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Observations on Ternary Quadratic Diophantine Equation $8(x^2+y^2)-15xy+7x+7y+49=40z^2$

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Abstract: The Ternary Quadratic Diophantine Equation is given by $8(x^2+y^2)-15xy+7x+7y+49=40z^2$. Number of non-zero distinct integer solutions. Some interesting relations among the solutions and the way of factorization, along with the illusion of non-zero distinct integer solutions to the above equations are obtained.
Keywords: Diophantine equation, Integral solutions, Quadratic equation with three unknowns, Ternary equation.

I. INTRODUCTION

Diophantine equation have provoked the interest of different mathematicians. It is simple to solve Diophantine equations with a degree greater than three by reducing them to equations degree 2 or 3. In [1-3], hypothesis of numbers is talked about. Diophantine quadratic equations are discussed in [5,6, and 12]. In [4,7-11] cubic, biquadratic and higher order equations are considered for its integer solutions

This correspondence deals with one more attracting ternary quadratic equation addressing a non-homogeneous cone for its infinitely non-zero integer points. In like, manner, a couple of fascinating relations among the arrangements are correlated.

II. NOTATIONS

$T_{8,n}$ =Octagonal number if rank 'n'.

$T_{10,n}$ = Decagonal number of rank 'n'.

$T_{18,n}$ = Octadecagonal number of rank 'n'.

$T_{22,n}$ = Icosidigonal number of rank 'n'.

$T_{26,n}$ = Icosihexagonal number of rank 'n'.

$T_{28,n}$ = Icosioctagonal number of rank 'n'

Gno_n = Gnomonic number of rank 'n'.

III. METHOD OF ANALYSIS

The ternary quadratic Diophantine equation for its non-zero integral solution is given by

$8(x^2+y^2)-15xy+7x+7y+49=40z^2$ (1)

The replacement of linear transformations $x=u+v$ and $y=u-v$ (2)

in (1) we get,

$(u+7)^2+31v^2=40z^2$ (3)

Below are four examples of distinct non-zero integer solutions to (1).

PATTERN: 1

Assume $z=z(a,b)=a^2+31b^2$ (4)

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wherein (a, b) is a pair of non-zero integers.

$$40 = (3 + i\sqrt{31})(3 - i\sqrt{31}) \qquad (5)$$

Using (4)and (5) in (3), by the method of factorization,

$$\left((u+7) + i\sqrt{31}v\right)\left((u+7) - i\sqrt{31}v\right) = (3 + i\sqrt{31})(3 - i\sqrt{31})\left[\left(a + i\sqrt{31}b\right)^2\left(a - i\sqrt{31}b\right)^2\right] \quad (6)$$

When we compare real and imaginary components and put like terms together, we get

$$u = u(a,b) = 3a^2 - 93b^2 - 62ab - 7$$
$$v = v(a,b) = a^2 - 31b^2 + 6ab$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x = x(a,b) = 4a^2 - 124b^2 - 56ab - 7$$
$$y = y(a,b) = 2a^2 - 62b^2 - 68ab - 7$$
$$z = z(a,b) = a^2 + 31b^2$$

OBSERVATIONS:

1. $x(a,a) - y(a,a) + z(a,a) + 4T_{10,a} \equiv 0\,(\text{mod}12)$
2. $y(1,1) - x(1,1) - z(1,1)$ is a perfect square.
3. $x(a,1) - y(a,1) + z(a,1) - T_{8,a} - 7Gno_a \equiv 0\,(\text{mod } 24)$
4. $x(a,a) - y(a,a) + 4T_{26,a} \equiv 0\,(\text{mod } 44)$
5. $x(a,a) - y(a,a) - z(a,a\cdot) + 10T_{18,a} \equiv 0\,(\text{mod } 70)$

PATTERN:2

Consider (5),we write 40 as

$$40 = \frac{(19 + i3\sqrt{31})(19 - i3\sqrt{31})}{16} \qquad (7)$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x = x(A,B) = 88A^2 - 2728B^2 - 592AB - 7$$
$$y = y(A,B) = 64A^2 - 1984B^2 - 896AB - 7$$
$$z = z(A,B) = 16A^2 + 496B^2$$

OBSERVATIONS:

1. $x(1,1) - y(1,1) + z(1,1)$ is a nasty number
2. $y(A,A) - x(A,A) - 32T_{28,A} \equiv 0\,(\text{mod}384)$

3. $x(A,A)-y(A,A)+z(A,A)-8T_{26,A}\equiv 0\,(\mathrm{mod}\,88)$
4. $x(A,A)-y(A,A)-z(A,A)+116T_{18,A}\equiv 0\,(\mathrm{mod}\,812)$
5. $x(A,1)-y(A,1)-2T_{26,A}-163Gno_A\equiv 0(\mathrm{mod}\,581)$

PATTERN: 3

Consider(5),we write 40 as,

$$40=\frac{(7+i9\sqrt{31})(7-i9\sqrt{31})}{64} \qquad (8)$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2)

$$x=x(A,B)=128A^2-3968B^2-4352AB-7$$
$$y=y(A,B)=-16A^2+496B^2-4576AB-7$$
$$z=z(A,B)=64A^2+1984B^2$$

OBSERVATIONS:

1. $y(1,1)-x(1,1)$ is a perfect square.
2. $x(A,A)-y(A,A)+z(A,A)+256T_{18,A}\equiv 0(\mathrm{mod}\,1792)$
3. $x(A,1)-y(A,1)-12T_{26,A}-178Gno_A\equiv 0\,(\mathrm{mod}\,4286)$
4. $x(A,A)-y(A,A)+1024T_{10,A}\equiv 0\,(\mathrm{mod}\,3072)$
5. $x(A,A)-y(A,A)-z(A,A)+512T_{26,A}\equiv 0\,(\mathrm{mod}\,5632)$

