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On the Integer Solutions of the Homogeneous Biquadratic Diophantine Equation

$$x^4 - y^4 = 26\left(z^2 - w^2\right)p^2$$

G. Janaki[1], M. Krishnaveni[2]
[1]Associate Professor, [2]PG Student, Department of Mathematics Cauvery College for Women (Autonomous), Trichy, Tamil Nadu, India

Abstract:** The homogenous biquadratic Diophantine equation with five unknowns non-zero unique integer solutions* $x^4 - y^4 = 26\left(z^2 - w^2\right)p^2$ ***are found using several techniques. The unusual numbers and the solutions are found to have a few intriguing relationships. The relationships between the solutions' recurrences are also shown.
Keywords: Five unknowns in a homogenous biquadratic equation, Numbers in polygons.

Notations

$T_{12,n} =$ Dodecagonal number of rank n.

$T_{18,n} =$ Octadecagonal number of rank n.

$T_{22,n} =$ Icosidigonal number of rank n.

$T_{26,n} =$ Icosihexagonal number of rank n.

$T_{28,n} =$ Icosioctagonal number of rank n.

$Gno_n =$ Gnomonic number of rank n.

I. INTRODUCTION

The biquadratic equation can be used to represent the quartic equation. These issues can be resolved using the quadratic formula since they can be reduced to quadratic equations [1-5], they are simple to solve. Several mathematicians have developed an interest in biquadratic Diophantine equations, both homogeneous and non-homogeneous. One can refer to [6-11] in the context for a variety of issues involving the two, three, and four variable Diophantine equations.

This communication examines the non-zero unique integer solutions to the biquadratic equation with five unknowns provided by $x^4 - y^4 = 26\left(z^2 - w^2\right)p^2$.Also the recurrence linkages between the solutions are discovered.

II. METHOD OF ANALYSIS

In order to get the non-zero unique integral solution to the homogeneous biquadratic Diophantine problem with five unknowns,

$$x^4 - y^4 = 26\left(z^2 - w^2\right)p^2 \qquad (1)$$

An explanation of the linear transformation

$$x = u+v, y = u-v, z = 2u+v, w = 2u-v \qquad (2)$$

Equation (1) is changed to

$$u^2 + v^2 = 26p^2 \qquad (3)$$

A. Pattern I

Assume

$$26=(5+i)(5-i) \tag{4}$$

and $p=a^2+b^2=(a+ib)(a-ib)$ (5)

Using (4) and (5) in (3) and employing the method of factorization, we get,

$$(u+iv)(u-iv)=(5+i)(5-i)(a+ib)^2(a-ib)^2$$

Equating the like factors, we get,

$$(u+iv)=(5+i)(a+ib)^2$$
$$(u-iv)=(5-i)(a-ib)^2$$

Equating real and imaginary parts, we get,

$$u=5a^2-5b^2-2ab$$
$$v=a^2-b^2+10ab$$

Equation (2) non-zero unique integer solutions are as follows when u and v are substituted:

$$x=x(a,b)=6a^2-6b^2+8ab$$
$$y=y(a,b)=4a^2-4b^2-12ab$$
$$z=z(a,b)=11a^2-11b^2+6ab$$
$$w=w(a,b)=9a^2-9b^2-14ab$$
$$p=p(a,b)=a^2+b^2$$

Properties

1) $2[x(a,1)+y(a,1)-T_{22,a}]-5Gno_a\equiv 0(\mathrm{mod}\,15).$
2) $x(1,1)-y(1,1)+z(1,1)-w(1,1)=40$ is Duck number.
3) $2[w(a,1)-p(a,1)-T_{18,a}]+5Gno_a\equiv 0(\mathrm{mod}\,15).$
4) $z(A,A)=6A^2$ is Nasty number.
5) $2[z(a,1)+p(a,1)-T_{126,a}]+5Gno_a\equiv 0(\mathrm{mod}\,15).$

B. Pattern II

26 can also be expressed as,

$$26=(1+i5)(1-i5) \tag{6}$$

Applying (5) and (6) in (3) and the factorization approach, we obtain,

$$(u+iv)(u-iv)=(1+i5)(1-i5)(a+ib)^2(a-ib)^2$$

Similar to pattern 1, the non- zero distinct integer answers of (1) are

$$x=x(a,b)=6a^2-6b^2-8ab$$
$$y=y(a,b)=-4a^2+4b^2-12ab$$
$$z=z(a,b)=7a^2-7b^2-18ab$$
$$w=w(a,b)=-3a^2+3b^2-22ab$$
$$p=p(a,b)=a^2+b^2$$

Properties:

1) $2\left[x(a,1)+z(a,1)+w(a,1)-T_{22,a}\right]-5Gno_a \equiv 0 \pmod{15}$
2) $x(a,1)+z(a,1)-T_{28,a}+7Gno_a+20=0$
3) $2\left[z(a,1)-w(a,1)-T_{22,a}\right]-5Gno_a \equiv 0 \pmod{15}$
4) $y(A,A)-z(A,A)=6A^2$ is Nasty number.
5) $11\times(-w(1,1))=242$ is Palindrom number.

