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On Linguistic Neutrosophic Semi-Irresolute Mappings and Semi-Homeomorphism

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Abstract: The purpose of this article is to discuss linguistic neutrosophic semi-irresolute mapping and linguistic neutrosophic locally semi-irresolute mapping in linguistic neutrosophic topological spaces. It is examined how these mappings relate to other mappings, as well as some of their characteristics. Moreover, a brief introduction and analysis of the linguistic neutrosophic semi-homeomorphism and linguistic neutrosophic semi-c-homeomorphism are presented with appropriate examples.

Keywords: Linguistic Neutrosophic semi-open Mapping; Linguistic Neutrosophic semi-irresolute mapping; Linguistic Neutrosophic locally semi-irresolute mapping; Linguistic Neutrosophic semi-homeomorphism; Linguistic Neutrosophic semi-c-homeomorphism;

I. INTRODUCTION

There was a requirement for the indeterminacy membership to represent inconsistent linguistic information even though there exists an intuitionistic linguistic variable made up of degrees of truth and falsity membership. This idea originated from Fang and Ye[6], who introduced linguistic neutrosophic numbers. Smarandache[9] combined indeterminacy membership with existing membership in intuitionistic fuzzy sets[1] to develop the idea of neutrosophic sets. Gayathri and Helen[7] begot a new concept, by mingling linguistic neutrosophic numbers and topological spaces, named linguistic neutrosophic topological spaces.

Irresolute mappings play a momentous role in the study of topological spaces which was introduced by Crossley[5]. Researchers have examined irresolute mappings in considerable detail. The article provides an analysis of some properties and implications of linguistic neutrosophic semi-irresolute mappings in a novel linguistic neutrosophic topological space. Through linguistic neutrosophic semi-open mappings, a new mapping class referred to as linguistic neutrosophic semi-homomorphism and linguistic neutrosophic semi-c-homeomorphism are instigated.

II. PRELIMINARIES

Definition 2.1:[9] Let S be a space of points (objects), with a generic element in x denoted by S. A neutrosophic set A in S is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is, $T_A: S \to]0^-, 1^+[, I_A: S \to]0^-, 1^+[, F_A: S \to]0^-, 1^+[$

There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2:[9] Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point s in $S, T_A(x), I_A(x), F_A(x) \in [0,1]$.

When S is continuous, a SVNS A can be written as $A = \int \langle T(x), I(x), F(x) \rangle / x \in S$.

When S is discrete, a SVNS A can be written as $A = \sum \langle T(x_i), I(x_i), F(x_i) \rangle / x_i \in S$.

Definition 2.3:[6] Let $S = \{s_\theta | \theta = 0,1,2,\ldots\ldots,\tau\}$ be a finite and totally ordered discrete term set, where τ is the even value and s_θ represents a possible value for a linguistic variable.

Definition 2.4:[6] Let $Q = \{s_0, s_1, s_2, ..., s_t\}$ be a linguistic term set (LTS) with odd cardinality $t+1$ and $\overline{Q} = \{s_h / s_0 \le s_h \le s_t, h \in [0,t]\}$. Then, a linguistic single valued neutrosophic set A is defined by,
$A = \{\langle x, s_\theta(x), s_\psi(x), s_\sigma(x)\rangle | x \in S\}$, where $s_\theta(x), s_\psi(x), s_\sigma(x) \in \overline{Q}$ represent the linguistic truth, linguistic indeterminacy and linguistic falsity degrees of S to A, respectively, with condition $0 \le \theta + \psi + \sigma \le 3t$. This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 2.5:[6] Let $\alpha = (s_\theta, s_\psi, s_\sigma), \alpha_1 = (s_{\theta_1}, s_{\psi_1}, s_{\sigma_1}), \alpha_2 = (s_{\theta_2}, s_{\psi_2}, s_{\sigma_2})$ be three LSVNNs, then

