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Numerical Investigation of Two-Stream Instability in Magnetized Inhomogeneous Dusty Plasma Under the Influence of Ionization

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Abstract: ***Dusty plasmas are multicomponent ionized media containing electrons, ions, neutral particles and charged dust grains. The simultaneous presence of dust, magnetic fields, plasma inhomogeneity and ionization modifies the collective response and affects electrostatic instability. In this study, a fluid model is formulated to investigate two-stream instability in a weakly ionized, magnetized and inhomogeneous dusty plasma. Electrons, singly charged positive ions and negatively charged dust grains are treated as interacting charged fluids, while ionization is represented by source terms in the continuity equations. A uniform magnetic field is applied along the z direction and a slowly varying equilibrium density gradient is included. Linear perturbation analysis with normal-mode dependence exp[i(k·r − ωt)] gives a complex dispersion condition D(ω,k,νI,B0,Ln−1,Zd,nd,V0)=0. The numerical analysis is performed in normalized variables to examine the effects of ionization frequency, magnetic-field strength, density-gradient parameter, dust density, dust charge and wave number. The calculated maximum growth rate increases from 0.20 to 0.37 as νI/ωpi increases from 0 to 0.6, and decreases from 0.42 to 0.18 as Ωi/ωpi increases from 0.2 to 2.5. The maximum growth rate also increases from 0.22 to 0.44 with the density-gradient parameter over the range 0.05–0.50. A non-monotonic dependence on dust density is obtained, with a maximum growth rate of 0.39 at nd/n0 = 0.10. Increasing dust charge from Zd = 500 to 5000 increases the maximum growth rate from 0.22 to 0.40. The wave-number spectrum has a maximum growth rate of 0.46 near kλD = 1.4 and decreases at larger wave numbers. The results demonstrate the combined influence of ionization, magnetic field, plasma inhomogeneity and dust parameters on the two-stream instability.***

Keywords: Dusty plasma; two-stream instability; magnetized plasma; ionization; plasma inhomogeneity; plasma waves; numerical simulation

I. INTRODUCTION

Plasma is an ionized medium in which the dynamics of charged particles are governed by collective electromagnetic interactions. Because the charged components respond to self-consistent fields, plasmas support a wide range of waves, transport processes and instabilities. Dusty plasma is a multicomponent form of plasma containing electrons, ions, neutral particles and charged solid grains. Dust grains introduce additional inertia and charge density and therefore modify the dielectric response and collective modes.

Dusty plasmas occur in laboratory discharges and processing devices as well as in planetary rings, cometary environments, interstellar media and other astrophysical settings. Dust charging, dust dynamics and plasma waves can be strongly coupled in these environments. Reviews of dusty plasma physics have described ion-streaming-driven instabilities, dust-acoustic modes and the dependence of instability thresholds on collisions, charging and plasma parameters.[1–6]

An external magnetic field introduces anisotropy into the plasma response. Electrons and ions can become magnetized while massive dust grains may remain weakly magnetized or effectively unmagnetized, depending on the field strength and particle properties. Magnetic fields therefore modify particle trajectories, transport and wave propagation and can alter the growth of unstable modes.[7,8]

Real dusty plasmas are also frequently inhomogeneous. Equilibrium density, temperature and dust concentration may vary spatially, and these gradients modify wave propagation and the local plasma response. Analytical and numerical studies of magnetized inhomogeneous dusty plasmas have demonstrated the importance of dust charge, dust density, magnetic field and collisional processes in determining instability characteristics.[9,10]

Two-stream instability arises when two charged populations have different drift velocities. The relative drift provides free energy to electrostatic perturbations and can produce growing modes when the plasma response satisfies the instability condition.

In dusty plasmas, the presence of massive charged grains introduces additional inertia, charge-balance constraints and modified collective modes.[3,11]

Ionization changes the number of charged particles and therefore affects the equilibrium plasma response. In a dusty plasma, ionization can also modify electron and ion currents associated with dust charging. The present work combines ionization, magnetic-field effects, equilibrium density inhomogeneity, dust dynamics and streaming within a single fluid framework and evaluates the resulting complex dispersion relation numerically.

II. PHYSICAL MODEL

The plasma is modeled as a weakly ionized, magnetized and inhomogeneous mixture of electrons, singly charged positive ions, negatively charged dust grains and neutral particles. The external magnetic field is uniform and directed along the z axis, $B_0 = B_0\,\hat{z}$. The equilibrium density varies slowly in the x direction and is characterized by a density-gradient scale L_n.

