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Observations on Ternary Quadratic Diophantine Equation $12(x^2+y^2)-23xy+2x+2y+4=56z^2$

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Abstract: The Ternary Quadratic Diophantine Equation $12(x^2+y^2)-23xy+2x+2y+4=56z^2$ is analyzed for its infinite number of non-zero integral solutions. Four interesting patterns satisfying the cone are identified. There are a few interesting connections between the solutions and some unique number patterns.
Keywords: Diophantine equation, Integral solutions, Quadratic equation with three unknowns, Ternary equation.

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I. INTRODUCTION

Diophantine equations have stimulated the interest of various mathematicians. Diophantine equations with higher degree greater than three can be reduced in to equations of degree 2 or 3 and it can be easily solved. In [1-3], theory of numbers is discussed. In [5,6 & 12], quadratic Diophantine equations are discussed. In [4, 7-11], cubic, biquadratic and higher order equations are considered for its integral solutions.

This communication concerns with yet another interesting ternary quadratic equation $12(x^2+y^2)-23xy+2x+2y+4=56z^2$ representing a non-homogeneous cone is determined for its infinitely many non-zero integral points. Likewise, a few interesting relations among the solutions are analyzed.

II. NOTATIONS

$T_{6,n}$ = Hexgonal number of rank 'n'.

$T_{8,n}$ = Octogonal number of rank 'n'.

$T_{10,n}$ = Decagonal number of rank 'n'.

$T_{14,n}$ = Tetradecogonal number of rank 'n'.

$T_{16,n}$ = Hexadecagonal number of rank 'n'.

$T_{18,n}$ = Octadecagonal number of rank 'n'.

$T_{22,n}$ = Icosidigonal number of rank 'n'.

$T_{24,n}$ = Icositetragonal number of rank 'n'.

$T_{26,n}$ = Icosihexagonal number of rank 'n'.

Gno_n = Gnomonic number rank 'n'.

III. METHOD OF ANALYSIS

The ternary quadratic diophantine equation to be tackled for its non-zero integral solution is

$$12(x^2+y^2)-23xy+2x+2y+4=56z^2 \qquad (1)$$

using the linear transformations

$$x=u+v \quad \text{and} \quad y=u-v \qquad (2)$$

in (1) leads to,

$$(u+2)^2+47v^2=56z^2 \qquad (3)$$

We illustrate below four different patterns of non-zero distinct integer solutions to (1)

A. Pattern : 1

Assume $z = z(a,b) = a^2 + 47b^2$ (4)

where a and b are non-zero integers.

and write $56 = (3 + i\sqrt{47})(3 - i\sqrt{47})$ (5)

Using (4) and (5) in (3), and using factorization method,

$$\left((u+2) + i\sqrt{47}v\right)\left((u+2) - i\sqrt{47}v\right) = (3 + i\sqrt{47})(3 - i\sqrt{47})\left[\left(a + i\sqrt{47}b\right)^2\left(a - i\sqrt{47}b\right)^2\right] \quad (6)$$

Equating the like terms and comparing real and imaginary parts, we get

$$u = u(a,b) = 3a^2 - 141b^2 - 94ab - 2$$

$$v = v(a,b) = a^2 - 47b^2 + 60ab$$

Substituting the above values of u & v in equation (2), the corresponding integer solutions of (1) are given by

$$x = x(a,b) = 4a^2 - 188b^2 - 88ab - 2$$

$$y = y(a,b) = 2a^2 - 94b^2 - 100ab - 2$$

$$z = z(a,b) = a^2 + 47b^2$$

Observations

1. $x(a,a) - y(a,a) + z(a,a) + 4T_{18,a} \equiv 0 \pmod{28}$
2. $2y(1,1) - 2x(1,1) - 2z(1,1)$ is a perfect square.
3. $y(a,a) + z(a,a) - x(a,a) - 32T_{10,a} \equiv 0 \pmod{96}$
4. $y(a,a) - x(a,a) - 8T_{22,a} \equiv 0 \pmod{72}$
5. $x(a,1) - y(a,1) + z(a,1) - T_{8,a} - 7Gno_a \equiv 0 \pmod{40}$
6. y(1,1)-x(1,1) is a duck number

B. Pattern : 2

We substitute (5) with 56 as

$$56 = \frac{(29 + i5\sqrt{47})(29 - i5\sqrt{47})}{36} \quad (7)$$

Following the methodology introduced over, the relating non-zero particular whole number answers for (1) are Gotten as

$$x = x(A,B) = 204A^2 - 9588B^2 - 2472AB - 2$$

$$y = y(A,B) = 144A^2 - 6768B^2 - 3168AB - 2$$

$$z = z(A,B) = 36A^2 + 1692B^2$$

Observations

1. $x(A,A) - y(A,A) + z(A,A) + 48T_{16,A} \equiv 0 \pmod{288}$
2. $x(A,1) - y(A,1) - 6T_{22,A} - 375Gno_A \equiv 0 \pmod{2445}$
3. $x(A,A) - y(A,A) + 172T_{26,A} \equiv 0 \pmod{1892}$
4. $y(A,1) - x(A,1) - z(A,1) + 16T_{14,A} + 388Gno_A \equiv 0 \pmod{740}$
5. Each of the following expressions represents a Duck number

i. $x(1,1)-2y(1,1)$

ii. $x(1,1)-2y(1,1)-z(1,1)$

iii. $-y(1,1)-z(1,1)$

C. Pattern : 3

We substitute (5) with 56 as

$$56=\frac{(67+i\sqrt{47})(67-i\sqrt{47})}{81} \tag{8}$$

Following the methodology introduced over, the relating non-zero particular whole number answers for (1) are gotten as

$$x=x(A,B)=612A^2-28764B^2-360AB-2$$

$$y=y(A,B)=594A^2-27918B^2-2052AB-2$$

$$z=z(A,B)=81A^2+38078B^2$$

Observations

1. $x(A,A)-y(A,A)+z(A,A)-132T_{26,A}-726Gno_A \equiv 0 \pmod{38885}$
2. $y(A,A)-x(A,A)+132T_{26,A}\equiv 0 \pmod{1452}$
3. $x(A,1)-z(A,1)-177T_{8,A}-357Gno_A\equiv 0 \pmod{66487}$
4. $4x(1,1)-4y(1,1)$ is a palindromic number
5. $z(1,1)-x(1,1)-y(1,1)$ is a duck number

D. Pattern : 4

We substitute (5) with 56 as

$$56=\frac{(83+i5\sqrt{47})(83-i5\sqrt{47})}{144} \tag{9}$$

Following the methodology introduced over, the relating non-zero particular whole number answers for (1) are gotten as

$$x=x(A,B)=1056A^2-49632B^2-3648AB-2$$

$$y=y(A,B)=936A^2-43992B^2-7632AB-2$$

$$z=z(A,B)=144A^2+6768B^2$$

Observations

1. $x(A,1)-y(A,1)-z(A,1)+2T_{26,A}-1981Gno_A\equiv 0 \pmod{10427}$
2. $x(A,1)-y(A,1)-10T_{24,A}-2047Gno_A\equiv 0 \pmod{3593}$
3. $y(1,1)-x(1,1)+z(1,1)$ is a palindromic number
4. $y(1,1)-x(1,1)$ is a nasty number.
5. $y(1,1)-z(1,1)-2x(1,1)$ is a duck number.

IV. CONCLUSION

This paper discusses four distinct patterns of non-zero distinct integer solutions to the non-homogeneous cone given by $12(x^2+y^2)-23xy+2x+2y-4=56z^2$. To conclude, one may search for other patterns of solutions and their corresponding properties.

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