied in which combustion is taking place inside the tube and air is being inducted because of this heat generation. Thus this problem is a coupled problem in which combustion, continuity, momentum and energy equations are solved simultaneously. As a first step of approach, numerical analysis of the free convection heat transfer with volumetric heat generation in an open vertical adiabatic tube is considered here. Although simplified, this approach is more practical from the point of view of chulha problem. Thus, this project aims to conduct a numerical study of natural convection in an open vertical tube for a range of wall temperatures and geometrical aspect ratio. Simulation results are used to generate correlations for non-dimensional mass flow rate (NDMF) in terms of relevant non-dimensional parameters.

II. REVIEW OF LITERATURE

Literature survey was carried in such a manner so as to include articles relating to the present geometry and similar geometry. Also, looking at the physics of this high temperature natural convection problem variable property formulation was studied. The articles reviewed include work with Boussinesq-constant property formulation, variable property formulation, phenomenon like backflow, transition from laminar to turbulent.

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III. GOVERNING EQUATION

The geometry under consideration is axisymmetric. Hence, equations are considered in cylindrical co-ordinates and the problem is solved as 2-dimensional steady state problem. Since the pressure differences in a natural convection problem are very small, the flow is considered to be incompressible and laminar. The governing equations are thus given by:

A. Continuity Equation

$$\frac{\partial v_x}{\partial x}+\frac{1}{r}\frac{\partial (rv_r)}{\partial r}=0$$

B. Axial Momentum Equation

$$\frac{\partial}{\partial x}\left(\rho v_x^2\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho v_r v_x\right)=-\frac{\partial p}{\partial x}+2\frac{\partial}{\partial x}\left(\mu\frac{\partial v_x}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu\left(\frac{\partial v_x}{\partial r}+\frac{\partial v_r}{\partial x}\right)\right]-\rho.g$$

C. Radial Momentum Equation

$$\frac{\partial}{\partial x}\left(\rho v_x v_r\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho v_r^{\ 2}\right)=-\frac{\partial p}{\partial r}+\frac{1}{r}\frac{\partial}{\partial x}\left[r\mu\left(\frac{\partial v_r}{\partial x}+\frac{\partial v_x}{\partial r}\right)\right]+\frac{2}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v_r}{\partial r}\right)-\frac{2}{3}\mu\frac{v_r}{r^2}$$

D. Energy Equation

$$\frac{\partial}{\partial x}\left(\rho c_p v_x T\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho C_p v_r T\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+Q'''_{gen}$$

Where, v_x and v_r are velocities in axial and radial directions respectively in m/s

ρ is density of the fluid inside the tube in Kg/m^3

μ is viscosity in N-s/ m^2

k is thermal conductivity in W/ m^2-K

C_p is specific heat of the fluid in J/Kg-K

Q'''_{gen} is volumetric heat generated in W/ m^3

E. Radiation Equation

Radiative heat transfer was included in this analysis as radiant heat flux is large compared to the heat transfer rate due to convection. Radiative heat transfer dominates in high temperature cases because radiative heat flux has a fourth order dependence on temperature.

The radiative transfer equation (RTE) for an absorbing, emitting, and scattering medium at position $\vec{r}$ in the direction $\vec{s}$ is given as:

$$\frac{dI(\vec{r},\vec{s})}{ds}+\left(a+\sigma_s\right)I(\vec{r},\vec{s})=an^2\frac{\sigma T^4}{\pi}+\frac{\sigma_s}{4\pi}\int_0^{4\pi} I((\vec{r},\vec{s}\,')\phi(\vec{s},\vec{s}\,'))d\Omega'$$

where $\vec{r}$ = position vector

$\vec{s}$ = direction vector

$\vec{s}\,'$ = scattering direction vector

s = path length

a = absorption coefficient
n = refractive index
σ_s = scattering coefficient
σ = Stefan-Boltzmann constant ($5.672\text{x}10^{-8}$ W/m^2 -K^4)
I = radiation intensity, which depends on position ($\vec{r}$) and direction ($\vec{s}$)
T = local temperature
ϕ = phase function
Ω' = solid angle

As already discussed, the fluid is assumed to be non-interfering medium where no scattering is taking place. So, scattering coefficient is taken as zero and refracting index is assumed to be unity. The equation for the change of radiation intensity, dI , along a path, ds, can be written as:

$$\frac{dI}{ds} + aI = \frac{a\sigma T^4}{\pi}$$

This equation (3.6) is integrated along a series of rays emanating from boundary faces. If a is constant along the ray:

$$I(s) = \frac{\sigma T^4}{\pi}\left(1 - e^{-as}\right) + I_o e^{-as}$$

As, only surface radiation was considered gas absorption coefficient was assumed to be zero, thus

$$I(s) = \frac{\sigma T^4}{\pi} + I_o$$

where I_o is the radiant intensity at the start of the incremental path, which is determined by the appropriate boundary condition (description of boundary condition is given in section

