



IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 10 Issue: VII Month of publication: July 2022

DOI: https://doi.org/10.22214/ijraset.2022.45544

www.ijraset.com

Call: 🕥 08813907089 🔰 E-mail ID: ijraset@gmail.com



Order Statistics of Doubly Truncated Additive Uniform Exponential Distribution

Uppu Venkata Subbarao¹, K. Srinivasa Rao², T. Narayana Rao³

¹Professor, Department of sciences &Humanities N.S.Raju Institute of Technology, Visakhapatnam, India ²Professor, Department of statistics, Andhra University, Visakhapatnam, India ³Assistant Professor, Department of management studies, N.S.Raju Institute of Technology, Visakhapatnam, India

Abstract: In this paper we truncated the Additive Uniform Exponential Distribution (AUED) proposed by Venkata Subbarao Uppu (2010). The probability density functions of r^{th} order Statistics, l^{th} moment of the r^{th} order Statistic, minimum, maximum order statistics, mean of the maximum and minimum order statistics, the joint density function of two order statistics of the truncated distribution were calculated and discussed in detailed.

Keywords: Additive Uniform Exponential Distribution, Truncation, moments, minimum order statistic, Maximum order statistic, joint density of the order Statistics, complete length of service

I. INTRODUCTION

To introduce the phenomenon of converting an infinite range into a finite range of the random variate we utilize the concept of truncation .The range of the Additive Uniform Exponential distribution is $(0, \infty)$. But in many practical situations arising at places like quality control, agricultural experiments, reliability studies, etc., the variate under study will have a finite range. For example in manpower modeling the complete length of service of an employee will have a finite range. Similarly in other areas like the incontrol times of processes, productions will have minimum and maximum limits. So, for these sorts of situations the random variable under study is to be considered as doubly truncated. Hence, in this paper we develop and analyze a Doubly Truncated Additive Uniform Exponential Distribution.

We discuss the probability density function of Doubly Truncated Additive Uniform Exponential Distribution as follows: The probability density function of additive uniform exponential distribution (AUED) is

$$g_{X}(x) = \frac{1}{a} \left[1 - e^{-\theta x} \right]; 0 \le X \le a$$

= $\frac{e^{-\theta x}}{a} \left[e^{a\theta} - 1 \right]; a \le X < \infty$ (1)
'a' and '\theta' are the parameters of the distribution, $a > 0$ and $\theta > 0$

Consider that the range of the random variable is finite say (L,U). Then the probability density function of the Doubly Truncated Additive Uniform Exponential Distribution (DTAUED) is

$$f_X(x) = \frac{g_X(x)}{G(U) - G(L)} \qquad L \le X \le U$$
(2)

Where $g_X(x)$ is given in equation (1) And $G(U) - G(L) = \int_L^a f(x) dx + \int_a^U f(x) dx = \int_L^a \frac{1}{a} [1 - e^{-\theta x}] dx + \int_a^U \frac{e^{-\theta x}}{a} [e^{a\theta} - 1] dx$ On simplification $G(U) - G(L) = \frac{1}{a} [(a - L) + \frac{1}{\theta} (1 - e^{-L\theta} + e^{-U\theta} - e^{\theta(a - U)}]$ Let the value of G(U) - G(L) = A

Therefore the probability density function of Doubly Truncated Additive Uniform Exponential distribution (DTAUED) is

$$f_X(x) = \frac{1}{A} [1 - e^{-\theta x}]; \quad L \le X \le a$$
$$= \frac{e^{-\theta x}}{A} [e^{a\theta} - 1]; \quad a \le X < U$$
(3)



International Journal for Research in Applied Science & Engineering Technology (IJRASET) ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538 Volume 10 Issue VII July 2022- Available at www.ijraset.com