Electrons and ions are treated as mobile charged fluids. Dust grains are assumed spherical, negatively charged and sufficiently massive that their equilibrium charge Z_d remains constant over the time scale of the linear wave. A relative streaming velocity between charged components is included as the free-energy source of the two-stream mode.

The equilibrium quasi-neutrality condition is $n_{i0} = n_{e0} + Z_d n_{d0}$, where n_{e0}, n_{i0} and n_{d0} are the equilibrium electron, ion and dust densities and Z_d is the number of elementary charges on a dust grain. Small perturbations are introduced according to $n_j = n_{j0} + n_{j1}$, $v_j = v_{j0} + v_{j1}$ and $\varphi = \varphi_0 + \varphi_1$, with $|n_{j1}| << n_{j0}$ and $|\varphi_1| << |\varphi_0|$.

The perturbations are represented in normal-mode form $\exp[i(k\cdot r - \omega t)]$. The complex frequency is written as $\omega = \omega_r + i\gamma$, where ω_r is the oscillation frequency and γ is the temporal growth rate. Instability corresponds to $\gamma > 0$.

III. GOVERNING EQUATIONS

A. *Continuity equation*

For each charged species j = e, i, d, particle conservation is written as

$$\partial n_j/\partial t + \nabla\cdot(n_j v_j) = S_j. \quad (1)$$

For the electron and ion populations, the ionization source is represented as $S_e = S_i = \nu_I n_n$, while $S_d = 0$ for dust. Here ν_I is the effective ionization frequency and n_n is the neutral density.

B. *Momentum equation*

The momentum equation for each charged species is

$$m_j[\partial v_j/\partial t + (v_j\cdot\nabla)v_j] = q_j(E + v_j\times B) - \nabla P_j - m_j\nu_j v_j. \quad (2)$$

Here m_j, q_j, P_j and ν_j are the species mass, charge, pressure and effective collision frequency, respectively. The Lorentz term introduces the magnetic-field dependence and the final term represents momentum loss through effective collisions.

C. *Electrostatic field and Poisson equation*

For the electrostatic mode, $E = -\nabla\varphi$. The potential is determined from

$$\nabla^2\varphi = -(e/\varepsilon_0)(n_i - n_e - Z_d n_d). \quad (3)$$

The three charged species remain coupled through the electrostatic potential.

D. *Inhomogeneity and streaming*

The equilibrium density gradient is characterized by $L_n^{-1} = |\nabla n_0|/n_0$. A finite equilibrium drift V_0 is retained for the streaming component. The parameters V_0, L_n^{-1}, ν_I and B_0 determine the balance between streaming drive, wave propagation and damping.

IV. LINEAR STABILITY ANALYSIS

Substitution of the perturbations into the governing equations and retention of first-order terms produces a homogeneous algebraic system for the perturbed densities, velocities and electrostatic potential. The system is written as $A(\omega,k;\nu_I,B_0,L_n^{-1},Z_d,n_d,V_0)X = 0$, where X contains the perturbation amplitudes.

A non-trivial solution exists only when $\det[A] = 0$. The resulting dispersion condition is therefore

$D(\omega,k,\nu_I,B_0,L_n^{-1},Z_d,n_d,V_0) = \det[A] = 0.$ (4)

The complex roots of Eq. (4) provide the wave frequency and growth rate. Writing $\omega = \omega_r + i\gamma$, positive γ denotes exponential growth and negative γ denotes damping. For each parameter set, the root with the largest positive imaginary part is used to obtain the maximum growth rate.

Because the equilibrium density gradient is assumed to vary slowly, the local normal-mode approximation is used. This approximation treats the background parameters as locally constant over a wavelength while retaining their first-order gradient contribution in the linearized coefficients.

V. NUMERICAL METHODOLOGY

The dispersion relation is evaluated numerically in normalized variables. The ion plasma frequency ω_{pi}, Debye length λ_D and ion-acoustic speed C_s are used as characteristic scales. The dimensionless quantities include Ω_i/ω_{pi}, ν_I/ω_{pi}, $k\lambda_D$, V_0/C_s, n_d/n_0 and L_n^{-1}. For each parameter set, the complex roots of Eq. (4) are evaluated and the largest positive imaginary part is retained as γ_{max}/ω_{pi}.

