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Nurbs Based Isogeometric Analysis to Perform Topology Optimisation of Continuum Structures Using Evolutionary Algorithms

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Abstract: This research investigates the application of Isogeometric Analysis (IGA) for topology optimisation of continuum structures under various loading and boundary conditions. IGA, which utilizes Non-Uniform Rational B-Splines (NURBS) for both geometric representation and solution approximation, offers a seamless integration of CAD and analysis phases. This significantly enhances the accuracy and efficiency of simulations compared to traditional Finite Element Analysis (FEA), where mesh generation remains a time-consuming and error-prone step.

The study focuses on minimizing structural weight while satisfying constraints on stress and displacement using the Firefly Algorithm for optimisation. Optimisation was carried out for three benchmark problems: a 2D cantilever plate, a 2D simply supported beam, and a 3D simply supported cube. Results were evaluated in terms of volume reduction and design efficiency, and validated through comparison with prior studies and commercial FEA software (MIDAS NFX®). The IGA-based approach demonstrated higher material savings and smoother convergence trends, confirming its potential in structural optimisation. Notably, the 3D cube retained only 11.5% of the initial volume, outperforming previous FEA results. This work reinforces the capability of IGA for advanced structural design and highlights its suitability for future integration into commercial analysis tools.

Keywords: Isogeometric Analysis (IGA), Topology Optimisation, NURBS, Firefly Algorithm, Structural Design, Finite Element Analysis (FEA), Continuum Structures, Material Efficiency, CAD Integration, Optimisation Algorithm.

I. INTRODUCTION

The primary purpose of a structure is to carry loads and transfer them safely to its supports. In doing so, the structure develops internal stresses that must be managed through proper design. Structural design ensures that the structure remains safe, durable, and functional under expected loads. Today, the demand for optimal engineering solutions is growing due to commercial needs and sustainability goals like reducing carbon footprint.

Modern designs aim to be lightweight, cost-effective, and high-performing. Traditional structural design evolved through trial and error, often by upgrading existing designs. However, current industrial competition demands faster, more efficient, and higher-quality design solutions. Structural optimisation has become a key method in achieving this.Structural optimisation involves improving the size, shape, or topology of a structure within design constraints. The goal is to maximize or minimize an objective function, such as weight or stress, while satisfying boundary conditions and performance requirements. Design variables may be continuous or discrete, including parameters like material density and dimensions.

Topology optimisation is the most general and advanced form of structural optimisation. It aims to determine the most efficient material layout and connectivity within a design space. Two main techniques exist: element-based and nodal-based approaches. Element-based methods use rectangular finite elements with constant variables, but struggle with geometry accuracy and stress constraints. In contrast, nodal-based approaches describe geometry more accurately and adapt better to changes in structure, loading, and boundary conditions. These are more suitable for meshless methods, such as Isogeometric Analysis (IGA). IGA eliminates the need for mesh generation by using the same basis functions for geometry and analysis.

Previously, CAD models used NURBS to define geometry, which then had to be approximated for Finite Element Analysis (FEA). This process consumed up to 80% of analysis time, mainly in mesh generation and geometry preparation as shown in fig.1. The inefficiency created a need to streamline the process by unifying geometry and analysis.

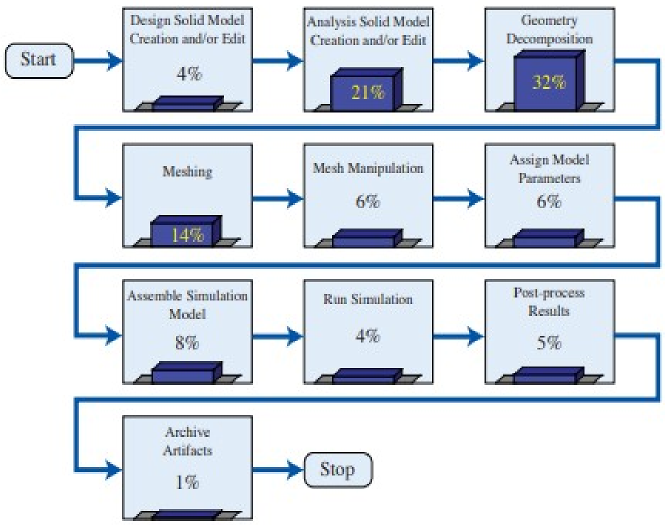

Fig 1 Example flowchart of engineering design which shows analysis becoming a time-consuming process

IGA, introduced by Hughes et al. in 2005, uses NURBS basis functions to represent both the CAD geometry and the analysis model. This integration reduces modeling time, improves accuracy, and avoids remeshing. IGA is especially effective for complex structures and optimisation problems requiring smooth and accurate geometry.