The lower and upper truncation points are L and U respectively. The degrees of truncation are G(L) and 1 - G(U). If L is replaced by 0 or U is replaced by ∞ the distribution is singly truncated from below or above respectively. Distribution function is defined as

$$F_X(x) = \int_L^x \frac{1}{A} \left[1 - e^{-\theta t} \right] dt = \frac{1}{A} \left[(x - L) + \frac{1}{\theta} \left(e^{-\theta x} - e^{-\theta L} \right) \right] \text{ for } L \le X \le a$$
(4)

$$F_{X}(x) = \int_{L}^{a} f(t)dt + \int_{a}^{x} f(t)dt = \int_{L}^{a} \frac{1}{A} [1 - e^{-\theta t}] dt + \int_{a}^{x} \frac{e^{-\theta t}}{A} [e^{a\theta} - 1] dt$$
$$= \frac{1}{A} [(a - L) + \frac{1}{\theta} [1 + e^{-\theta x} - e^{-L\theta} - e^{(a - x)\theta}]] \text{ ;for } a \le X \le U$$
(5)

II. ORDER STATISTICS OF DOUBLY TRUNCATED ADDITIVE UNIFORM EXPONENTIAL DISTRIBUTION

We derived the distribution of extreme order statistics and the joint distribution of the order statistics and some properties. The distribution of extreme order statistics are very important for studying the inferences related to the maximum, minimum and median of the data sets. For example in manpower modeling the complete length of service of an employee is a random variable and studying its dynamics is very important for several operating policies regarding welfare and pensionable benefits to find the probability distribution of the maximum duration of a state of an employee in an organization can be derived through order statistics.

Let $X_{1:n} \leq X_{2:n} \leq X_{3:n} \dots \leq X_{m:n}$ be the order statistics obtained from a random sample of size n from DTAUED having the

density function as given in (3)

The probability density function of the rth order statistics is given by (David 1981) is

$$f_{r:n}(u) = \mathsf{D}_{r:n}[F(u)]^{r-1}[1 - F(u)]^{n-r}f(u); \ -\infty < u < \infty$$
(6)
Where $D_{r:n} = \frac{n!}{(-1)!(n-1)!}$
(7)

Where
$$D_{r:n} = \frac{1}{(r-1)!(n-r)!}$$

Substituting the values from equations (3) (4) and (5)

$$f_{r:n}(x) = \frac{D_{r:n}}{A^n} \left[(\theta(x-L) + (e^{-\theta x} - e^{-L\theta}) \right]^{r-1} \left[(A-x+L)\theta - (e^{-\theta x} - e^{-L\theta}) \right]^{n-r} (1-e^{-\theta u})$$

For $L \le X \le a$ (8)
$$f_{r:n}(u) = \frac{D_{r:n}}{A^n \theta^{n-1}} \left[\theta(a-L) + 1 + e^{-\theta x} - e^{-L\theta} - e^{\theta(a-x)}) \right]^{r-1} \left[\theta(A-a+L) \right]^{n-r} (1-e^{-\theta x})$$
(9)

For $a \leq X < U$

Where $D_{r:n}$ is given in equation (7) The l^{th} moment of the rth order statistic is

$$E(X^{l}_{r:n}) = \int_{L}^{a} x^{l} \frac{\mathsf{D}_{r:n}}{A^{n}} \left[(\theta(x-L) + (e^{-\theta x} - e^{-L\theta}) \right]^{r-1} \left[(A-x+L)\theta - (e^{-\theta x} - e^{-L\theta}) \right]^{n-r} \\ \left(1 - e^{-\theta u}\right) d + \frac{\mathsf{D}_{r:n}}{A^{n}\theta^{n-1}} \int_{a}^{U} u^{l} \left[\theta(a-L) + 1 + e^{-\theta x} - e^{-L\theta} - e^{\theta(a-x)} \right]^{r-1} \left[\theta(A-a+L) \right]^{n-r} (1 - e^{-\theta x}) dx$$