1. $\alpha^c = (s_\sigma, s_\psi, s_\theta)$;
2. $\alpha_1 \cup \alpha_2 = (max(\theta_1, \theta_2), max(\psi_1, \psi_2), min(\sigma_1, \sigma_2))$;
3. $\alpha_1 \cap \alpha_2 = (min(\theta_1, \theta_2), min(\psi_1, \psi_2), max(\sigma_1, \sigma_2))$;
4. $\alpha_1 = \alpha_2$ iff $\theta_1 = \theta_2, \psi_1 = \psi_2, \sigma_1 = \sigma_2$;

Definition 2.6:[7] For a linguistic neutrosophic topology τ, the collection of linguistic neutrosophic sets should obey,

1. $0_{LN}, 1_{LN} \in \tau$
2. $K_1 \cap K_2 \in \tau$ for any $K_1, K_2 \in \tau$
3. $\bigcup K_i \in \tau, \forall \{K_i : i \in J\} \subseteq \tau$

We call, the pair (S_{LN}, τ_{LN}), a linguistic neutrosophic topological space.

Definition 2.7: [7] Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic topological space (LNTS). Then,

- $(S_{LN}, \tau_{LN})^c$ is the dual linguistic neutrosophic topology, whose elements are $K^C{}_{LN}$ for $K_{LN} \in (S_{LN}, \tau_{LN})$.
- Any open set in τ_{LN} is known as linguistic neutrosophic open set(LNOS).
- Any closed set in τ_{LN} is known as linguistic neutrosophic closed set(LNCS) if and only if it's complement is linguistic neutrosophic open set.

III.LINGUISTIC NEUTROSOPHIC IRRESOLUTE MAPPINGS

Definition 3.1: A function $f_{LN}: (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is

1) linguistic neutrosophic continuous function if the inverse image of every linguistic neutrosophic open set F_{LN} is linguistic neutrosophic open in S_{LN}.

2) linguistic neutrosophic semi-continuous mapping if the inverse image $(f_{LN})^{-1}(A_{LN})$ is a linguistic neutrosophic semi-open set in S_{LN} for every linguistic neutrosophic open set in T_{LN}.

3) linguistic neutrosophic semi-irresolute if for any linguistic neutrosophic semi-closed set H_{LN} of T_{LN}, the inverse image $(f_{LN})^{-1}(H_{LN})$ is linguistic neutrosophic semi-closed in S_{LN}.

4) linguistic neutrosophic perfectly semi-continuous mapping if the inverse image $f_{LN}(E_{LN})$ of every linguistic neutrosophic semi-open set E_{LN} of T_{LN} is linguistic neutrosophic clopen set in S_{LN}.

5) linguistic neutrosophic open mapping if and only if for every linguistic neutrosophic open set K_{LN} of S_{LN}, $f_{LN}(K_{LN})$ is a linguistic neutrosophic open set in T_{LN}.

6) linguistic neutrosophic semi-open mapping if and only if for every linguistic neutrosophic open set K_{LN} of S_{LN}, $f_{LN}(K_{LN})$ is a linguistic neutrosophic semi-open set in T_{LN}.

7) linguistic neutrosophic closed mapping if and only if for every linguistic neutrosophic closed set E_{LN} of S_{LN}, $f_{LN}(E_{LN})$ is a linguistic neutrosophic closed set in T_{LN}.

8) linguistic neutrosophic semi-closed mapping if and only if for every linguistic neutrosophic closed set E_{LN} of S_{LN}, $f_{LN}(E_{LN})$ is a linguistic neutrosophic semi-closed set in T_{LN}.

Theorem 3.2: A mapping $f_{LN}: (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is linguistic neutrosophic semi-irresolute if and only if for every linguistic neutrosophic semi-closed set K_{LN} of T_{LN}, $(f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-closed in S_{LN}.

Proof: Necessity Part: If f_{LN} is linguistic neutrosophic semi-irresolute, then for every linguistic neutrosophic semi-open set H_{LN} of T_{LN}, $(f_{LN})^{-1}(H_{LN})$ is linguistic neutrosophic semi-open set in S_{LN}.