This study applies IGA for topology optimisation under specific material, loading, and boundary conditions. The main objectives are to optimize a plate structure under in-plane loading, and to optimize a simply supported 3D cube structure.

II. ISOGEOMETRIC ANALYSIS

Isogeometric Analysis (IGA) is a numerical technique first proposed by Hughes et al. in 2005. It was developed to solve partial differential equations more efficiently and accurately. IGA stands out from traditional methods because it uses the same mathematical functions to represent both geometry and analysis. These functions are called Non-Uniform Rational B-Splines (NURBS). NURBS offer smooth and precise surface representation, making them ideal for structural optimisation problems.

In conventional methods like Finite Element Analysis (FEA), geometry from CAD software must be approximated and remeshed, which is time-consuming and can reduce accuracy. IGA eliminates this issue by directly using the CAD geometry for analysis. This integration reduces modeling effort and improves the fidelity of the analysis results. The term "Isogeometric" reflects the use of identical basis functions for both geometry definition and structural computation. "Iso" means "same," and "geometric" refers to the geometry representation. As a result, there is no need for separate mesh generation or conversion when transitioning from design to analysis. This makes IGA a powerful and efficient tool for solving complex engineering problems, as illustrated in Fig. 2.

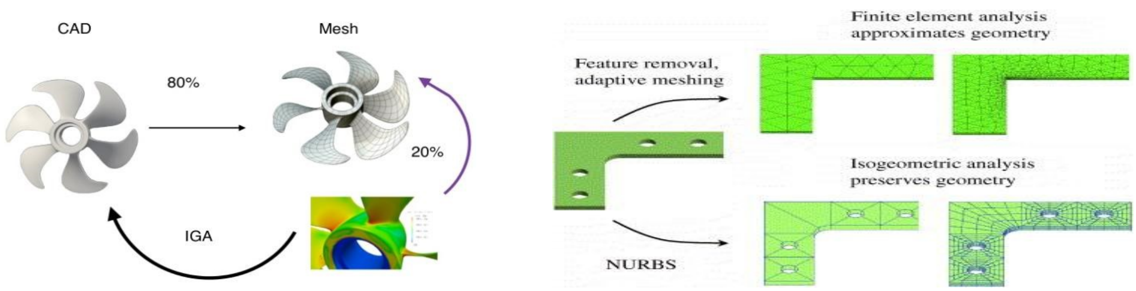

Fig.2 Isogeometric analysis

III. LITERATURE REVIEW

Several researchers have significantly contributed to the development of topology optimisation techniques, particularly for three-dimensional structures. Olhoff, Ronholt, and Scheel presented a notable study on 3D topology optimisation using optimum microstructures [63]. Prior research largely focused on 2D structures, with Bendsoe and Kikuchi's 1988 work being a key foundation. Their formulations were built on earlier analytical results by Gibianski and Cherkaev in 1987. One major challenge in structural optimisation is the non-existence or non-uniqueness of solutions, often due to poorly defined design spaces.

To overcome this, the introduction of periodic or perforated microstructures as admissible materials was proposed. This approach led to more feasible designs, such as a cubic domain with four concentrated loads and simply supported corners, which resulted in a quadropod-like optimal structure. It was also observed that optimal designs in 3D may be non-symmetric under certain load conditions. Importantly, using microstructures can help avoid local optima, making sensitivity-based optimisation procedures more effective.

Further development was contributed by Bendsoe and Sigmund, who studied various interpolation schemes for topology optimisation [58]. These include the Solid Isotropic Material with Penalization (SIMP), Voigt bound, and Hashin-Shtrikman bound methods. Their differences were highlighted using design examples, which showed material distribution results under different penalization powers. Most methods, except the Voigt bound, resulted in clear black-and-white material layouts. The paper also explored homogenization approaches for orthotropic and anisotropic composite materials, addressing two- and three-material designs and non-linear problems.

Additionally, Cazacu and Grama reviewed five major topology optimisation methods: SIMP, Evolutionary Structural Optimisation (ESO), Soft-Kill Option (SKO), Level-Set, and Evolutionary Algorithms (EA) [71]. These methods were analyzed for their capabilities, advantages, and limitations in achieving design goals such as stress minimization, weight reduction, and stiffness maximization. Among these, SIMP remains the most widely used approach.

Recent improvements in EA techniques include smoother geometry representations using NURBS and spline curves, allowing direct optimisation on CAD models. This overcomes earlier limitations of discrete representations. Our project emphasizes Evolutionary Algorithms due to their suitability for large, complex, and multi-modal problems. These algorithms work by generating an initial population, evaluating fitness, and applying selection, recombination, and mutation to evolve better solutions. The process iterates until a defined termination criterion, such as the maximum number of generations, is reached [70].