F. *Non Dimensional Governing Equation*

The above equations can be non-dimensionalized in different ways. In literature it was found that either Grashof number or Rayleigh number are the relevant non-dimensional parameters that are governing the buoyancy flows. Considering this fact, non-dimensionalization is done in such a manner that Grashof number term appears in the equation. Here two sets of non-dimensionalized equations are obtained.

G. *First Method of Dimensionalisation*

Above equations were non-dimensionalized below using following non-dimensional variables corresponding to reference temperature.

$$R = \frac{r}{r_o}, \hat{\mu} = \frac{\mu}{\mu_o}, \hat{k} = \frac{k}{k_o}, \hat{C}_p = \frac{C_p}{C_{po}},$$

$$X = \frac{x}{r_o}, \hat{\rho} = \frac{\rho}{\rho_o}, V = \frac{v_r r_o}{v_o}, U = \frac{v_x r_o}{v_o}$$

$$\theta = \frac{T - T_o}{T_{ref} - T_o}, \hat{p}_d = \frac{\left(p - p_a\right) r_o^2}{\rho_o v_o^2}$$

Where,

$$\Delta T_{ref} = T_{ref} - T_o = \frac{Q'''_{gen} r_o^2}{k_o} \quad \text{and,}$$

$p = p_d + p_a$, p_a being ambient pressure follows the relation $\frac{\partial p_a}{\partial x} = -\rho_o g$.

H. Non-dimensional Equations

Continuity equation

$$\frac{\partial U}{\partial X}+\frac{1}{R}\frac{\partial (RV)}{\partial R}=0$$

Axial Momentum equation

$$\frac{\partial}{\partial X}\left(\hat{\rho}U^2\right)+\frac{1}{R}\frac{\partial}{\partial R}\left(R\hat{\rho}UV\right)=-\frac{\partial \hat{p}_d}{\partial X}+2\frac{\partial}{\partial X}\left(\hat{\mu}\frac{\partial U}{\partial X}\right)+\frac{1}{R}\frac{\partial}{\partial R}\left[R\hat{\mu}\left(\frac{\partial U}{\partial R}+\frac{\partial V}{\partial X}\right)\right]+\theta.Gr$$

Radial Momentum equation

$$\frac{\partial(\hat{\rho}UV)}{\partial X}+\frac{1}{R}\frac{\partial(\hat{\rho}RV^2)}{\partial R}=-\frac{\partial \hat{p}_d}{\partial R}+\frac{\partial}{\partial R}\left[\hat{\mu}\left(\frac{\partial V}{\partial X}+\frac{\partial U}{\partial R}\right)\right]-\frac{2}{3}\frac{\hat{\mu}V}{R^2}+\frac{2}{R}\frac{\partial}{\partial R}\left(R\hat{\mu}\frac{\partial V}{\partial R}\right)$$

Energy Equation

$$\frac{\partial}{\partial X}\left(\hat{\rho}\hat{C}_pU\theta\right)+\frac{1}{R}\frac{\partial}{\partial R}\left(R\hat{\rho}\hat{C}_pV\theta\right)=\frac{1}{\mathrm{Pr}_o}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\hat{k}R\frac{\partial\theta}{\partial R}\right)+\frac{\partial}{\partial X}\left(\hat{k}\frac{\partial\theta}{\partial X}\right)+1\right]$$

Where, Prandtl number and Grashof number are defined as

$$Gr=\frac{g\beta \mathrm{Q}'''_{\mathrm{gen}}r_o^5}{k_o\nu_o^2},\ \mathrm{Pr}=\frac{\nu_o}{\alpha_o}$$

where β is defined as $\beta=\frac{1}{T_o}$

T_o = reference temperature (293.16 K)

Second method of non-dimensionalization

In the second method of non-dimensionalization the axial momentum equation and energy equation were modified as:

I. Non-dimensional Equations

Axial Momentum equation

$$\frac{\partial}{\partial X}\left(\hat{\rho}U^2\right)+\frac{1}{R}\frac{\partial}{\partial R}\left(R\hat{\rho}UV\right)=-\frac{\partial \hat{p}_d}{\partial X}+2\frac{\partial}{\partial X}\left(\hat{\mu}\frac{\partial U}{\partial X}\right)+\frac{1}{R}\frac{\partial}{\partial R}\left[R\hat{\mu}\left(\frac{\partial U}{\partial R}+\frac{\partial V}{\partial X}\right)\right]+\frac{\theta-1}{\theta}\frac{g.r_o^3}{\nu_o^2}$$