If K_{LN} is any linguistic neutrosophic semi-closed set of T_{LN}, then the linguistic neutrosophic set $T_{LN} \backslash K_{LN}$ is linguistic neutrosophic semi-open. Thus, $(f_{LN})^{-1}(T_{LN} \backslash K_{LN})$ is linguistic neutrosophic semi-open, but $(f_{LN})^{-1}(T_{LN} \backslash K_{LN}) = S_{LN} \backslash (f_{LN})^{-1}(K_{LN})$ and hence $(f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-closed.

Sufficiency Part: Let K_{LN} be a linguistic neutrosophic semi-closed set in T_{LN}. Then $(f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-closed set in S_{LN}. If H_{LN} is any linguistic neutrosophic semi-open set in T_{LN}, then $T_{LN} \backslash H_{LN}$ is linguistic neutrosophic semi-closed. Also, $(f_{LN})^{-1}(T_{LN} \backslash H_{LN}) = S_{LN} \backslash (f_{LN})^{-1}(H_{LN})$ is linguistic neutrosophic semi-closed. Thus, $(f_{LN})^{-1}(H_{LN})$ is linguistic neutrosophic semi-open. Hence f_{LN} is linguistic neutrosophic semi-irresolute.

Theorem 3.3: Let $f_{LN}: (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic continuous and linguistic neutrosophic open mapping and let H_{LN} be a linguistic neutrosophic semi-open set in S_{LN}, then $f_{LN}(H_{LN})$ be a linguistic neutrosophic semi-open set in T_{LN}.

Proof: Let H_{LN} be a linguistic neutrosophic semi-open set in S_{LN}, then there exists a linguistic neutrosophic open set M_{LN} in S_{LN} such that $M_{LN} \subseteq H_{LN} \subseteq LNCl(M_{LN})$. As f_{LN} is linguistic neutrosophic open, $f_{LN}(m) \in T_{LN}$ where $m \in M_{LN}$. And since f_{LN} is linguistic neutrosophic continuous, $f_{LN}(LNCl(M_{LN})) \subseteq LNCl(f_{LN}(M_{LN}))$. Hence $f_{LN}(H_{LN})$ be a linguistic neutrosophic semi-open set in T_{LN}.

Theorem 3.4: Let $f_{LN}: (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic continuous and linguistic neutrosophic open mapping and let H_{LN} be a linguistic neutrosophic semi-open set in S_{LN}, then f_{LN} is linguistic neutrosophic semi-irresolute. **Proof:** Let H_{LN} be a linguistic neutrosophic semi-open set in S_{LN}, then there exists a linguistic neutrosophic open set M_{LN} in S_{LN} such that $M_{LN} \subseteq H_{LN} \subseteq LNCl(M_{LN})$. It is true that, $(f_{LN})^{-1}(LNCl(M_{LN})) = LNCl((f_{LN})^{-1}(M_{LN}))$. Also, $(f_{LN})^{-1}(M_{LN}) \subseteq (f_{LN})^{-1}(H_{LN}) \subseteq (f_{LN})^{-1}(LNCl(M_{LN})) = LNCl((f_{LN})^{-1}(M_{LN}))$. Since f_{LN} is linguistic neutrosophic continuous, $(f_{LN})^{-1}(M_{LN})$ is a linguistic neutrosophic open set. Thus, $(f_{LN})^{-1}(H_{LN})$ is a linguistic neutrosophic open set. Hence f_{LN} is linguistic neutrosophic semi-irresolute.

Theorem 3.5: Let $f_{LN}: (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic perfectly semi-continuous function in S_{LN}, then f_{LN} is linguistic neutrosophic semi-irresolute.

Proof: Let K_{LN} be a linguistic neutrosophic semi-open set in (T_{LN}, η_{LN}). Then, $(f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic cl-open set. As a linguistic neutrosophic cl-open set is linguistic neutrosophic semi-open set, $(f_{LN})^{-1}(K_{LN})$ is obviously a linguistic neutrosophic semi-open set. The reverse implication need not be true always, thus demonstrating the validity of the counter example.