IV. METHODOLOGY

A. Problem Statement

The aim of the project is to find an optimal linearly elastic structure with the help of Isogeometric analysis which gives more accurate solutions compared to finite element analysis. Optimal solution refers to a design which is light but strong and rigid at the same time i.e. mathematically our goal is to minimize the weight of the structure that is indirectly maximizing the total forces that are acting on the structure.

An objective function describes the main aim of the model whether to minimize or maximize the structure so here our objective function is to minimize the weight of the structure subject to constraints

Problem statement is only to perform optimisation for plate problems Minimize, the weight of the structure subjected to

σ_{max} *principal* $\leq \sigma_{allowable}$

$^{u}max_{nodal} \leq {}^{u}allowable$

$$\left(\sum_{i=1}^{n} \rho_i v_i\right) * \bar{\rho} > 0$$

where, ρ_i is the density parameter of the element, v_i is the volume of the element, and $\bar{\rho}$ is the density of the material subjected to $V_a * v - \left(\sum_{i=1}^{n} \rho_i v_i\right) \geq 0$

where, V_a is the allowable material in percentage, and v is the volume of the structure

$0 \leq \rho_{min} \leq \rho_e \leq 1$

Where σ is stress constraint, u is displacement constraint, v is volume constraint and ρ is relative density with given state variables Young modulus 'E' and Poisson's ratio '$\boldsymbol{v}$' along with Neumann and Dirichlet boundary conditions.

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B. Objectives

1. The objective is to find the minimum weight of the structure subjected to the constraint on the amount of material used and the elemental stresses and nodal displacements in the structure.
2. To perform topology optimisation of a plate structure carrying in-plane loading.
3. To perform topology optimisation for a three-dimensional structure i.e. simply supported cube.

The methodology followed in this study is as follows:

The process begins with defining the problem, including geometry, material properties, loads, and boundary conditions. After formulation and assumptions are established, boundary smoothing is done in AutoCAD to refine the topology. The design is then modeled in SolidWorks® for stress and displacement analysis. If results are acceptable, shape optimisation is performed using Design Works.

This study focuses on IGA-based topology optimisation for:

- A 2D cantilever plate with a point load.
- A 2D simply supported plate with a central load.
- A 3D simply supported cube.

Each case is first mathematically formulated. NURBS basis functions define both geometry and displacement fields, illustrating IGA's integration of modeling and analysis within a single framework.

As shown in Fig.3(b), each problem is first mathematically formulated and then analyzed using the Isogeometric Analysis (IGA) approach. NURBS basis functions define both the domain geometry and are used to compute displacements and coordinates. The degree and knot vector for the NURBS are defined initially, followed by the derivation of governing equations.

A solution process Fig.3(c) is implemented in C++ to solve these equations. The optimisation flowchart (connector C) outlines the process, starting with initial values for design variables. Parameters such as minimum elemental stress and relative density are predefined. Material properties (Young's modulus, Poisson's ratio) are input, and initial relative densities are assigned.

Optimisation uses the Firefly Algorithm, where each fly represents a design solution. The objective functionstructural weightis evaluated, and flies move toward better solutions using Lévy flights. Updated relative densities are then analyzed via finite element analysis (FEA), checking for stress and displacement compliance.

This cycle constitutes one iteration and is repeated until convergence, based on the set number of iterations. A convergence graph plots objective function versus iteration count.

The IGA-based topology optimisation is executed via a C++ program on a notebook with an Intel i7 (4-core, 3.4 GHz) and 4 GB RAM.

C. Flowcharts

The flowchart for Integrated Topology optimisation is as shown below.

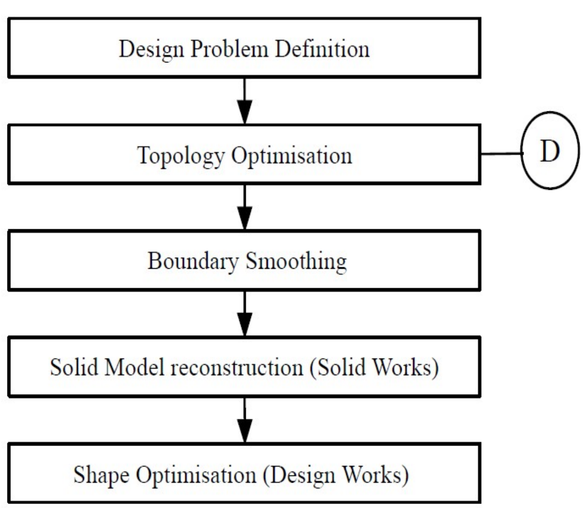

Fig.3(a) Integrated Optimisation Process

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The flow chart showing the approach for this study is as shown below.