Energy Equation

$$\frac{\partial}{\partial X}\left(\hat{\rho}\hat{C}_pU\theta\right)+\frac{1}{R}\frac{\partial}{\partial R}\left(R\hat{\rho}\hat{C}_pV\theta\right)=\frac{1}{\mathrm{Pr}_o}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\hat{k}R\frac{\partial\theta}{\partial R}\right)+\frac{\partial}{\partial X}\left(\hat{k}\frac{\partial\theta}{\partial X}\right)\right]+\frac{Q'''_{gen}r_o^2\alpha_o}{\nu_ok_oT_o}$$

Here, $\theta=\frac{T_{ref}}{T_o}$ and rest of the variables represents the same quantities as defined earlier in above section.

After, non-dimensionalizing the governing equations possibility of $\frac{Q'''_{gen} r_o^2 \alpha_o}{v_o k_o T_o}$ and $\frac{g.r_o^3}{v_o^2}$ as relevant non-dimensional numbers was found. Here, the term $\frac{Q'''_{gen} r_o^2 \alpha_o}{v_o k_o T_o}$ represents the temperature aspect. Examination of non-dimensional number $\frac{g.r_o^3}{v_o^2}$ reveals that when it is divided by aspect ratio i.e. $\frac{L}{r_o}$ it becomes a function of geometry of the tube.

Thus Nd1 $=\frac{g.r_o^3}{v_o^2}\frac{r_o}{L}$, Nd2$=\frac{Q'''_{gen} r_o^2 \alpha_o}{v_o k_o T_o}$

Simulations with heat generation and considering surface radiation

At high temperatures radiation cannot be neglected and thus effect of radiation was included. Thus the same problem is numerically analyzed considering surface radiation and variable property effect. Before starting the simulations for determining correlations, it was decided to study the velocity and temperature profiles of the buoyancy induced flow inside the adiabatic tube, where volumetric heat generation is causing the flow. Length of the simulated tube was 183.3 mm and radius was 18.33 mm, volumetric heat generation inside the tube was 15×10^5 W/m^3.