And on simplification

$$\sum_{s=0}^{n-r} \sum_{k=0}^{r-1} \sum_{m=0}^{n-s-k-1} \sum_{j=0}^{s} \sum_{i=0}^{k} \sum_{q=1}^{\infty} \sum_{h=0}^{\infty} \frac{\mathsf{D}_{r:n}}{A^n \theta^{n-1}} \binom{n-1}{s} \binom{r-1}{k} \binom{n-s-k}{m} \binom{s}{j} \binom{k}{i} \frac{\theta^{q+h}}{q!h!}$$

$$\frac{(-1)^{2n-r-s-m-i+q+h-1}e^{-a\theta(n-s-k-m-1)}}{(l+s-j+k+q+h+1)}[a^{l+s-j+k+q+h+1} - L^{l+s-j+k+q+h+1}] + \frac{D_{r:n}}{A^{n}\theta^{n-1}}\sum_{s=0}^{r-1}\sum_{k=0}^{n-r}\sum_{m=0}^{\infty} \binom{n-r}{k}\binom{r-1}{s}[\theta(a-L) + 1 - e^{-L\theta}]^{r-1}[\theta(A-a+L) + e^{-L\theta}]^{k-1}$$



International Journal for Research in Applied Science & Engineering Technology (IJRASET) ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538

Volume 10 Issue VII July 2022- Available at www.ijraset.com

$$(e^{a\theta} - 1)^{r-s} (-1)^{r+m-s-1} (1 + e^{a\theta})^{n-r-k-1}$$
(10)

The probability density function of the first order statistic of DTAUED can be obtained by taking r=1 in equations (5) and (6)

For
$$L \le X \le a$$

$$f_{1:n}(\mathbf{x}) = \frac{n}{A^n \theta^{n-1}} \left[\theta (A - \mathbf{x} + L) + e^{-\theta \mathbf{x}} + e^{-L\theta} \right]^{n-1} (1 - e^{-\theta \mathbf{x}})$$
(11)

For
$$a \le X \le U$$

 $f_{1:n}(u) = \frac{n}{A^n \theta^{n-1}} \left[\theta(A - a + L) - 1 - e^{-\theta x} + e^{-L\theta} + e^{\theta(a-x)} \right]^{n-1} (e^{a\theta} - 1)e^{-\theta x}$ (12)

Where $D_{1:n}$ is given in equation (7)

The mean of the first order statistic is given by

$$E(X_{(1)}) = \int_{a}^{L} x \frac{n}{A^{n} \theta^{n-1}} \left[\theta(A - x + L) + e^{-\theta x} + e^{-L\theta} \right]^{n-1} (1 - e^{-\theta x}) dx$$
$$+ \int_{a}^{U} \frac{n}{A^{n} \theta^{n-1}} \left[\theta(A - a + L) - 1 - e^{-\theta x} + e^{-L\theta} + e^{\theta(a-x)} \right]^{n-1} (e^{a\theta} - 1)e^{-\theta x} dx$$

On simplification we get

 $E(X_{(1)}) = \frac{1}{A^{n}\theta^{n-1}}\sum_{r=0}^{n-1}\sum_{s=1}^{\infty}\sum_{m=0}^{n-r-1}\sum_{k=0}^{\infty} \binom{n-1}{r}\binom{n-r-1}{m}[\theta(A+L) + e^{-L\theta}]^{r}\frac{\theta^{s+m+k}(-1)^{n-r+s+k}}{s!}\left[\frac{a^{m+s+k+2}-L^{m+s+k+2}}{m+s+k+2}\right] + \frac{1}{A^{n}\theta^{n-1}}\sum_{r=0}^{n-1}\binom{n-1}{r}[\theta(A-a+L-1) + e^{-L\theta}]^{r}(e^{a\theta}-1)^{n-r}\left[\frac{Ue^{-\theta U(n-r)}-ae^{-a\theta(n-r)}}{(r-n)\theta} - \frac{e^{-\theta U(n-r)}+e^{-a\theta(n-r)}}{(r-n)^{2}\theta^{2}}\right]$ (13)

The probability density function of the highest order statistic of AUED can be obtained by taking r = n in equations (5) and (6) $f_{n:n}(X) = \frac{n}{A^n \theta^{n-1}} \left[(x - L) + e^{-\theta x} - e^{-L\theta} \right]^{n-1} (1 - e^{-\theta x}) \quad \text{; For } L \le X \le a \quad (14)$ And

$$f_{n:n}(X) = \frac{n}{A^n \theta^{n-1}} \left[a\theta + e^{-\theta u} (1 - e^{a\theta}) \right]^{n-1} e^{-\theta u} (e^{a\theta} - 1); \text{ For } a \le X < U$$
(15)
Where $D_{n:n} = n$ calculated by using equation (7)