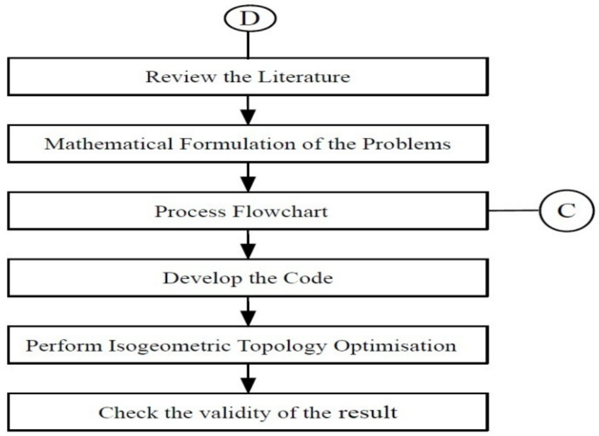

Fig.3(b) Approach flowchart

The process flow chart is as shown below.

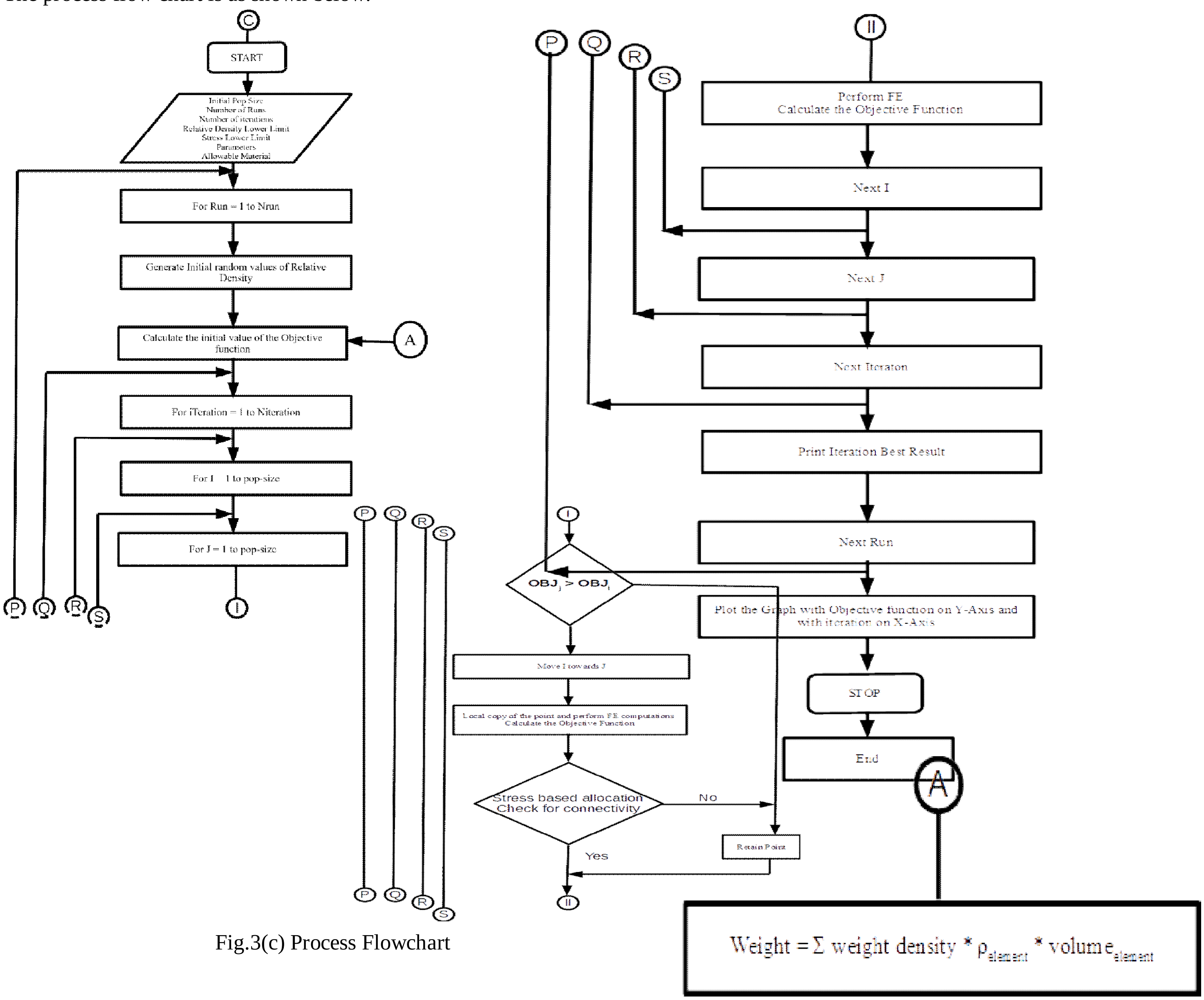

Fig.3(c) Process Flowchart

V. RESULT

This section presents the results of isogeometric topology optimisation applied to two-dimensional and three-dimensional continuum structures. The goal of the study was to minimize structural volume while ensuring structural performance under applied loads. Simulations were conducted using linear NURBS basis functions and results were compared with existing literature and commercial finite element software (MIDAS NFX®).

A. Two-Dimensional Structures

1) Cantilever Plate with Midpoint Load

A cantilever plate with dimensions 9600 mm × 3300 mm was analyzed under a concentrated load of 5028 N applied at the mid-point of the right edge. Due to symmetry, only one half of the structure was modeled. The domain was discretized using 352 four-node quadrilateral plane