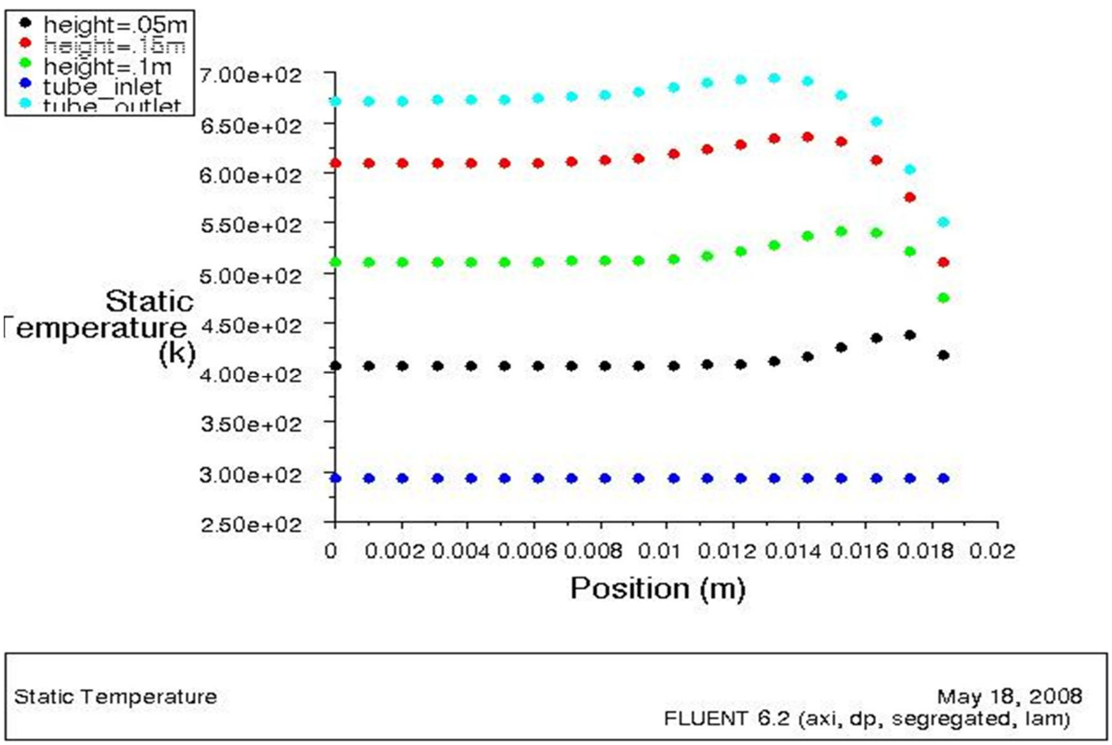

Figure Temperature profile of the buoyancy induced flow inside the tube

Above Figure shows temperature profiles of the buoyancy induced flow due to the volume heat generation inside an adiabatic tube considering surface radiation in account. It was found that temperature increases along with the height of tube. When, this profile was analyzed for a particular height it was found that maximum temperature doesn't exist at the wall unlike the earlier case when radiation effect was neglected. It was also observed that maximum temperature and average temperature were lower when we considered radiation effect in account. Velocity vectors showing different temperature with different colors are shown below (Figure)

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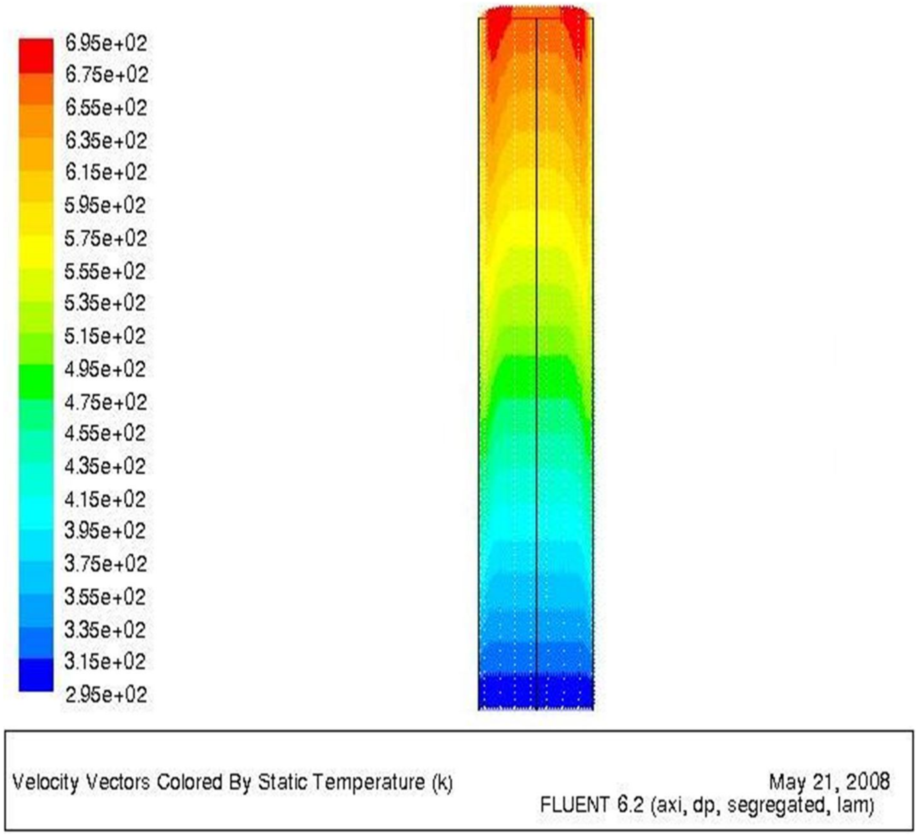

Figure Temperature distributions inside the tube considering radiation

Velocity profiles were also studied for the same case (Shown in Figure 5.19). It was found that although, velocity profiles were similar but velocities were lower in this case as compared to the case neglecting radiation effect. Velocity vectors showing different velocities with different colors are also shown below.(Figure)

