iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 12 Issue: III Month of publication: March 2024

DOI: https://doi.org/10.22214/ijraset.2024.58887

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 12 Issue III Mar 2024- Available at www.ijraset.com

Observations on Ternary Quadratic Diophantine Equation $8(x^2+y^2)-15xy+7x+7y+49=40z^2$

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Abstract:** The Ternary Quadratic Diophantine Equation is given by* $8(x^2+y^2)-15xy+7x+7y+49=40z^2$. ***Number of non-zero distinct integer solutions. Some interesting relations among the solutions and the way of factorization, along with the illusion of non-zero distinct integer solutions to the above equations are obtained.
***Keywords:** Diophantine equation, Integral solutions, Quadratic equation with three unknowns, Ternary equation.*

I. INTRODUCTION

Diophantine equation have provoked the interest of different mathematicians. It is simple to solve Diophantine equations with a degree greater than three by reducing them to equations degree 2 or 3. In [1-3], hypothesis of numbers is talked about. Diophantine quadratic equations are discussed in [5,6, and 12]. In [4,7-11] cubic, biquadratic and higher order equations are considered for its integer solutions

This correspondence deals with one more attracting ternary quadratic equation addressing a non-homogeneous cone for its infinitely non-zero integer points. In like, manner, a couple of fascinating relations among the arrangements are correlated.

II. NOTATIONS

$T_{8,n}$ =Octagonal number if rank ‘n’.

$T_{10,n}$ = Decagonal number of rank ‘n’.

$T_{18,n}$ = Octadecagonal number of rank ‘n’.

$T_{22,n}$ = Icosidigonal number of rank ‘n’.

$T_{26,n}$ = Icosihexagonal number of rank ‘n’.

$T_{28,n}$ = Icosioctagonal number of rank ‘n’

Gno_n = Gnomonic number of rank ‘n’.

III. METHOD OF ANALYSIS

The ternary quadratic Diophantine equation for its non-zero integral solution is given by

$8(x^2+y^2)-15xy+7x+7y+49=40z^2$ (1)

The replacement of linear transformations $x=u+v$ and $y=u-v$ (2)

in (1) we get,

$(u+7)^2+31v^2=40z^2$ (3)

Below are four examples of distinct non-zero integer solutions to (1).

PATTERN: 1

Assume $z=z(a,b)=a^2+31b^2$ (4)

wherein (a, b) is a pair of non-zero integers.

$$40 = (3 + i\sqrt{31})(3 - i\sqrt{31}) \qquad (5)$$

Using (4)and (5) in (3), by the method of factorization,

$$\left((u+7) + i\sqrt{31}v\right)\left((u+7) - i\sqrt{31}v\right) = (3 + i\sqrt{31})(3 - i\sqrt{31})\left[\left(a + i\sqrt{31}b\right)^2 \left(a - i\sqrt{31}b\right)^2\right] \quad (6)$$

When we compare real and imaginary components and put like terms together, we get

$$u = u(a,b) = 3a^2 - 93b^2 - 62ab - 7$$
$$v = v(a,b) = a^2 - 31b^2 + 6ab$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x = x(a,b) = 4a^2 - 124b^2 - 56ab - 7$$
$$y = y(a,b) = 2a^2 - 62b^2 - 68ab - 7$$
$$z = z(a,b) = a^2 + 31b^2$$

OBSERVATIONS:

1. $x(a,a) - y(a,a) + z(a,a) + 4T_{10,a} \equiv 0 \pmod{12}$
2. $y(1,1) - x(1,1) - z(1,1)$ is a perfect square.
3. $x(a,1) - y(a,1) + z(a,1) - T_{8,a} - 7Gno_a \equiv 0 \pmod{24}$
4. $x(a,a) - y(a,a) + 4T_{26,a} \equiv 0 \pmod{44}$
5. $x(a,a) - y(a,a) - z(a,a\cdot) + 10T_{18,a} \equiv 0 \pmod{70}$

PATTERN:2

Consider (5),we write 40 as

$$40 = \frac{(19 + i3\sqrt{31})(19 - i3\sqrt{31})}{16} \qquad (7)$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x = x(A,B) = 88A^2 - 2728B^2 - 592AB - 7$$
$$y = y(A,B) = 64A^2 - 1984B^2 - 896AB - 7$$
$$z = z(A,B) = 16A^2 + 496B^2$$

OBSERVATIONS:

1. $x(1,1) - y(1,1) + z(1,1)$ is a nasty number
2. $y(A,A) - x(A,A) - 32T_{28,A} \equiv 0 \pmod{384}$