stress elements, and 396 control points were used. The material had a Young's modulus of 2×10^5 N/mm², a Poisson's ratio of 0.3, and a thickness of one unit, with a weight density of 7800 kg/m³. Linear NURBS basis functions were used for the analysis.

The optimal material distribution obtained using the IGA method is shown in Fig.4(b). A flipped vertical version of the image is presented in Fig.4(a) for comparison with results obtained by Hasasni et al. (2012), who used 1617 control points, shown in Fig.4(b). The convergence behavior is captured in Fig.5, which plots the weight of the structure against iteration number, showing a smooth convergence trend.

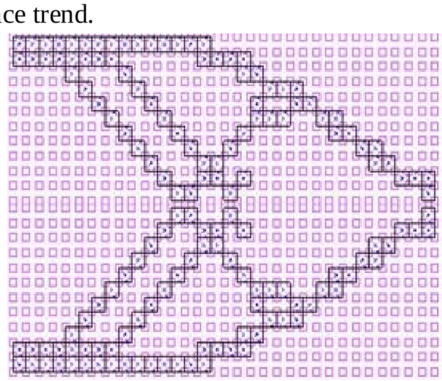

Fig. 4(a) showing the flip vertical image

Fig. 4(b) Optimal layout using 1617 control points

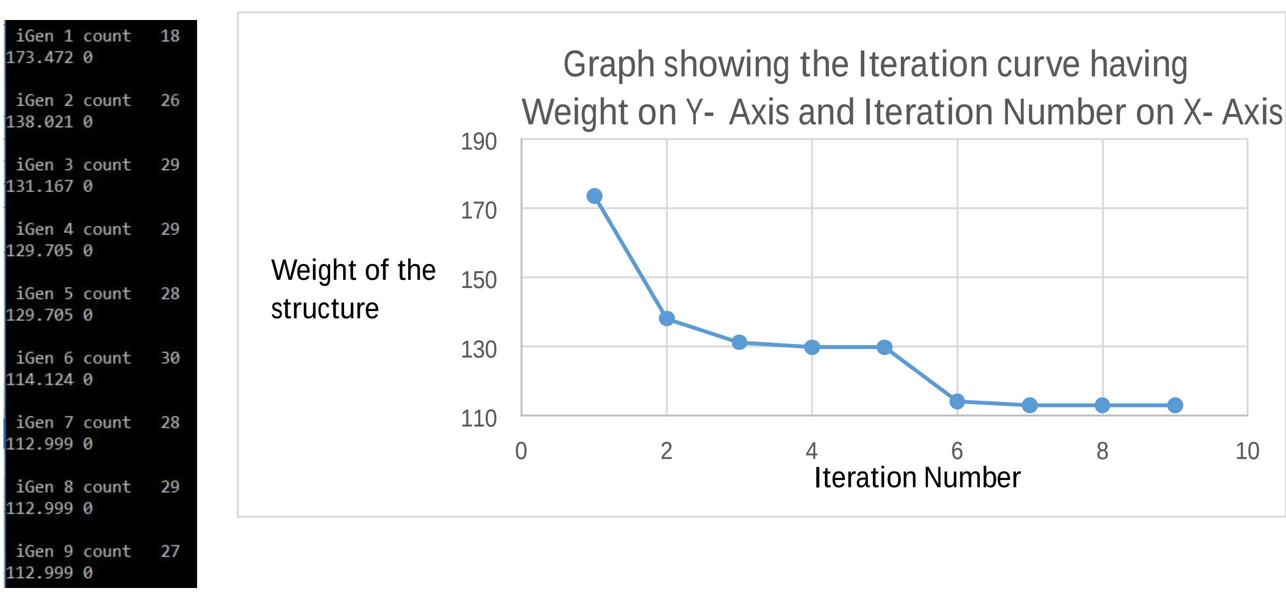

Fig.5 Showing the Iteration history and the Iteration curve with weight on Y-Axis and the Iteration Number on X-Axis