The mean of the highest order statistic is given by

 $E(U_{(n)}) =$

$$\int_{L}^{a} \frac{n}{A^{n} \theta^{n-1}} \left[(x-L) + e^{-\theta x} - e^{-L\theta} \right]^{n-1} (1-e^{-\theta x}) \quad dx + \int_{a}^{U} \frac{n}{A^{n} \theta^{n-1}} \left[a\theta + e^{-\theta u} (1-e^{a\theta}) \right]^{n-1} e^{-\theta u} (e^{a\theta} - 1) \, dx$$

And on simplification we get

$$\frac{n}{A^{n}\theta^{n-1}} \sum_{r=0}^{n-1} \sum_{s=0}^{n-r-1} \sum_{m=1}^{\infty} \sum_{k=0}^{\infty} \binom{n-1}{r} \binom{n-r-1}{s} (-1)^{r+m+k} \frac{(\theta s)^{k+m}}{k! \, m!} \left[\frac{a^{m+k+n-r-s+1} - L^{m+k+n-r-s+1}}{m+k+n-r-s+1} \right] \\ + \frac{n(e^{a\theta} - 1)}{A^{n}\theta^{n-1}} \sum_{r=0}^{n-1} (-1)^{n-r-1} (1 - e^{a\theta})^{n-r-1} [\theta(a-L) + 1 - e^{-L\theta}]^{r} \\ \left[\frac{ae^{-a\theta(n-r)} - Ue^{-\theta U(n-r)}}{(n-r)\theta} - \frac{e^{-\theta U(n-r)} + e^{-a\theta(n-r)}}{(r-n)^{2\theta^{2}}} \right]$$
(16)
The joint density function of the order statistics $U_{m:n}$ and $U_{s:n}$ (m< S) is given by (David 1981) $f(u, v)_{m,s:n} = D_{m,s:n} [F(U)]^{m-1} [F(V) - F(U)]^{s-m-1} [1 - F(V)]^{n-s} f(u) f(v)$ (17)

Where
$$D_{r:n} = \frac{n!}{(r-1)!(n-r)!}$$



International Journal for Research in Applied Science & Engineering Technology (IJRASET) ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538

Volume 10 Issue VII July 2022- Available at www.ijraset.com

Here ,We have three cases based up on the sample space of the variates they are

- (i) $L \leq X_m < X_s \leq a$
- (ii) $L \leq X_m \leq a; a \leq X_s < U$
- (iii) $a \leq U_m < U_s < U$

For case (i) the joint probability density function of the order statistics $X_{m:n}$ and $X_{s:n}$ is

For $L \leq X_m < X_s \leq a$

$$f(X,Y)_{m,s:n} = \frac{D_{m,s:n}}{A^n \theta^{n-2}} \left[\theta(x-L) + e^{-\theta x} - e^{-\theta L} \right]^{m-1} \left[(y-x)\theta + e^{-\theta y} - e^{-\theta x} \right]^{s-m-1} \left[(A-y+L)\theta - e^{-\theta y} + e^{-\theta L} \right]^{n-s} (1-e^{-\theta x}) (1-e^{-\theta y})$$

$$(18)$$
Where $D_{m,s:n}$ is given in equation (4)

For case (ii) For $L \le X_m \le a$; $a \le X_s < U$

$$f(X,Y)_{m,s:n} = \frac{D_{m,s:n}}{A^{n}\theta^{n-2}} \left[\theta(x-L) + e^{-\theta x} - e^{-\theta L}\right]^{m-1} \left[\theta(a-x) + e^{-\theta y} - e^{-\theta x} - e^{-\theta(a-y)} + 1\right]^{s-m-1} \left[(A-a+L)\theta + e^{-\theta y} - e^{-\theta L} - e^{-\theta(a-y)} + 1\right]^{n-s} (1-e^{-\theta x})e^{-\theta x} (e^{a\theta} - 1)$$
(19)