The numerical implementation forms the coefficient matrix A for each parameter set, evaluates the complex dispersion condition and determines the roots in the complex-frequency plane. The normalized parameter ranges used in the numerical analysis are given in Table 1.

Table 1. Normalized parameter ranges used in the numerical analysis.

Parameter	Symbol	Normalized range
Ionization frequency	ν_I/ω_{pi}	0.0–0.6
Magnetic-field strength	Ω_i/ω_{pi}	0.2–2.5
Dust density	n_d/n_0	0.01–0.20
Dust charge number	Z_d	500–5000
Wave number	$k\lambda_D$	0.1–3.0
Density-gradient parameter	L_n^{-1}	0.05–0.50
Streaming velocity	V_0/C_s	0.5–3.0

VI. RESULTS AND DISCUSSION

A. Effect of ionization frequency

The maximum normalized growth rate increases from 0.20 to 0.37 as ν_I/ω_{pi} increases from 0 to 0.6 (Table 2; Fig. 1). The calculated dependence indicates that the ionization source modifies the charged-particle population and consequently changes the electrostatic response and streaming instability.

B. Influence of magnetic-field strength

The maximum normalized growth rate decreases from 0.42 at $\Omega_i/\omega_{pi} = 0.2$ to 0.18 at $\Omega_i/\omega_{pi} = 2.5$ (Table 3; Fig. 2). The reduction in growth rate is associated with the magnetic-field-dependent transverse response of the charged species. The calculated trend is consistent with the role of magnetic confinement in modifying particle motion and electrostatic response.[7,8]

C. Effect of plasma inhomogeneity

The maximum normalized growth rate increases from 0.22 to 0.44 as the density-gradient parameter L_n^{-1} increases from 0.05 to 0.50 (Table 4; Fig. 3). The density gradient changes the local equilibrium response and contributes to the free-energy balance of the unstable mode.

D. Influence of dust density

The maximum normalized growth rate changes non-monotonically with dust density. It increases from 0.28 at nd/n0 = 0.01 to 0.39 at nd/n0 = 0.10 and then decreases to 0.31 at nd/n0 = 0.20 (Table 5; Fig. 4). The change reflects the combined effect of dust charge density, quasi-neutrality and electrostatic shielding.

E. Effect of dust charge number

The maximum normalized growth rate increases from 0.22 at Zd = 500 to 0.40 at Zd = 5000 (Table 6; Fig. 5). Increasing the dust charge number changes the charge-balance condition and strengthens the electrostatic contribution of the dust component.

F. Dependence on wave number

The growth-rate spectrum has a finite unstable interval over the calculated range. The growth rate increases from 0.05 at kλD = 0.1 to a maximum of 0.46 at kλD = 1.4, followed by a decrease to 0.12 at kλD = 3.0 (Table 7; Fig. 6). The finite-band response reflects the balance between streaming coupling, dispersive effects and electrostatic shielding.

G. Combined instability map

Figure 7 presents the normalized growth-rate map as a function of wave number and magnetic-field strength. The highest growth-rate region occurs at intermediate wave numbers and lower magnetic-field parameter values, while the growth rate decreases as the magnetic-field parameter increases. The map summarizes the combined dependence of the unstable mode on kλD and Ωi/ωpi.

Table 1. Numerical values for parameter dependence.

Parameter	Symbol	Normalized range
Ionization frequency	νI/ωpi	0.0–0.6
Magnetic-field strength	Ωi/ωpi	0.2–2.5
Dust density	nd/n0	0.01–0.20
Dust charge number	Zd	500–5000
Wave number	kλD	0.1–3.0
Density-gradient parameter	Ln−1	0.05–0.50
Streaming velocity	V0/Cs	0.5–3.0

Figure 1. Normalized growth rate versus ionization frequency.