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The structure was also analyzed using MIDAS NFX®, a commercial finite element analysis package. The resulting optimal material distribution, shown in Fig.6 and its flipped version in Fig.7, was found to closely match the IGA results obtained through the C++ program.

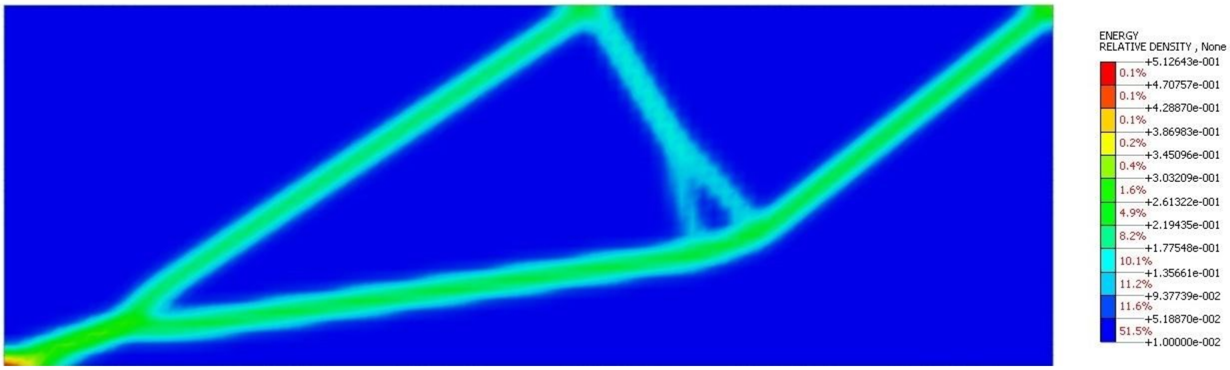


Fig 6 Showing the optimal distribution of material using MIDAS NFX - a Standard Finite

Element -Analysis Package and first order four node quadrilateral element

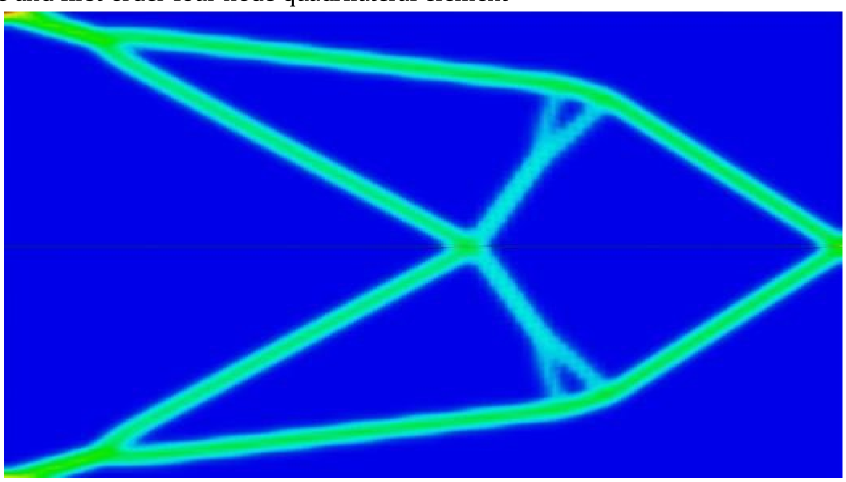
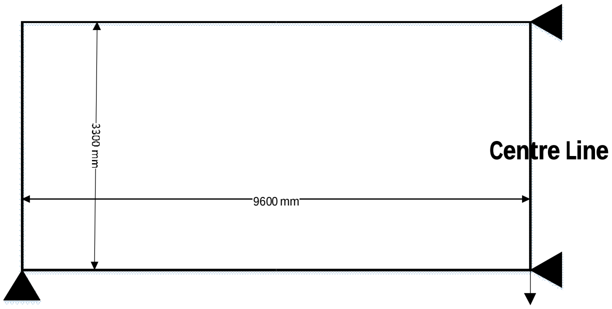


Fig.7 Showing the Flip Vertical Image of the distribution of material in the design domain using a Standard Finite Element Analysis Package MIDAS NFX and using first order four node quadrilateral elements.