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3. $x(A,A) - y(A,A) + z(A,A) - 8T_{26,A} \equiv 0 \pmod{88}$
4. $x(A,A) - y(A,A) - z(A,A) + 116T_{18,A} \equiv 0 \pmod{812}$
5. $x(A,1) - y(A,1) - 2T_{26,A} - 163Gno_A \equiv 0 \pmod{581}$

PATTERN: 3

Consider(5),we write 40 as,

$$40 = \frac{(7 + i9\sqrt{31})(7 - i9\sqrt{31})}{64} \quad (8)$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2)

$$x = x(A,B) = 128A^2 - 3968B^2 - 4352AB - 7$$
$$y = y(A,B) = -16A^2 + 496B^2 - 4576AB - 7$$
$$z = z(A,B) = 64A^2 + 1984B^2$$

OBSERVATIONS:

1. $y(1,1) - x(1,1)$ is a perfect square.
2. $x(A,A) - y(A,A) + z(A,A) + 256T_{18,A} \equiv 0 \pmod{1792}$
3. $x(A,1) - y(A,1) - 12T_{26,A} - 178Gno_A \equiv 0 \pmod{4286}$
4. $x(A,A) - y(A,A) + 1024T_{10,A} \equiv 0 \pmod{3072}$
5. $x(A,A) - y(A,A) - z(A,A) + 512T_{26,A} \equiv 0 \pmod{5632}$

PATTERN: 4

Consider(5),we write 40 as

$$40 = \frac{(63 + i\sqrt{31})(63 - i\sqrt{31})}{100} \quad (9)$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x = x(A,B) = 640A^2 - 19840B^2 + 640AB - 7$$
$$y = y(A,B) = 620A^2 - 19220B^2 - 1880AB - 7$$
$$z = z(A,B) = 100A^2 + 3100B^2$$

OBSERVATIONS:

1. $x(A,A) - y(A,A) + z(A,A) - 640T_{18,A} \equiv 0 \pmod{4480}$
2. $x(A,A) - y(A,A) - 160T_{26,A} \equiv 0 \pmod{1760}$
3. $x(A,1) - y(A,1) - z(A,1) + 10T_{18,A} - 1225Gno_A \equiv 0 \pmod{2495}$
4. $x(A,A) - y(A,A) - z(A,A) + 128T_{22,A} \equiv 0 \pmod{1152}$

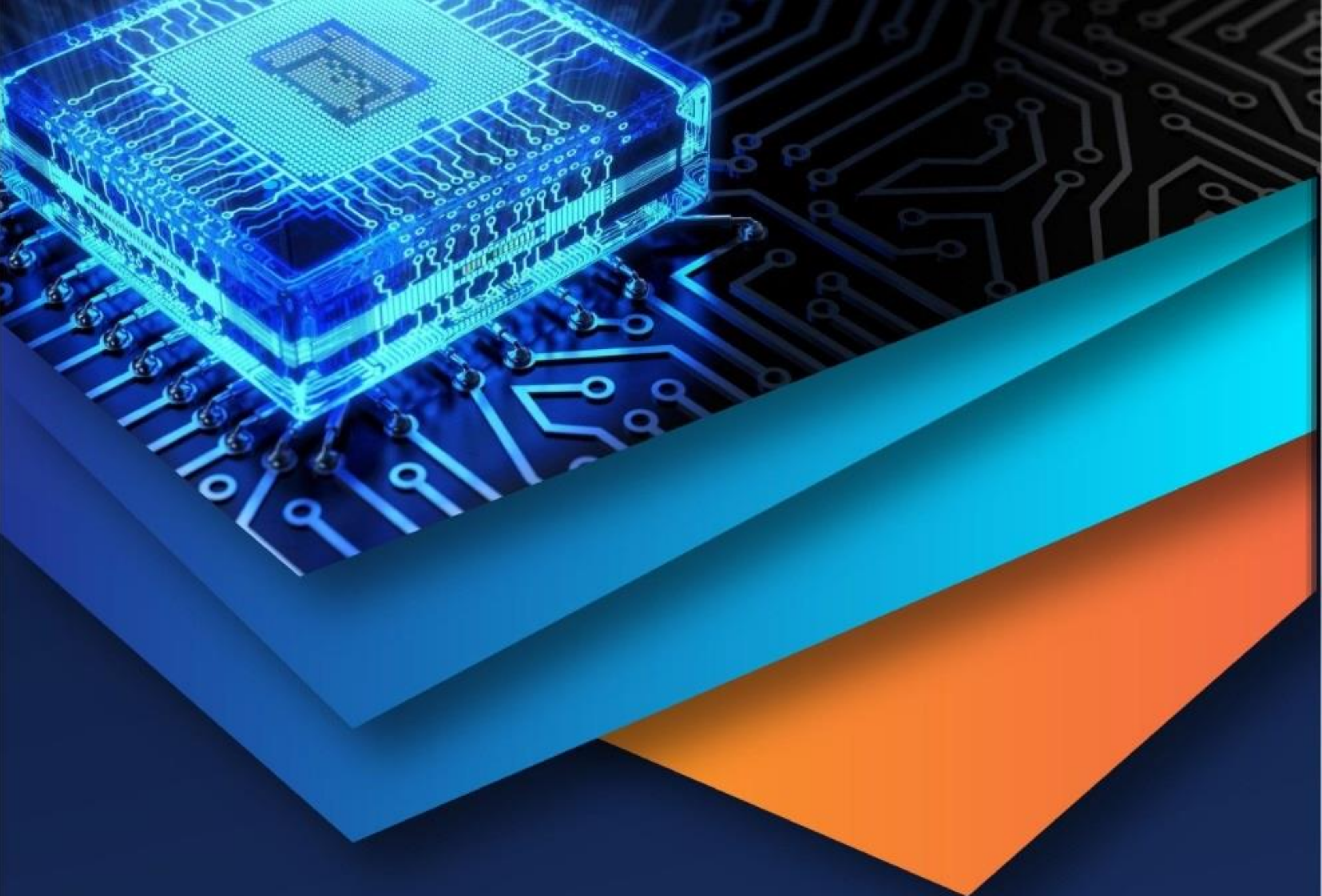
5. $x(A,1) - z(A,1) - 54T_{22,A} - 563Gno_A \equiv 0 \pmod{22384}$

