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Observations on Ternary Quadratic Diophantine

Equation $12(x^2 + y^2) - 23xy + 2x + 2y + 4 = 56z^2$

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Abstract: The Ternary Quadratic Diophantine Equation $12(x^2 + y^2) - 23xy + 2x + 2y + 4 = 56z^2$ is analyzed for its infinite number of non-zero integral solutions. Four interesting patterns satisfying the cone are identified. There are a few interesting connections between the solutions and some unique number patterns.

Keywords: Diophantine equation, Integral solutions, Quadratic equation with three unknowns, Ternary equation.

I. INTRODUCTION

Diophantine equations have stimulated the interest of various mathematicians. Diophantine equations with higher degree greater than three can be reduced in to equations of degree 2 or 3 and it can be easily solved. In [1-3], theory of numbers is discussed. In [5,6 & 12], quadratic Diophantine equations are discussed. In [4, 7-11], cubic, biquadratic and higher order equations are considered for its integral solutions.

This communication concerns with yet another interesting ternary quadratic equation $12(x^2 + y^2) - 23xy + 2x + 2y + 4 = 56z^2$ representing a non-homogeneous cone is determined for its infinitely many non-zero integral points. Likewise, a few interesting relations among the solutions are analyzed.

II. NOTATIONS

- $T_{6,n}$ = Hexagonal number of rank 'n'.
- $T_{8,n}$ = Octogonal number of rank 'n'.
- $T_{10,n}$ = Decagonal number of rank 'n'.
- $T_{14,n}$ = Tetradecagonal number of rank 'n'.
- $T_{16,n}$ = Hexadecagonal number of rank 'n'.
- $T_{18,n}$ = Octadecagonal number of rank 'n'.
- $T_{22,n}$ = Icosidigonal number of rank 'n'.
- $T_{24,n}$ = Icositetragonal number of rank 'n'.
- $T_{26,n}$ = Icosihexagonal number of rank 'n'.
- Gno_n = Gnomonic number rank 'n'.

III. METHOD OF ANALYSIS

The ternary quadratic diophantine equation to be tackled for its non-zero integral solution is

$$12(x^2 + y^2) - 23xy + 2x + 2y + 4 = 56z^2 \tag{1}$$

using the linear transformations

$$x = u + v \quad \text{and} \quad y = u - v \tag{2}$$

in (1) leads to, $(u + 2)^2 + 47v^2 = 56z^2 \tag{3}$

We illustrate below four different patterns of non-zero distinct integer solutions to (1)

A. Pattern : 1

Assume $z = z(a,b) = a^2 + 47b^2$ (4)

where a and b are non-zero integers.

and write $56 = (3 + i\sqrt{47})(3 - i\sqrt{47})$ (5)

Using (4) and (5) in (3), and using factorization method,

$$\left((u + 2) + i\sqrt{47}v \right) \left((u + 2) - i\sqrt{47}v \right) = (3 + i\sqrt{47})(3 - i\sqrt{47}) \left[(a + i\sqrt{47}b)^2 (a - i\sqrt{47}b)^2 \right] \tag{6}$$

Equating the like terms and comparing real and imaginary parts, we get

$$u = u(a,b) = 3a^2 - 141b^2 - 94ab - 2$$

$$v = v(a,b) = a^2 - 47b^2 + 60ab$$

Substituting the above values of u & v in equation (2), the corresponding integer solutions of (1) are given by

$$x = x(a,b) = 4a^2 - 188b^2 - 88ab - 2$$

$$y = y(a,b) = 2a^2 - 94b^2 - 100ab - 2$$

$$z = z(a,b) = a^2 + 47b^2$$

Observations

1. $x(a,a) - y(a,a) + z(a,a) + 4T_{18,a} \equiv 0 \pmod{28}$
2. $2y(1,1) - 2x(1,1) - 2z(1,1)$ is a perfect square.
3. $y(a,a) + z(a,a) - x(a,a) - 32T_{10,a} \equiv 0 \pmod{96}$
4. $y(a,a) - x(a,a) - 8T_{22,a} \equiv 0 \pmod{72}$
5. $x(a,1) - y(a,1) + z(a,1) - T_{8,a} - 7Gno_a \equiv 0 \pmod{40}$
6. $y(1,1) - x(1,1)$ is a duck number