2) Simply Supported Beam (Michelle Structure)

A simply supported beam with dimensions 9600 mm × 3300 mm, carrying a point load of 10056 N at the center of its bottom edge, was also analyzed. Due to symmetry, only half of the structure was considered. The mesh and control points used were the same as in the previous example (352 elements and 396 control points). The material properties were the same, and linear basis functions were used. The optimal material distribution, obtained using the Firefly algorithm, is shown in Fig.8(b). Fig.9(a) shows the horizontally flipped version of the layout, and Fig.9(b) shows the corresponding optimal result by Hartman (2012) using 1617 control points. A similar output was presented by Abholbashari (2017) using FEA, shown in Fig.10.



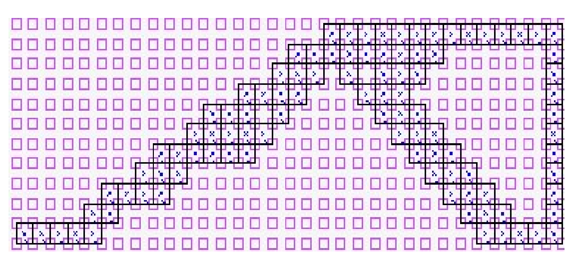


Fig.8(a) showing the design domain

8(b) Optimal design using C++ IGA (98/352)

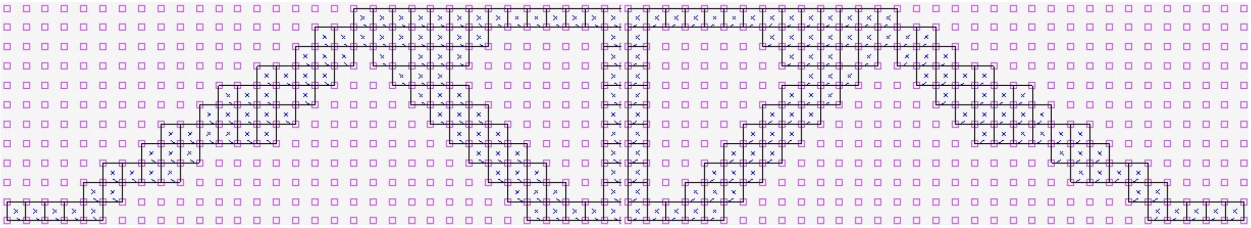


Fig.9(a) Showing the Flip Horizontal Image of the distribution for the entire beam which is symmetrical about the center

Fig.9(b) Optimal distribution, 1617 ctrl pts [26]

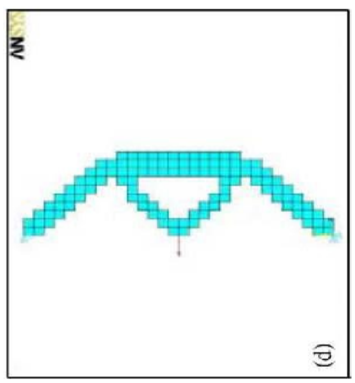


Fig.10 A similar output by [55] 1250 Q4 FEA - 10m x 5m x 0.1m

	Using FFA (2017)	Hassani et. al. (2012)
Minimum Volume (V/V_0) %	27.84 %	20%
Number of Elements	352	1600

Table 1 showing the comparison of the Volume fraction

A comparison of volume fractions is presented in Table 1. The Firefly algorithm achieved a volume fraction of 27.84%, whereas Hassani et al. (2012) obtained 20% using 1600 elements. The structure was also analyzed using MIDAS NFX®, and the distribution results, shown in Figs.11 and 12, confirmed a close match with the C++ IGA results and the FEA results by Abholbashari.

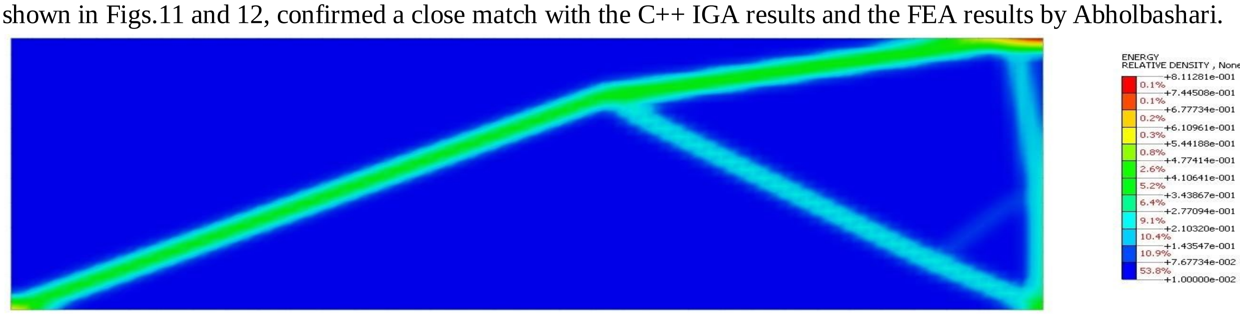


Fig.11 The optimal distribution of material in the design domain using a Standard Finite Element Analysis Package MIDAS NFX® using first order four node quadrilateral elements.