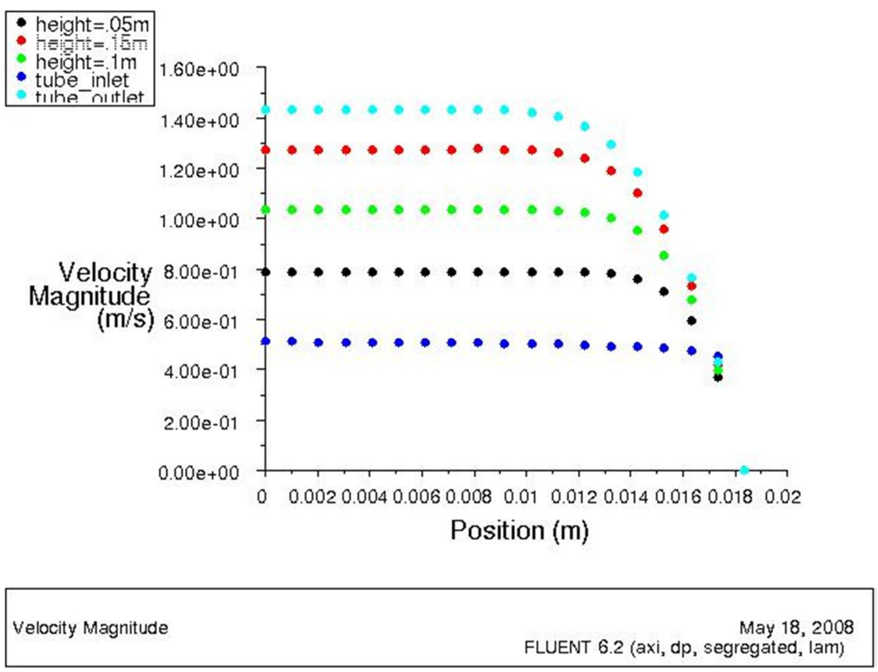

Figure Velocity profile of the buoyancy induced flow inside the tube

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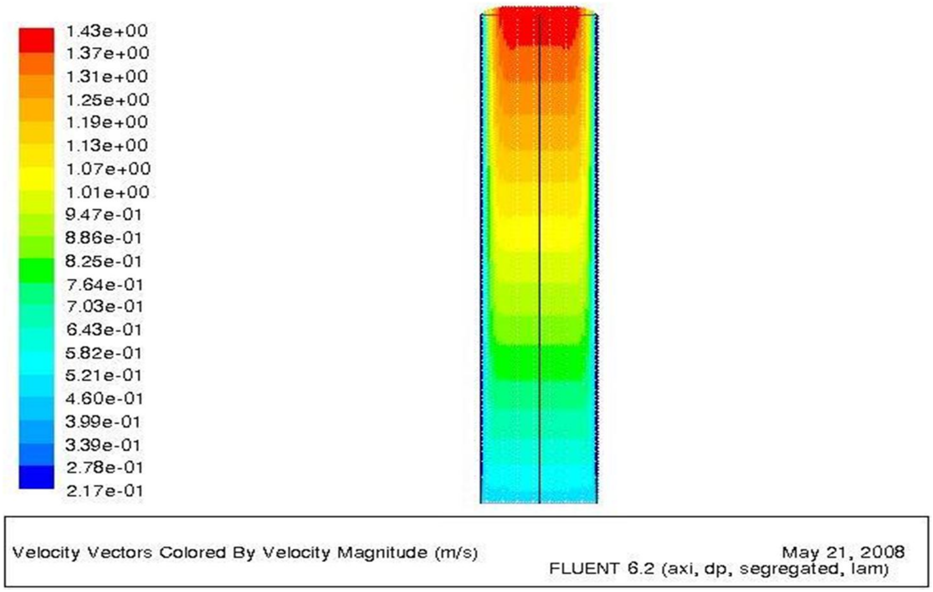

Figure Velocity distributions inside the tube considering radiation

A comparison of variation of NDMF with Nd2 without radiation and with radiation is given in Figure (below). Data for this simulation is given in Appendix B.3

