



iJRASET

International Journal For Research in
Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 12 Issue: III Month of publication: March 2024

DOI: <https://doi.org/10.22214/ijraset.2024.58887>

www.ijraset.com

Call:  08813907089

E-mail ID: ijraset@gmail.com

Observations on Ternary Quadratic Diophantine

Equation $8(x^2 + y^2) - 15xy + 7x + 7y + 49 = 40z^2$

C.Saranya¹, R.Salini²

¹Assistant Professor, ²PG student, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous)
(Affiliated to Bharathidasan University), Tiruchirappalli, Tamil Nadu, India

Abstract: The Ternary Quadratic Diophantine Equation is given by $8(x^2 + y^2) - 15xy + 7x + 7y + 49 = 40z^2$. Number of non-zero distinct integer solutions. Some interesting relations among the solutions and the way of factorization, along with the illusion of non-zero distinct integer solutions to the above equations are obtained.

Keywords: Diophantine equation, Integral solutions, Quadratic equation with three unknowns, Ternary equation.

I. INTRODUCTION

Diophantine equation have provoked the interest of different mathematicians. It is simple to solve Diophantine equations with a degree greater than three by reducing them to equations degree 2 or 3. In [1-3], hypothesis of numbers is talked about. Diophantine quadratic equations are discussed in [5,6, and 12]. In [4,7-11] cubic, biquadratic and higher order equations are considered for its integer solutions

This correspondence deals with one more attracting ternary quadratic equation addressing a non-homogeneous cone for its infinitely non-zero integer points. In like, manner, a couple of fascinating relations among the arrangements are correlated.

II. NOTATIONS

$T_{8,n}$ = Octagonal number of rank 'n'.

$T_{10,n}$ = Decagonal number of rank 'n'.

$T_{18,n}$ = Octadecagonal number of rank 'n'.

$T_{22,n}$ = Icosidigonal number of rank 'n'.

$T_{26,n}$ = Icosihexagonal number of rank 'n'.

$T_{28,n}$ = Icosioctagonal number of rank 'n'

Gno_n = Gnomonic number of rank 'n'.

III. METHOD OF ANALYSIS

The ternary quadratic Diophantine equation for its non-zero integral solution is given by

$$8(x^2 + y^2) - 15xy + 7x + 7y + 49 = 40z^2 \quad (1)$$

The replacement of linear transformations $x = u + v$ and $y = u - v$ (2)

in (1) we get,

$$(u + 7)^2 + 31v^2 = 40z^2 \quad (3)$$

Below are four examples of distinct non-zero integer solutions to (1).

PATTERN: 1

Assume $z = z(a, b) = a^2 + 31b^2$ (4)

wherein (a, b) is a pair of non-zero integers.

$$40 = (3 + i\sqrt{31})(3 - i\sqrt{31}) \quad (5)$$

Using (4) and (5) in (3), by the method of factorization,

$$\left((u + 7) + i\sqrt{31}v \right) \left((u + 7) - i\sqrt{31}v \right) = (3 + i\sqrt{31})(3 - i\sqrt{31}) \left[(a + i\sqrt{31}b)^2 (a - i\sqrt{31}b)^2 \right] \quad (6)$$

When we compare real and imaginary components and put like terms together, we get

$$u = u(a, b) = 3a^2 - 93b^2 - 62ab - 7$$

$$v = v(a, b) = a^2 - 31b^2 + 6ab$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x = x(a, b) = 4a^2 - 124b^2 - 56ab - 7$$

$$y = y(a, b) = 2a^2 - 62b^2 - 68ab - 7$$

$$z = z(a, b) = a^2 + 31b^2$$

OBSERVATIONS:

1. $x(a, a) - y(a, a) + z(a, a) + 4T_{10,a} \equiv 0 \pmod{12}$
2. $y(1,1) - x(1,1) - z(1,1)$ is a perfect square.
3. $x(a,1) - y(a,1) + z(a,1) - T_{8,a} - 7Gno_a \equiv 0 \pmod{24}$
4. $x(a, a) - y(a, a) + 4T_{26,a} \equiv 0 \pmod{44}$
5. $x(a, a) - y(a, a) - z(a, a) + 10T_{18,a} \equiv 0 \pmod{70}$

PATTERN:2

Consider (5), we write 40 as

$$40 = \frac{(19 + i3\sqrt{31})(19 - i3\sqrt{31})}{16} \quad (7)$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$x = x(A, B) = 88A^2 - 2728B^2 - 592AB - 7$$

$$y = y(A, B) = 64A^2 - 1984B^2 - 896AB - 7$$

$$z = z(A, B) = 16A^2 + 496B^2$$

OBSERVATIONS:

1. $x(1,1) - y(1,1) + z(1,1)$ is a nasty number
2. $y(A, A) - x(A, A) - 32T_{28,A} \equiv 0 \pmod{384}$

3. $x(A, A) - y(A, A) + z(A, A) - 8T_{26,A} \equiv 0 \pmod{88}$
4. $x(A, A) - y(A, A) - z(A, A) + 116T_{18,A} \equiv 0 \pmod{812}$
5. $x(A,1) - y(A,1) - 2T_{26,A} - 163Gno_A \equiv 0 \pmod{581}$