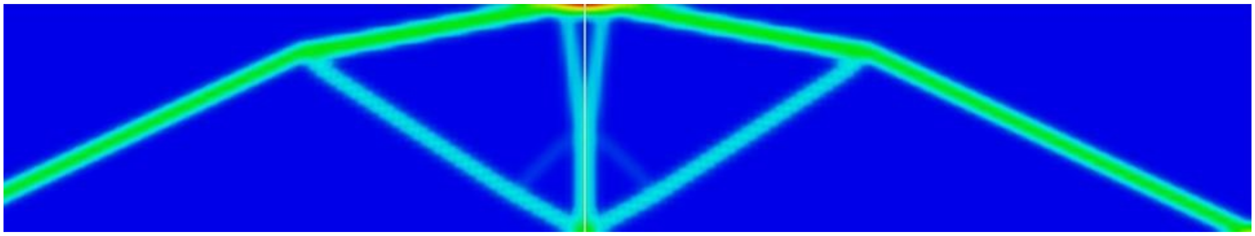


Fig.12 Showing the flip horizontal image of the distribution of material within the design domain of the Michelle structure using a Standard Finite Element Analysis Package MIDAS NFX® using first order four node quadrilateral elements.

B. Three-Dimensional Structure

1) Simply Supported Cube

A three-dimensional cube domain with 5m side length was analyzed. The structure was simply supported at the four bottom corners and carried a vertical upward point load of 1000 N at the center of the top face. The domain was discretized using 1000 first-order eight-node hexahedral elements and 1331 control points. The material had a Young's modulus of 1×10^5 MPa and a Poisson's ratio of 0.3. A knot vector of length 15 was used along each direction with NURBS basis functions of degree 1.

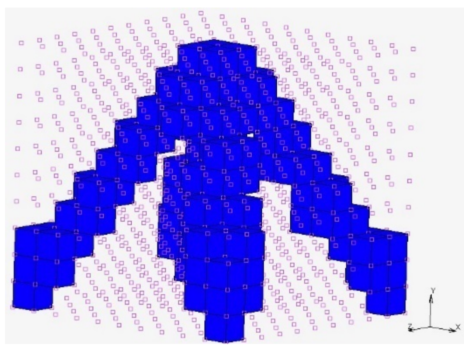


Fig.13 Optimized structure of a three-dimensional cube

The number of iterations taken for convergence was 196, and the optimized material layout retained only 115 elements out of 1000, corresponding to 11.5% of the initial volume. The optimized layout is shown in Fig.13. For comparison, a similar problem was solved by Abolbashari (2006) using FEA with the same number of elements, achieving a material saving of 82.4%. In contrast, the IGA method used in this study achieved 88.5% material saving, as summarized in Table 2.

	Abolbashari FEA(2006) [55]	This study using IGA
%Volume reduction	82.4	88.5
Final volume (% of tot. vol.)	17.6	11.5

Table 2 Comparison of percentage of volume

VI. CONCLUSION

This study applied the Firefly Algorithm, inspired by firefly behavior and introduced by X.S. Yang (2009), for topology optimisation of continuum structures under in-plane loading. Two 2D and one 3D problems were optimized.

The 2D cases included a cantilever beam and a simply supported (Michell) beam. Both were optimized using 396 control points, significantly fewer than the 1617 used by Hassani et al., yet yielding similar material distributions. The cantilever results were also verified using MIDAS NFX®. The Michell beam, optimized for symmetry, achieved a final volume of 27.84% compared to Hassani's 20%, likely due to mesh differences.

The 3D problem was solved using linear B-splines in all directions and achieved a volume of 11.5%, outperforming the 17.6% volume from Abolbashari's 2006 study using traditional FEA.

These results confirm that Isogeometric Analysis (IGA) is a highly efficient and accurate approach for structural mechanics, as it uses the same NURBS basis for geometry and displacement fields. IGA's ability to precisely represent geometry enhances solution accuracy over standard FEA. Given its advantages, IGA is expected to be integrated into commercial FEA tools, with LS-DYNA® already exploring its implementation.

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