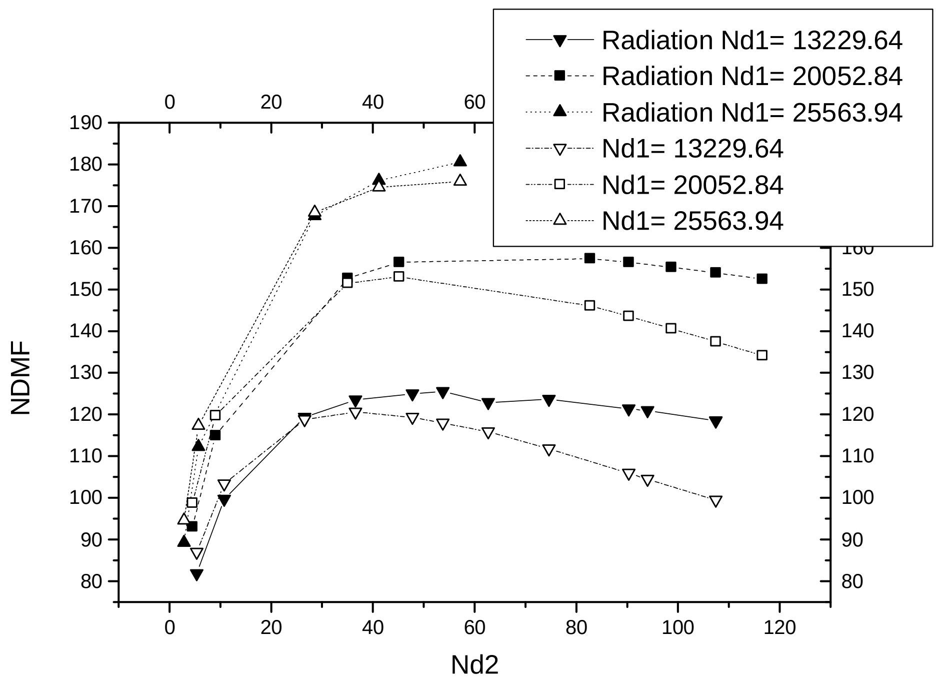

Figure Variation of NDMF with Nd2 without radiation and with radiation

When radiation effect was considered, it was observed that

- NDMF is a function Nd1 and Nd2 and the relationship is exponential with respect to Nd2
- NDMF is lower for lower temperatures and higher at higher temperatures than earlier predicted without considering radiation, this is explained below (Figure 5.22).
- After the peak the NDMF was found to be almost constant.

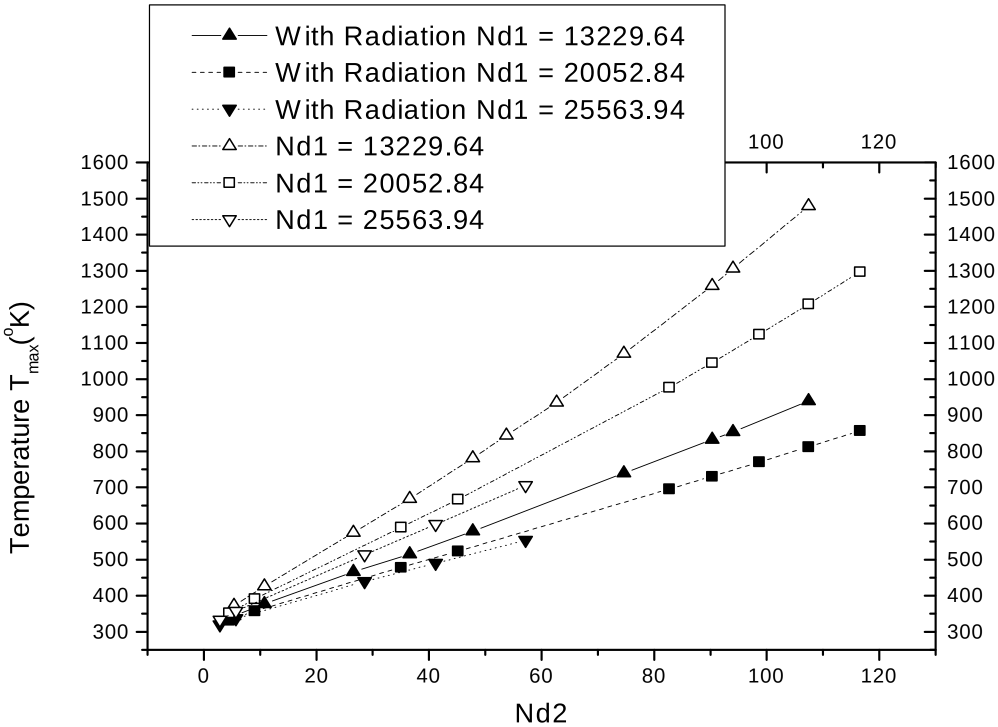

Figure Variation of maximum temperature with Nd2 at different Nd1

A graph was plotted between maximum temperature inside the tube and ND2 (Figure 5.22), using the data which is plotted in above figure, it was found that maximum temperatures prevailing inside the tube were lower for the same Nd2 when radiation effect was considered. When radiation effect is neglected, maximum temperature increases more rapidly for the same increase in Nd2. Before reaching to the peak value higher temperature causes mass flux to increase and after reaching peak value mass flux decreases with temperature thus case considering radiation effect gives higher values of mass flux after reaching the peak value.