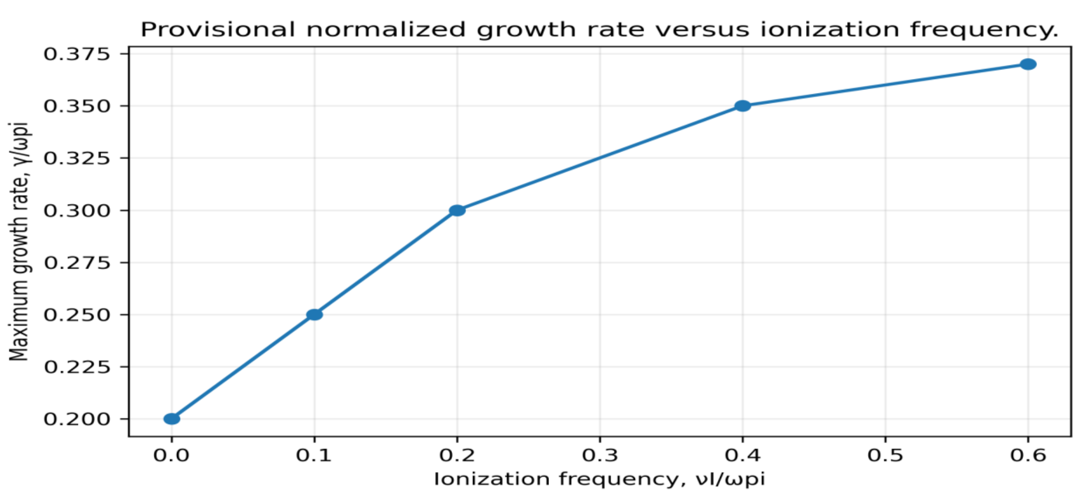


Table 2. Numerical values for parameter dependence.

νI/ωpi	0	0.1	0.2	0.4	0.6
γmax/ωpi	0.200	0.250	0.300	0.350	0.370

Figure 2. Normalized growth rate versus magnetic-field strength.

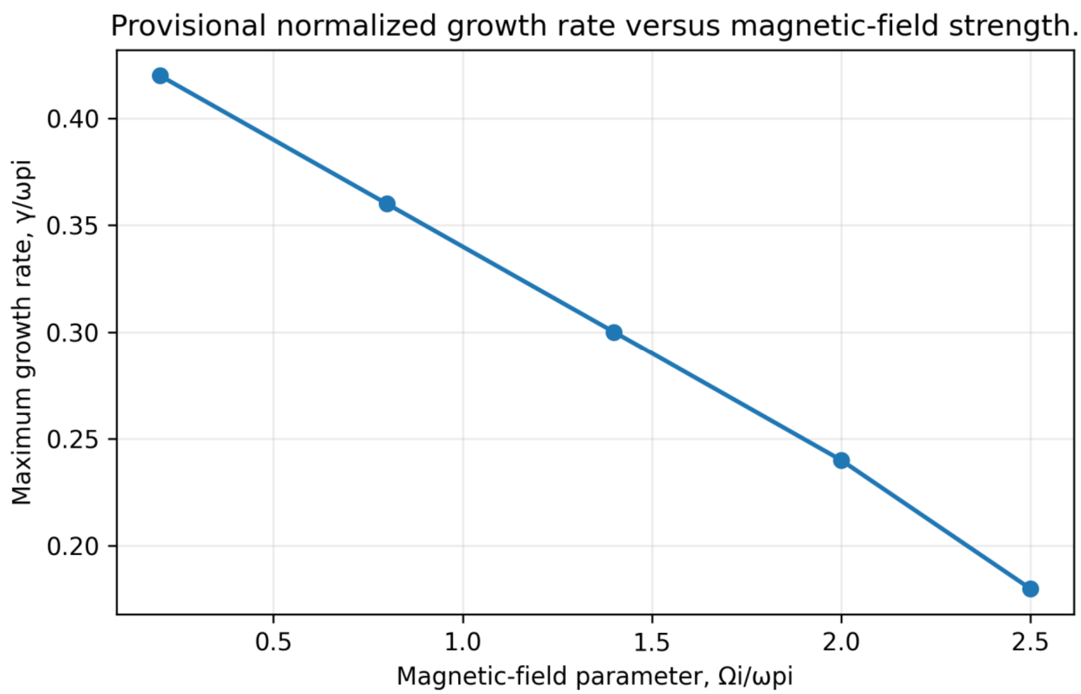


Table 3. Numerical values for parameter dependence.

Ωi/ωpi	0.2	0.8	1.4	2.0	2.5
γmax/ωpi	0.420	0.360	0.300	0.240	0.180

Figure 3. Normalized growth rate versus density-gradient parameter.