IV. CONCLUSION

Four distinct patterns of non-zero distinct integer solutions to the non-homogeneous cone given by are presented in this paper.
To finish up, one might look for different examples of non-zero number unmistakable arrangements and their relating properties for different decisions of ternary quadratic Diophantine conditions.

REFERENCES

[1] Carmichael, R.D., The theory of numbers and Diophantine Analysis, Dover Publications,New York, 1959.
[2] Dickson L.E, History of Theory of Numbers, Vol.11, Chelsea Publishing company, New York,1952.
[3] Mordell. L.J, Diophantine equations, Academic Press,London,1969 Telang, S.G., Number theory, Tata McGraw Hill publishing company, New Delhi, 1996.
[4] Gopalan.M.A., Vidhyalakshmi.S and Umarani.J., "On ternary Quadratic Diophantine equation $6(x^2+y^2)-8xy=21z^2$", Sch.J. Eng. Tech. 2(2A); 108- 112,2014.
[5] Janaki.G and Saranya.C., Observations on the Ternary Quadratic Diophantine Equation $6(x^2+y^2)-11xy+3x+3y+9=72z^2$, International Journal of Innovative Research in Science, Engineering and Technology, Vol-5, Issue-2, Pg.no: 2060-2065, Feb 2016.
[6] Janaki.G and Vidhya.S., On the integer solutions of thehomogeneous biquadratic diophantine equation $x^4-y^4=82(z^2-w^2)p^2$, International Journal of Engineering Science and Computing, Vol. 6, Issue 6, pp.7275-7278, June, 2016.
[7] Gopalan.M.A and Janaki.G, Integral solutions of $(x^2-y^2)(3x^2+3y^2-2xy=2(z^2-w^2)p^3$, Impact J.Sci.,Tech., 4(1), 97-102, 2010.
[8] Janaki.G and Saranya.P., On the ternary Cubic diophantineequation $5(x^2+y^2)-6xy+4(x+y)+4=40z^3$, International Journal of Science and Research- online, Vol 5, Issue 3, Pg.No:227-229, March 2016.
[9] Janaki.G and Saranya.C., Integral Solutions of the non-homogeneous heptic equation with five unknowns $5(x^3-y^3)-7(x^2+y^2)+4(z^3-w^3+3wz-xy+1)=972p^7$, International Journal of Engineering Science and Computing, Vol. 6, Issue 5, pp.5347-5349, May, 2016.
[10] Janaki.G and Saranya.C., Integral Solutions of the ternary cubic equation $3(x^2+y^2)-4xy+2(x+y+1)=972\,z^3$, International Research Journal of Engineering and Technology, Vol. 4, Issue 3, pp.665-669, March, 2017.
[11] Janaki.G and Saranya.C., Integral Solutions of the homogeneous biquadratic diophantine equation $3(x^4-y^4)-2xy(x^2-y^2)=972\,(z+w)p^3$, International Journal for Research in Applied Science and Engineering Technology, Vol. 5, Issue 8, pp.1123-1127, Aug 2017.



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