PATTERN: 4

Consider(5),we write 40 as

$$40=\frac{(63+i\sqrt{31})(63-i\sqrt{31})}{100} \qquad (9)$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x=x(A,B)=640A^2-19840B^2+640AB-7$$
$$y=y(A,B)=620A^2-19220B^2-1880AB-7$$
$$z=z(A,B)=100A^2+3100B^2$$

OBSERVATIONS:

1. $x(A,A)-y(A,A)+z(A,A)-640T_{18,A}\equiv 0(\mathrm{mod}\,4480)$
2. $x(A,A)-y(A,A)-160T_{26,A}\equiv 0\,(\mathrm{mod}\,1760)$
3. $x(A,1)-y(A,1)-z(A,1)+10T_{18,A}-1225Gno_A\equiv 0\,(\mathrm{mod}\,2495)$
4. $x(A,A)-y(A,A)-z(A,A)+128T_{22,A}\equiv 0\,(\mathrm{mod}\,1152)$

5. $x(A,1) - z(A,1) - 54T_{22,A} - 563Gno_A \equiv 0 \pmod{22384}$

IV. CONCLUSION

Four distinct patterns of non-zero distinct integer solutions to the non-homogeneous cone given by are presented in this paper.
To finish up, one might look for different examples of non-zero number unmistakable arrangements and their relating properties for different decisions of ternary quadratic Diophantine conditions.

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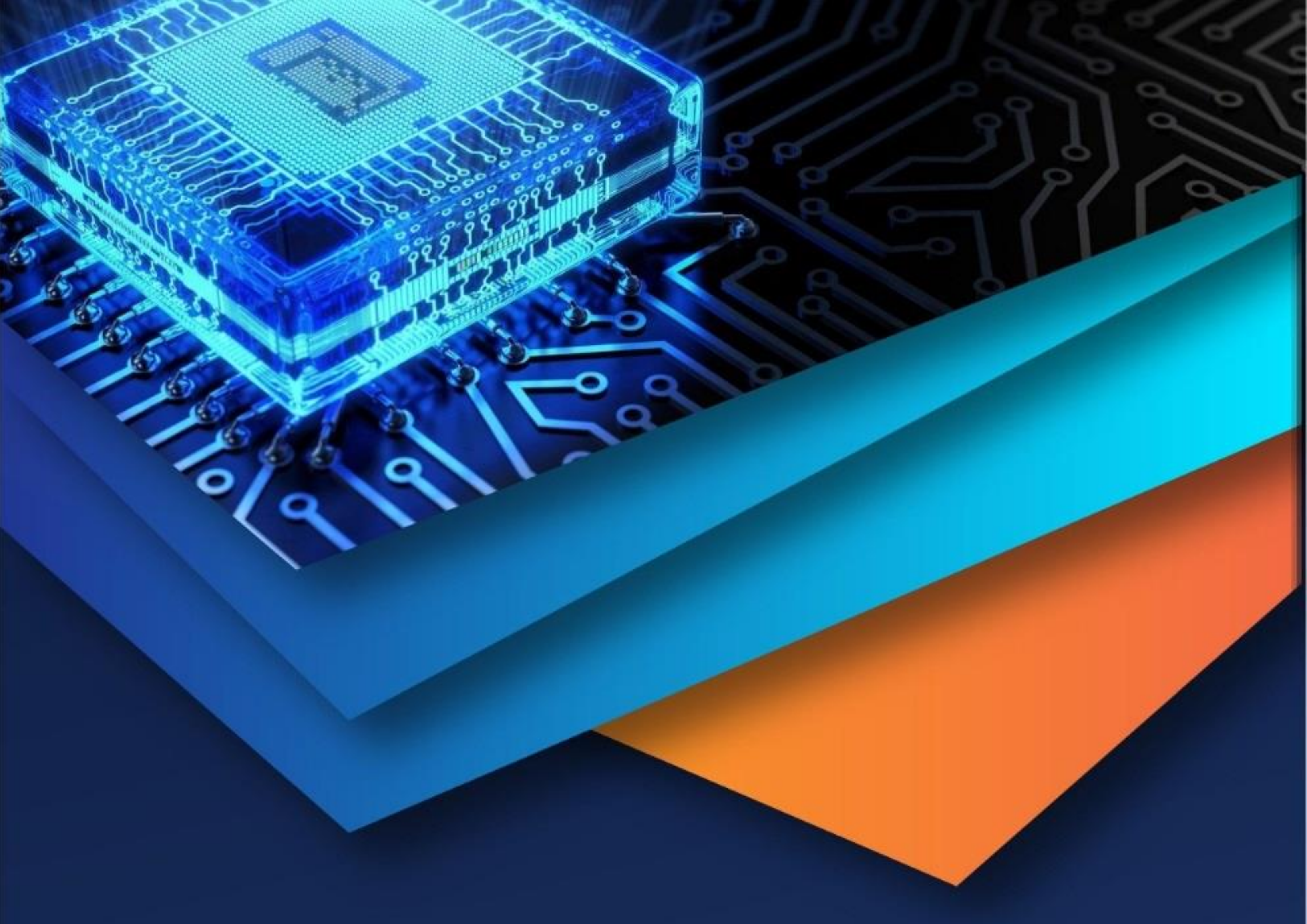

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