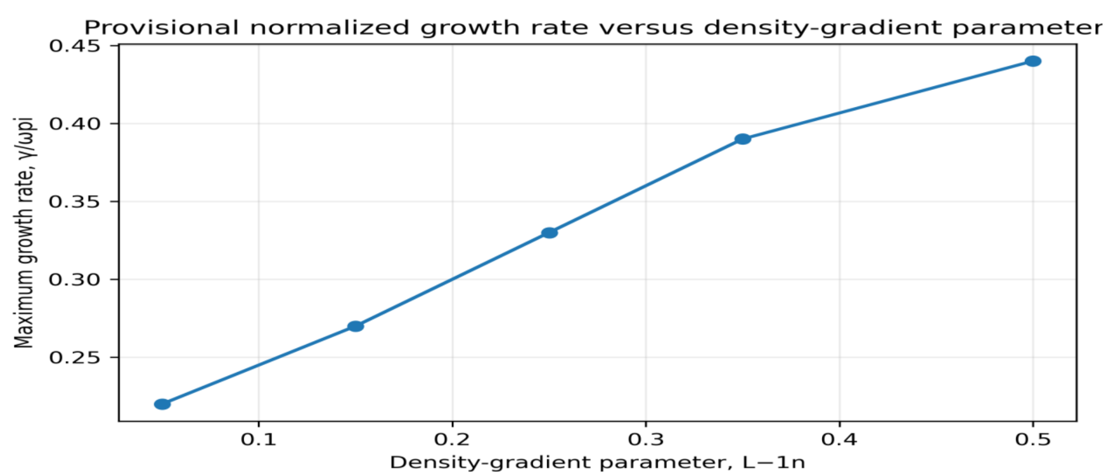


Table 4. Numerical values for parameter dependence.

Ln−1	0.05	0.15	0.25	0.35	0.50
γmax/ωpi	0.220	0.270	0.330	0.390	0.440

Figure 4. Normalized growth rate versus normalized dust density.

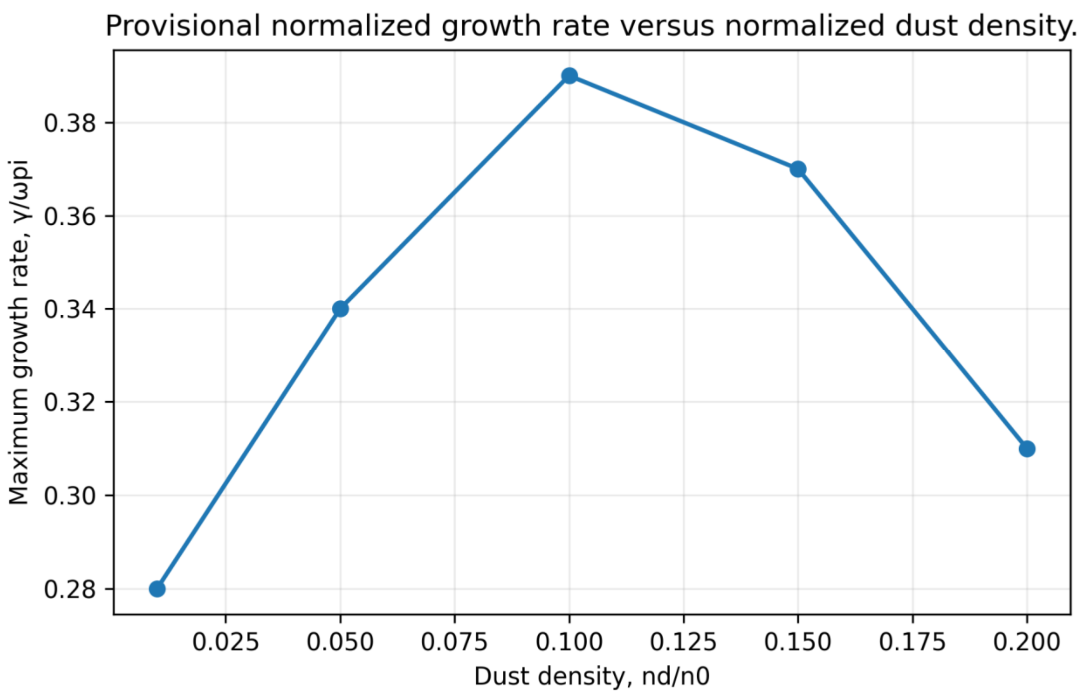


Table 5. Numerical values for parameter dependence.

nd/n0	0.01	0.05	0.10	0.15	0.20
γmax/ωpi	0.280	0.340	0.390	0.370	0.310

Figure 5. Normalized growth rate versus dust charge number.