J. Determining Correlation

To find correlation computational runs were conducted for laminar flow in an open ended tube with volumetric heat generation and considering surface radiation. A curve was fitted in the obtained data and was found to be exponential:

$$\tilde{m} = a_1 (Nd_1)^{a2} \left[1 - \exp(-bNd_2)\right]$$

Where coefficients are given as:

$a_1 = 3.48$, $a_2 = 0.39$, $b = 0.14$

Range to check validity of correlation

Simulations were carried for low temperature cases using Boussinesq constant property formulation. It was found that the correlation (without considering radiation) was giving erroneous results for low temperature cases (T_{max} less than 340 K) . Results are given in appendix B.4.

When non-dimensional mass flow rates calculated by using correlations were compared with the non-dimensional mass flow rates calculated by simulation using variable property formulation, results were found to be agreeing (Equation 5.4 and Equation 5.5). Validity of the above correlations was checked for a long range (Nd1 = 133229 to 259720 and Nd2 = 5.37 to 210.47) and correlations were found to be agreeing with simulated results. Scatter graphs are shown below for both cases i.e. neglecting radiations and considering radiations(Figure 5.23 and Figure) respectively.

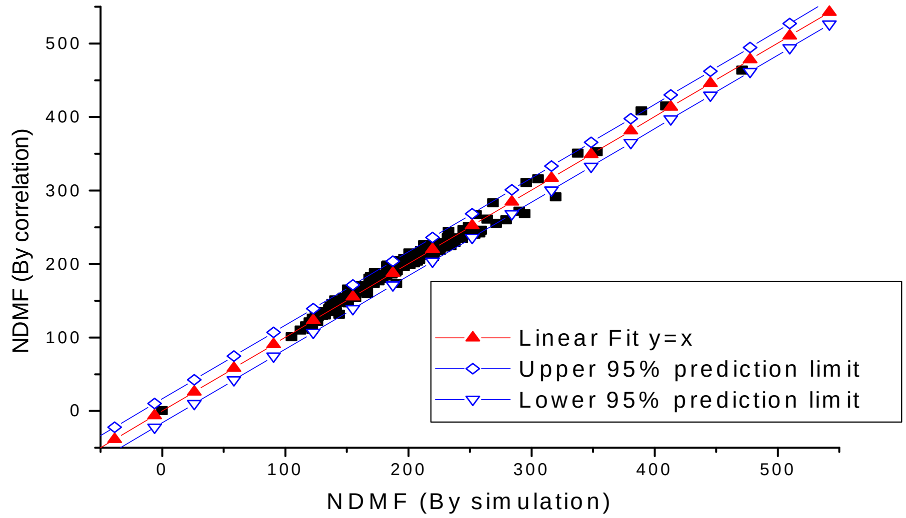

Figure Scatter plot between NDMF (By simulation) and NDMF (By correlation) [Neglecting radiation]

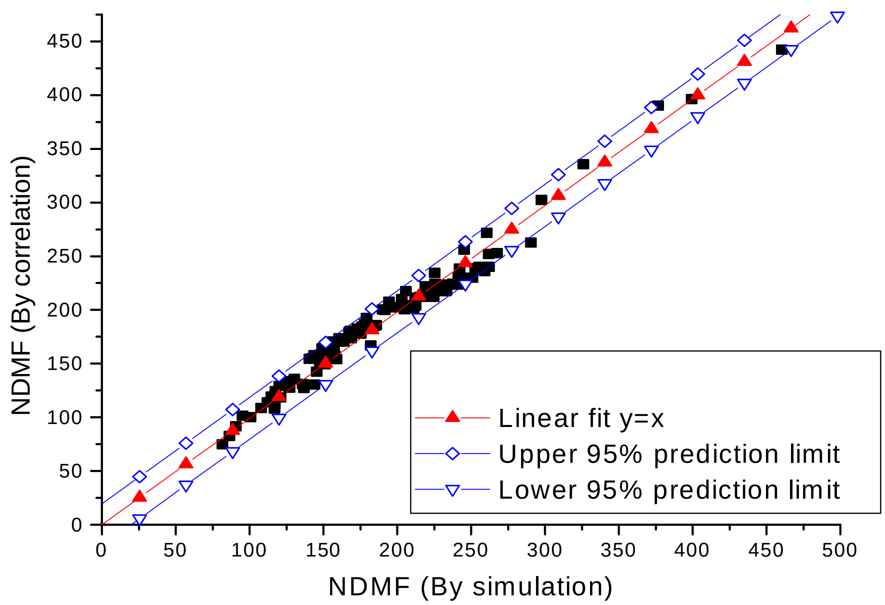

Figure Scatter plot between NDMF (By simulation) and NDMF (By correlation) [considering radiation]