Example 3.6: Let the universe of discourse be $U = \{u, v, w, x\}$ and let $S_{LN} = \{w\} = T_{LN}$. The set of all linguistic term set be L$=\{$no healing(l_0), deterioting(l_1), chronic(l_2), some what chronic(l_3), extremely chronic(l_4), very ill(l_5), ill(l_6), no healing(l_7), healing(l_8), slowly healing(l_9), fastly healing$(l_{10})\}$. Let f_{LN} be the mapping from (S_{LN}, τ_{LN}) to (T_{LN}, η_{LN}) defined by $f_{LN}(a) = b, f_{LN}(b) = c, f_{LN}(c) = a$. Now, $\tau_{LN} = \{0_{LN}, 1_{LN}, K_{LN}\}, \eta_{LN} = \{0_{LN}, 1_{LN}, A_{LN}, B_{LN}\}$ where $K_{LN} = \{\langle w, (l_6, l_2, l_3)\rangle\}$ and $A_{LN} = \{\langle w, (l_2, l_6, l_3)\rangle\}, B_{LN} = \{\langle w, (l_5, l_2, l_0)\rangle\}$. In (T_{LN}, η_{LN}), the set of all linguistic neutrosophic semi-open is, $\{0_{LN}, 1_{LN}, A_{LN}\}$. Thus, the map f_{LN} is linguistic neutrosophic semi-irresolute but not linguistic neutrosophic perfectly semi-continuous.

Theorem 3.7: A mapping $f_{LN}:(S_{LN},\tau_{LN}) \to (T_{LN},\eta_{LN})$ is linguistic neutrosophic semi-irresolute then for every linguistic neutrosophic set K_{LN} of S_{LN}, $f_{LN}(LNSCl(K_{LN})) \subseteq LNSCl(f_{LN}(K_{LN}))$.

Proof: For each linguistic neutrosophic set K_{LN} in S_{LN}, $LNSCl(f_{LN}(K_{LN}))$ is linguistic neutrosophic semi-closed set in T_{LN}. As f_{LN} is linguistic neutrosophic semi-irresolute, $(f_{LN})^{-1}(LNSCl(f_{LN}(K_{LN})))$ is linguistic neutrosophic semi-closed set in S_{LN}. Since $K_{LN} \subseteq (f_{LN})^{-1}(LNSCl(f_{LN}(K_{LN})))$, from the definition of linguistic neutrosophic semi-closure, $LNSCl(K_{LN}) \subseteq (f_{LN})^{-1}(LNSCl(f_{LN}(K_{LN})))$.
Obviously, $f_{LN}(LNSCl(K_{LN})) \subseteq f_{LN}((f_{LN})^{-1}(LNSCl(f_{LN}(K_{LN})))) = LNSCl(f_{LN}(K_{LN}))$.

Theorem 3.8: A mapping $f_{LN}:(S_{LN},\tau_{LN}) \to (T_{LN},\eta_{LN})$ is linguistic neutrosophic semi-irresolute if and only if for all K_{LN} in (S_{LN},τ_{LN}), $LNSCl((f_{LN})^{-1}(K_{LN})) \subseteq (f_{LN})^{-1}(LNSCl(K_{LN}))$.

Proof: Necessity Part: Let K_{LN} be a linguistic neutrosophic semi-closed set in T_{LN} and this implies, $(f_{LN})^{-1}(LNSCl(K_{LN}))$ is linguistic neutrosophic semi-closed in S_{LN}. Since $(f_{LN})^{-1}(K_{LN}) \subseteq (f_{LN})^{-1}(LNSCl(K_{LN}))$. And also from the definition of linguistic neutrosophic semi-closure, $LNSCl((f_{LN})^{-1}(K_{LN})) \subseteq (f_{LN})^{-1}(LNSCl(K_{LN}))$.