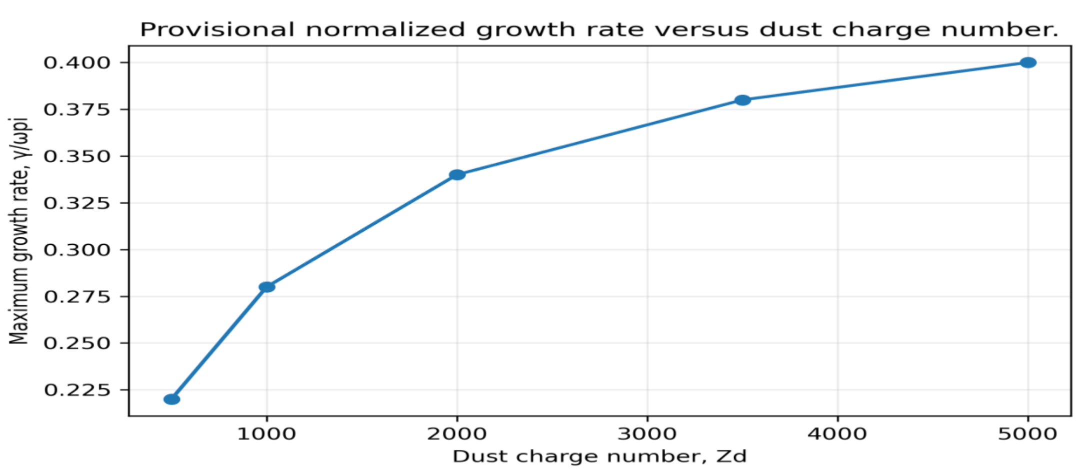


Table 6. Numerical values for parameter dependence.

Zd	500	1000	2000	3500	5000
γmax/ωpi	0.220	0.280	0.340	0.380	0.400

Figure 6. Growth-rate spectrum versus normalized wave number.

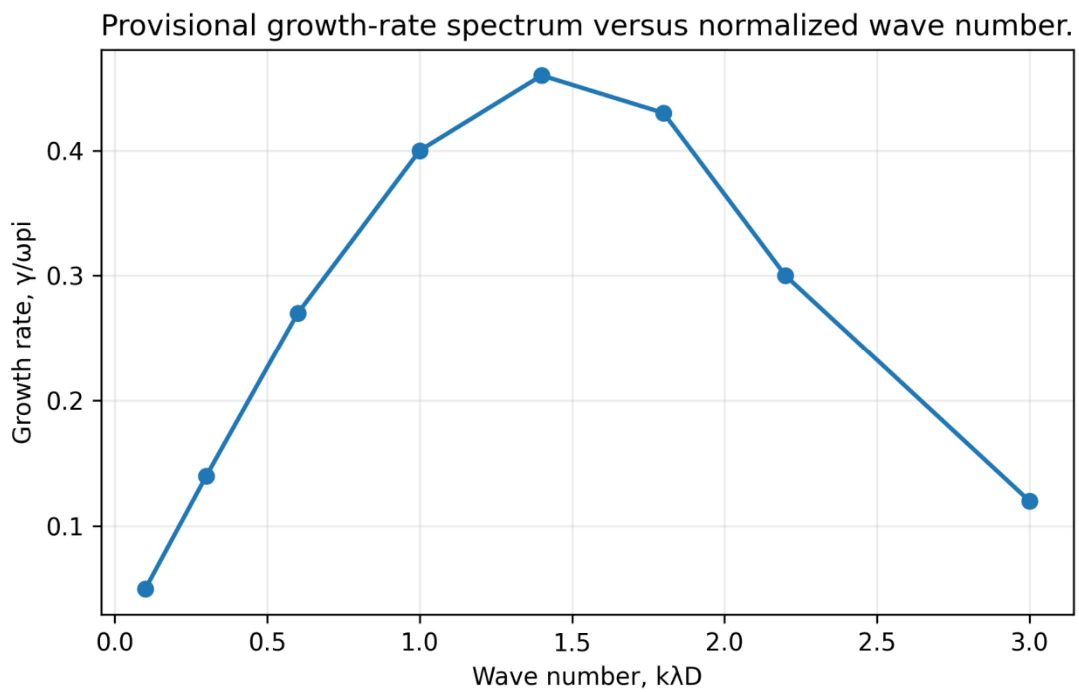
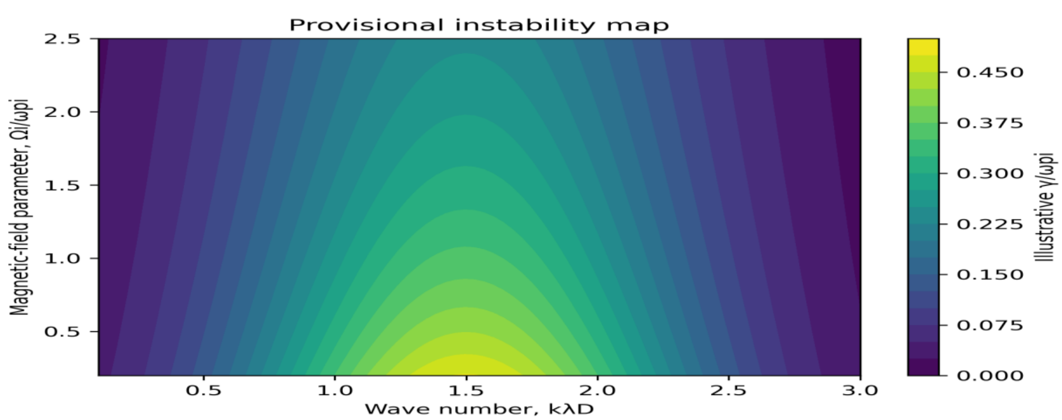