IV. CONCLUSION AND FUTURE SCOPE

1) A numerical investigation for a two-dimensional natural convection in open ended vertical tube has been performed with volumetric heat generation. A wide range of heat generation rate, radius and aspect ratio have been considered to find out the appropriate non-dimensional numbers governing the process and correlations between them.
2) Many sets of simulations were carried out varying dimensional quantities within range, and it was found that T_{max} has a strong dependence on Q_{gen}, for both the cases, considering and neglecting radiation
3) In the present work, laminar flow in an open vertical tube with volumetric generation was studied, firstly neglecting radiation and then taking radiation into account. In future turbulent flows could be analyzed both with and without inclusion of radiation. Also, criterion for transition from laminar to turbulent flows could be studied for the present case

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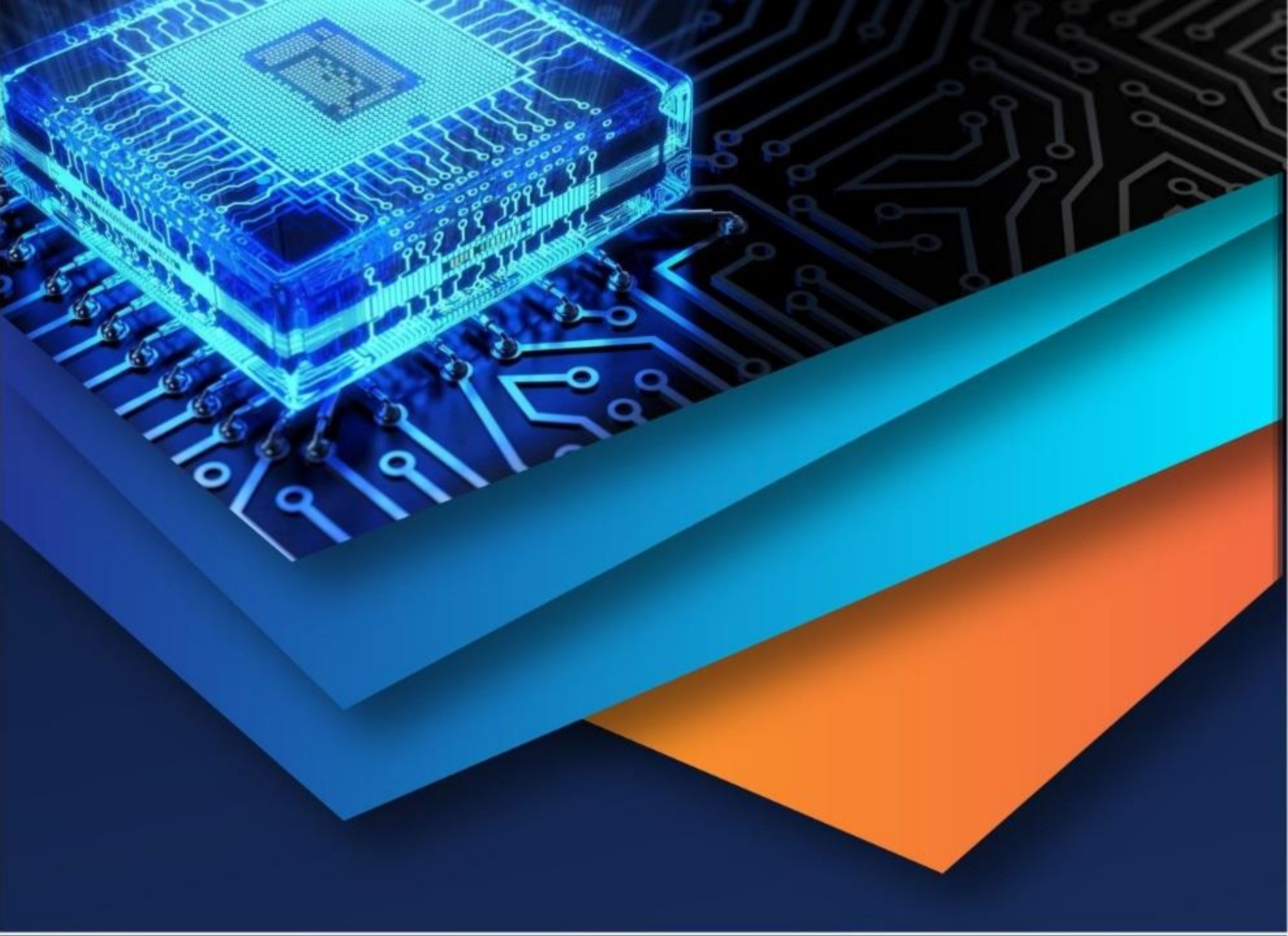

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