PATTERN: 3

Consider(5),we write 40 as,

$$40 = \frac{(7 + i9\sqrt{31})(7 - i9\sqrt{31})}{64} \tag{8}$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2)

$$\begin{aligned} x &= x(A, B) = 128A^2 - 3968B^2 - 4352AB - 7 \\ y &= y(A, B) = -16A^2 + 496B^2 - 4576AB - 7 \\ z &= z(A, B) = 64A^2 + 1984B^2 \end{aligned}$$

OBSERVATIONS:

1. $y(1,1) - x(1,1)$ is a perfect square.
2. $x(A, A) - y(A, A) + z(A, A) + 256T_{18,A} \equiv 0 \pmod{1792}$
3. $x(A,1) - y(A,1) - 12T_{26,A} - 178Gno_A \equiv 0 \pmod{4286}$
4. $x(A, A) - y(A, A) + 1024T_{10,A} \equiv 0 \pmod{3072}$
5. $x(A, A) - y(A, A) - z(A, A) + 512T_{26,A} \equiv 0 \pmod{5632}$

PATTERN: 4

Consider(5),we write 40 as

$$40 = \frac{(63 + i\sqrt{31})(63 - i\sqrt{31})}{100} \tag{9}$$

The corresponding integer solutions of equation (1) are given by using the values of u & v in equation (2).

$$\begin{aligned} x &= x(A, B) = 640A^2 - 19840B^2 + 640AB - 7 \\ y &= y(A, B) = 620A^2 - 19220B^2 - 1880AB - 7 \\ z &= z(A, B) = 100A^2 + 3100B^2 \end{aligned}$$

OBSERVATIONS:

1. $x(A, A) - y(A, A) + z(A, A) - 640T_{18,A} \equiv 0 \pmod{4480}$
2. $x(A, A) - y(A, A) - 160T_{26,A} \equiv 0 \pmod{1760}$
3. $x(A,1) - y(A,1) - z(A,1) + 10T_{18,A} - 1225Gno_A \equiv 0 \pmod{2495}$
4. $x(A, A) - y(A, A) - z(A, A) + 128T_{22,A} \equiv 0 \pmod{1152}$

$$5. \quad x(A,1) - z(A,1) - 54T_{22,A} - 563Gno_A \equiv 0 \pmod{22384}$$

IV. CONCLUSION

Four distinct patterns of non-zero distinct integer solutions to the non-homogeneous cone given by are presented in this paper. To finish up, one might look for different examples of non-zero number unmistakable arrangements and their relating properties for different decisions of ternary quadratic Diophantine conditions.

REFERENCES

- [1] Carmichael, R.D., The theory of numbers and Diophantine Analysis, Dover Publications, New York, 1959.
- [2] Dickson L.E, History of Theory of Numbers, Vol.11, Chelsea Publishing company, New York, 1952.
- [3] Mordell. L.J, Diophantine equations, Academic Press, London, 1969 Telang, S.G., Number theory, Tata McGraw Hill publishing company, New Delhi, 1996.
- [4] Gopalan.M.A., Vidhyalakshmi.S and Umarani.J., "On ternary Quadratic Diophantine equation $6(x^2 + y^2) - 8xy = 21z^2$ ", Sch.J. Eng. Tech. 2(2A), 108- 112, 2014.
- [5] Janaki.G and Saranya.C., Observations on the Ternary Quadratic Diophantine Equation $6(x^2 + y^2) - 11xy + 3x + 3y + 9 = 72z^2$, International Journal of Innovative Research in Science, Engineering and Technology, Vol-5, Issue-2, Pg.no: 2060-2065, Feb 2016.
- [6] Janaki.G and Vidhya.S., On the integer solutions of the homogeneous biquadratic diophantine equation $x^4 - y^4 = 82(z^2 - w^2)p^2$, International Journal of Engineering Science and Computing, Vol. 6, Issue 6, pp.7275-7278, June, 2016.
- [7] Gopalan.M.A and Janaki.G, Integral solutions of $(x^2 - y^2)(3x^2 + 3y^2 - 2xy) = 2(z^2 - w^2)p^3$, Impact J.Sci.,Tech., 4(1), 97-102, 2010.
- [8] Janaki.G and Saranya.P., On the ternary Cubic diophantine equation $5(x^2 + y^2) - 6xy + 4(x + y) + 4 = 40z^3$, International Journal of Science and Research- online, Vol 5, Issue 3, Pg.No:227-229, March 2016.
- [9] Janaki.G and Saranya.C., Integral Solutions of the non-homogeneous heptic equation with five unknowns $5(x^3 - y^3) - 7(x^2 + y^2) + 4(z^3 - w^3 + 3wz - xy + 1) = 972p^7$, International Journal of Engineering Science and Computing, Vol. 6, Issue 5, pp.5347-5349, May, 2016.
- [10] Janaki.G and Saranya.C., Integral Solutions of the ternary cubic equation $3(x^2 + y^2) - 4xy + 2(x + y + 1) = 972z^3$, International Research Journal of Engineering and Technology, Vol. 4, Issue 3, pp.665-669, March, 2017.
- [11] Janaki.G and Saranya.C., Integral Solutions of the homogeneous biquadratic diophantine equation $3(x^4 - y^4) - 2xy(x^2 - y^2) = 972(z + w)p^3$, International Journal for Research in Applied Science and Engineering Technology, Vol. 5, Issue 8, pp.1123-1127, Aug 2017.



10.22214/IJRASET



45.98



IMPACT FACTOR:
7.129



IMPACT FACTOR:
7.429



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24*7 Support on Whatsapp)