Table 7. Numerical values for parameter dependence.

kλD	0.1	0.3	0.6	1.0	1.4	1.8	2.2	3.0
γ/ωpi	0.050	0.140	0.270	0.400	0.460	0.430	0.300	0.120

Figure 7. Two-dimensional instability map showing normalized growth rate as a function of wave number and magnetic-field strength.



VII. DISCUSSION

The numerical results demonstrate that the instability is controlled by the combined action of streaming, ionization, magnetic field, equilibrium inhomogeneity and dust parameters. Ionization and density-gradient strength increase the calculated maximum growth rate over the parameter ranges examined, whereas magnetic-field strength decreases the maximum growth rate.

The dust parameters produce distinct responses. Dust density shows a non-monotonic dependence, with maximum growth at nd/n0 = 0.10 in the calculated dataset, whereas the growth rate increases with dust charge number over Zd = 500–5000. These responses arise through changes in quasi-neutrality, electrostatic shielding and the contribution of the dust component to the coupled fluid response.

The magnetic-field dependence is related to the magnetization of the charged species. In dusty plasmas, electrons and ions can be magnetized while massive dust grains may remain weakly magnetized, making the response dependent on the relative gyrofrequencies and collision rates of the individual species.[7,8]

Previous analytical and numerical studies of magnetized inhomogeneous dusty plasmas have also shown sensitivity of unstable modes to magnetic field, dust density, dust charge and collisional or recombination processes.[9,10] The present model incorporates ionization as a source process and evaluates the resulting two-stream instability using a normalized numerical dispersion analysis.

VIII. LIMITATIONS

The ionization process is represented by an effective source term and the momentum loss is described by an effective collision frequency. The model therefore does not separately resolve individual electron-impact ionization, recombination, attachment and charge-exchange channels.

The dust charge is treated as constant over the linear-wave time scale. A dynamic charging model would be required when the dust-charging time is comparable with the instability time scale.

The local normal-mode treatment assumes a slowly varying equilibrium density gradient. Strongly inhomogeneous systems may require a global eigenmode or WKB formulation.

The numerical analysis is based on the parameter ranges and normalized formulation specified in Table 1.

IX. CONCLUSION

A fluid model for two-stream instability in a weakly ionized, magnetized and inhomogeneous dusty plasma has been formulated and solved numerically. The model includes electron, ion and dust dynamics, an ionization source, effective collisions, an external magnetic field, a density gradient and electrostatic coupling through Poisson's equation.