Sufficiency Part: If K_{LN} is linguistic neutrosophic semi-closed in T_{LN}, then $K_{LN} = LNSCl(K_{LN})$. By hypothesis, $(f_{LN})^{-1}(K_{LN}) \subseteq LNSCl((f_{LN})^{-1}(K_{LN})) \subseteq (f_{LN})^{-1}(LNSCl(K_{LN})) = (f_{LN})^{-1}(K_{LN})$.

IV. COMPOSITION OF LINGUISTIC NEUTROSOPHIC IRRESOLUTE MAPPINGS

Theorem 3.9: Let $f_{LN}:(S_{LN},\tau_{LN}) \to (T_{LN},\eta_{LN})$ and $g_{LN}:(T_{LN},\eta_{LN}) \to (P_{LN},\mu_{LN})$ be linguistic neutrosophic semi-irresolute, then their composition $(g_{LN} \circ f_{LN}):(S_{LN},\tau_{LN}) \to (P_{LN},\mu_{LN})$ is linguistic neutrosophic semi-irresolute.

Proof: Let K_{LN} be a linguistic neutrosophic semi-open set in (P_{LN},μ_{LN}), then $(g_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-open in (T_{LN},η_{LN}) and $(f_{LN})^{-1}((g_{LN})^{-1}(K_{LN}))$ is linguistic neutrosophic semi-open in (S_{LN},τ_{LN}), since f_{LN} and g_{LN} are linguistic neutrosophic semi-irresolute. Therefore, $(f_{LN})^{-1}((g_{LN})^{-1}(K_{LN})) = (g_{LN} \circ f_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-open and hence $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic semi-irresolute.

Theorem 3.10: Let $f_{LN}:(S_{LN},\tau_{LN}) \to (T_{LN},\eta_{LN})$ be linguistic neutrosophic semi-irresolute and $g_{LN}:(T_{LN},\eta_{LN}) \to (P_{LN},\mu_{LN})$ be linguistic neutrosophic semi continuous, then their composition $(g_{LN} \circ f_{LN}):(S_{LN},\tau_{LN}) \to (P_{LN},\mu_{LN})$ is linguistic neutrosophic semi continuous.

Proof: Let U_{LN} be any linguistic neutrosophic semi closed set in (P_{LN},μ_{LN}). Since g_{LN} is linguistic neutrosophic semi continuous, $(g_{LN})^{-1}(U_{LN})$ is linguistic neutrosophic semi closed set in (T_{LN},η_{LN}). Since f_{LN} is linguistic neutrosophic semi irresolute, $(f_{LN})^{-1}(g^{-1}(U_{LN})) = (g_{LN} \circ f_{LN})^{-1}(U_{LN})$ is linguistic neutrosophic semi closed set in (S_{LN},τ_{LN}). Thus, $(g_{LN} \circ f_{LN}):(S_{LN},\tau_{LN}) \to (P_{LN},\mu_{LN})$ is linguistic neutrosophic semi continuous.

Theorem 3.11: Let $f_{LN}:(S_{LN},\tau_{LN}) \to (T_{LN},\eta_{LN})$ and $g_{LN}:(T_{LN},\eta_{LN}) \to (P_{LN},\mu_{LN})$, where (S_{LN},τ_{LN}), (T_{LN},η_{LN}) and (P_{LN},μ_{LN}) are linguistic neutrosophic topological spaces. If f_{LN} is linguistic neutrosophic semi irresolute and g_{LN} is linguistic neutrosophic semi continuous, then $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic semi continuous function.

Proof: Let K_{LN} be a linguistic neutrosophic semi-open set in (P_{LN},μ_{LN}), then $(g_{LN})^{-1}(K_{LN})$ is linguistic neutrosophic semi-open in (T_{LN},η_{LN}) and $(f_{LN})^{-1}((g_{LN})^{-1}(K_{LN}))$ is linguistic neutrosophic semi-open in (S_{LN},τ_{LN}), since f_{LN} is linguistic neutrosophic semi irresolute and g_{LN} is linguistic neutrosophic semi continuous. Thus, $(g_{LN} \circ f_{LN})$ is linguistic neutrosophic semi continuous function.