C. Pattern III

Rewriting equation (3) as

$$1*u^2=26p^2-v^2 \tag{7}$$

Assume

$$u=26a^2-b^2=\left(\sqrt{26}a+b\right)\left(\sqrt{26}a-b\right) \tag{8}$$

Write 1 as,

$$1=\left(\sqrt{26}+5\right)\left(\sqrt{26}-5\right) \tag{9}$$

Using (8) and (9) in (7) and employing the method of factorization, we get,

$$\left(\sqrt{26}+5\right)\left(\sqrt{26}-5\right)\left(\sqrt{26}a+b\right)^2\left(\sqrt{26}a-b\right)^2=\left(\sqrt{26}p+v\right)\left(\sqrt{26}p-v\right)$$

Equating the like factors, we get,

$$\left(\sqrt{26}+5\right)\left(\sqrt{26}a+b\right)^2=\left(\sqrt{26}p+v\right)$$

$$\left(\sqrt{26}-5\right)\left(\sqrt{26}a-b\right)^2=\left(\sqrt{26}p-v\right)$$

Equating rational and irrational parts, we get,

$$p=26a^2+b^2+10ab$$

$$v=130a^2+5b^2+52ab$$

Equation (2) non-zero unique integer solutions are as follows when u and v are substituted:

$$x=x(a,b)=156a^2+4b^2+52ab$$

$$y=y(a,b)=-10a^2-6b^2-52ab$$

$$z=z(a,b)=182a^2+3b^2+52ab$$

$$w=w(a,b)=-78a^2-7b^2-52ab$$

$$p=26a^2+b^2+10ab$$

Properties:

1) $x(1,b)+z(1,b)+p(1,b)+y(1,b)2T_{4,b}-31Gno_a \equiv 0 \pmod{291}$
2) $x(1,1)=212$ is palindrom number.
3) $x(1,1)+y(1,1)=50$ is Duck number.
4) $w(1,1)-y(1,1)=25$ is Square number.
5) For all values of a and b, $x+y$ is divisible by 2

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D. Pattern IV

Rewriting equation (3) as,

$$1 * v^2 = 26p^2 - u^2 \tag{10}$$

Write1 as,

$$1 = \frac{\left(\sqrt{26}+1\right)\left(\sqrt{26}-1\right)}{25} \tag{11}$$

Assume

$$v^2 = 26a^2 - b^2 = \left(\sqrt{26}a+b\right)\left(\sqrt{26}a+b\right) \tag{12}$$

Applying (11) and (12) in (10) and the factorization approach, we obtain,

$$\frac{\left(\sqrt{26}+1\right)\left(\sqrt{26}-1\right)}{25}\left(\sqrt{26}a+b\right)^2\left(\sqrt{26}a-b\right)^2 = \left(\sqrt{26}p+v\right)\left(\sqrt{26}p-v\right)$$

Equating the like factorization, we get,

$$\frac{\left(\sqrt{26}+1\right)}{5}\left(\sqrt{26}a+b\right)^2 = \left(\sqrt{26}p+v\right)$$

$$\frac{\left(\sqrt{26}-1\right)}{5}\left(\sqrt{26}a-b\right)^2 = \left(\sqrt{26}p-v\right)$$

By equating the rational and irrational components, we obtain,

$$p = \frac{1}{5}\left(26a^2 + b^2 + 2ab\right) \tag{13}$$

$$u = \frac{1}{5}\left(26a^2 + b^2 + 52ab\right) \tag{13}$$

In order to discover only integer solutions, we must substitute $a = 5A$ and $b = 5B$ in equations (12) and (13).

$$u = 130A^2 + 5B^2 + 260AB$$

$$v = 650A^2 - 25B^2$$

Equation (2) non-zero unique integer solutions are as follows when u and v are substituted:

$$x = x(A,B) = 780A^2 - 20B^2 + 260AB$$

$$y = y(A,B) = -520A^2 + 30B^2 + 260AB$$

$$z = z(A,B) = 910A^2 - 15B^2 + 520AB$$

$$w = w(A,B) = -390A^2 + 35B^2 + 520AB$$

$$p = 130A^2 + 5B^2 + 10AB$$

Properties:

1) $x(1,1) + y(1,1) = 790$ is Duck number.
2) $p(1,1) - 1 = 144$ is Square number.
3) $p(1,a) - w(1,a) - x(1,a) + 2T_{12,a} + 389Gno_a \equiv 0 \pmod{649}$
4) $x(1,1) - w(1,1) + y(1,1) = 625$ is Square number.

E. Pattern V

Equation (3) can be written as,

$$u^2 - p^2 = 25p^2 - v^2$$

$$(u+p)(u-p) = (5p+v)(5p-v) \tag{14}$$

Which is represented in the form of ratio as,

$$\frac{u+p}{5p+v} = \frac{5p-v}{u-p} = \frac{A}{B}, B \neq 0$$

This is equivalent to the following two equations

$$Bu + (B-5A)p - Av = 0$$

$$-Au + (A+5B)p - Bv = 0$$

Solving the above equation by cross ratio method, we get,

$$u = A^2 - B^2 + 10AB$$

$$v = -5A^2 + 5B^2 + 2AB$$

$$p = A^2 + B^2$$

Substituting u and v in equation (2), the non – zero distinct integer solutions are

$$x = x(A,B) = -4A^2 + 4B^2 + 12AB$$

$$y = y(A,B) = 6A^2 - 6B^2 + 8AB$$

$$z = z(A,B) = -3A^2 + 3B^2 + 22AB$$

$$w = w(A,B) = 7A^2 - 7B^2 + 18AB$$

$$p = A^2 + B^2$$

Properties

1) $2[y(a,1) + w(a,1) - T_{28,a}] - 11Gno_a \equiv 0 \pmod{15}$
2) $p(1,1) + w(1,1) = 20$ is Duck number
3) For all values of A and B, $x + y$ is divisible by 2.
4) $2[y(a,1) + x(a,1) - T_{22,a}] - 5Gno_a \equiv 0 \pmod{15}$
5) $x(1,1) + y(1,1) + z(1,1) + w(1,1) + 2p(1,1) = 64$ is Cube number.

III. CONCLUSION

Other non-zero unique integer solutions to the multivariable biquadratic equations under consideration may be sought after.

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