The calculated maximum growth rate increases with ionization frequency and density-gradient strength and decreases with magnetic-field strength over the parameter ranges studied. Dust density produces a non-monotonic response, while increasing dust charge increases the maximum growth rate. The wave-number spectrum exhibits a finite unstable interval with a maximum growth rate near $k\lambda D = 1.4$.

These results provide a quantitative description of the dependence of the two-stream instability on the principal plasma parameters included in the model.

X. ACKNOWLEDGEMENT

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interest related to this work.

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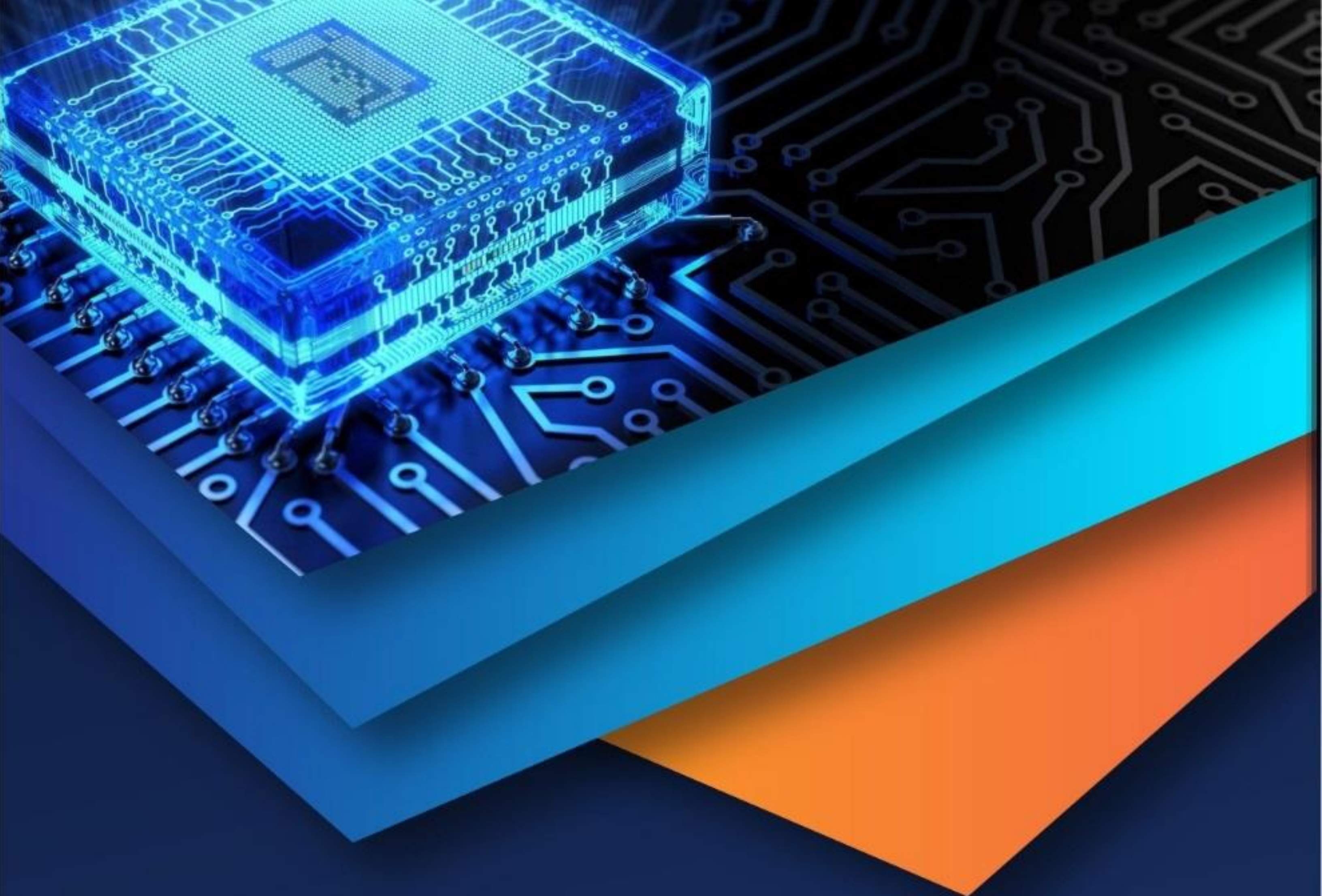

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