B. Pattern : 2

We substitute (5) with 56 as

$$56 = \frac{(29 + i5\sqrt{47})(29 - i5\sqrt{47})}{36} \tag{7}$$

Following the methodology introduced over, the relating non-zero particular whole number answers for (1) are Gotten as

$$x = x(A,B) = 204A^2 - 9588B^2 - 2472AB - 2$$

$$y = y(A,B) = 144A^2 - 6768B^2 - 3168AB - 2$$

$$z = z(A,B) = 36A^2 + 1692B^2$$

Observations

1. $x(A,A) - y(A,A) + z(A,A) + 48T_{16,A} \equiv 0 \pmod{288}$
2. $x(A,1) - y(A,1) - 6T_{22,A} - 375Gno_A \equiv 0 \pmod{2445}$
3. $x(A,A) - y(A,A) + 172T_{26,A} \equiv 0 \pmod{1892}$
4. $y(A,1) - x(A,1) - z(A,1) + 16T_{14,A} + 388Gno_A \equiv 0 \pmod{740}$
5. Each of the following expressions represents a Duck number

- i. $x(1,1) - 2y(1,1)$
- ii. $x(1,1) - 2y(1,1) - z(1,1)$
- iii. $-y(1,1) - z(1,1)$

C. Pattern : 3

We substitute (5) with 56 as

$$56 = \frac{(67 + i\sqrt{47})(67 - i\sqrt{47})}{81} \tag{8}$$

Following the methodology introduced over, the relating non-zero particular whole number answers for (1) are gotten as

$$x = x(A, B) = 612A^2 - 28764B^2 - 360AB - 2$$

$$y = y(A, B) = 594A^2 - 27918B^2 - 2052AB - 2$$

$$z = z(A, B) = 81A^2 + 38078B^2$$

Observations

- 1. $x(A, A) - y(A, A) + z(A, A) - 132T_{26,A} - 726Gno_A \equiv 0 \pmod{38885}$
- 2. $y(A, A) - x(A, A) + 132T_{26,A} \equiv 0 \pmod{1452}$
- 3. $x(A, 1) - z(A, 1) - 177T_{8,A} - 357Gno_A \equiv 0 \pmod{66487}$
- 4. $4x(1,1) - 4y(1,1)$ is a palindromic number
- 5. $z(1,1) - x(1,1) - y(1,1)$ is a duck number

D. Pattern : 4

We substitute (5) with 56 as

$$56 = \frac{(83 + i5\sqrt{47})(83 - i5\sqrt{47})}{144} \tag{9}$$

Following the methodology introduced over, the relating non-zero particular whole number answers for (1) are gotten as

$$x = x(A, B) = 1056A^2 - 49632B^2 - 3648AB - 2$$

$$y = y(A, B) = 936A^2 - 43992B^2 - 7632AB - 2$$

$$z = z(A, B) = 144A^2 + 6768B^2$$

Observations

- 1. $x(A, 1) - y(A, 1) - z(A, 1) + 2T_{26,A} - 1981Gno_A \equiv 0 \pmod{10427}$
- 2. $x(A, 1) - y(A, 1) - 10T_{24,A} - 2047Gno_A \equiv 0 \pmod{3593}$
- 3. $y(1,1) - x(1,1) + z(1,1)$ is a palindromic number
- 4. $y(1,1) - x(1,1)$ is a nasty number.
- 5. $y(1,1) - z(1,1) - 2x(1,1)$ is a duck number.

IV. CONCLUSION

This paper discusses four distinct patterns of non-zero distinct integer solutions to the non-homogeneous cone given by $12(x^2 + y^2) - 23xy + 2x + 2y - 4 = 56z^2$. To conclude, one may search for other patterns of solutions and their corresponding properties.



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