Where $D_{m,s:n}$ is given in equation (4) For case (iii) $a \le U_m < U_s < U$

$$f(X,Y)_{m,s:n} = \frac{D_{m,s:n}}{A^{n}\theta^{n-2}} \Big[\theta(a-L) + e^{-\theta x} - e^{-\theta L} - e^{\theta(a-x)} + 1\Big]^{m-1} \Big[\Big(e^{-\theta y} - e^{-\theta x} - e^{-\theta(a-y)} + e^{\theta(a-x)}\Big) \Big]^{s-m-1} \Big[A - \theta(a-L) + e^{-\theta y} - e^{-\theta L} - e^{\theta(a-y)} - 1\Big]^{n-s} e^{-2\theta x} (e^{a\theta} - 1)^2$$
(20)

Where $D_{m,s:n}$ is given in equation (4)

The joint density function of the smallest and highest order statistics can be obtained by taking m=1 and S = n in the equations (15),(16),(17)

Therefore the joint probability density function of the extreme order statistics X_{1:n} and X_{n:n}
Then for case (i)

$$f(X,Y)_{1,n:n} = \frac{D_{1,n:n}}{A^{n}\theta^{n-2}} [(y-x)\theta + (e^{-\theta y} - e^{-\theta x}]^{n-2}(1 - e^{-\theta x})(1 - e^{-\theta y})$$
(21)
For case (ii)

$$f(X,Y)_{1,n:n} = \frac{D_{1,n:n}}{A^{n}\theta^{n-2}} [(a-x)\theta + 1 + e^{-\theta y} - e^{-\theta x} - e^{(a-y)\theta}]^{n-2}(1 - e^{-\theta x})e^{-\theta x}(e^{a\theta} - 1)$$
(22)
For case (iii)

$$f(X,Y)_{1,n:n} = \frac{D_{1,n:n}}{A^{n}\theta^{n-2}} \left[e^{-\theta y} - e^{-\theta x} - e^{(a-y)\theta} + e^{(a-x)\theta} \right]^{n-2} e^{-2\theta x} (e^{a\theta} - 1)^2$$
(23)

III. CONCLUSIONS

The above order statistics are very useful in manpower planning models, especially the minimum and maximum order statistics are used to calculate the pensionable benefits of an employee in an organization by treating the complete length of service as a random variable which is additive in nature. Order statistics were employed in many ways in acceptance sampling. First-order statistics are used to improve the robustness of sampling plans by variables. In life testing, these are much useful to shorten testing times to produce many lifetime distributions. In actuarial sciences, these have tremendous potential in joint life insurance aspects to calculate the distribution of life span and insurance risk. Order statistics are concerned with the ranks as well as the magnitude of the observations we can use them in the grouping of continuous data into frequency classification.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)



ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538

Volume 10 Issue VII July 2022- Available at www.ijraset.com

REFERENCES

- [1] BARRY, C.A; BALAKRISHNAN.N; NAGARAJA,H.N(2008), "A first course in order Statistics "Society for Industrial and Applied mathematics
- [2] DAVID, H.A; NAGARAJA,H.N(2003), "Order Statistics "third edition, Wiley –Interscience ,A John Wiley & Sons, Inc
- [3] M.P. RAMAYYA (2005), "Contributions on merged uniform exponential distribution" Ph.D. thesis: Andhra University. India
- [4] PRAKASH V.V.S. (1997), "Some manpower Models in discrete time with truncated and Compound distributions", Ph.D. Thesis, Andhra University, Visakhapatnam, India
- [5] RANGA RAO, V. (1994). "Some truncated distributions with applications to manpower modeling", Ph.D. Thesis, Andhra University, Visakhapatnam, India.
- [6] SUBBARAO,U. V. (2010). "Studies on Additive Uniform Exponential distribution with applications to manpower models", Ph.D. Thesis, Andhra University, Visakhapatnam, India
- [7] SILCOK, H (1954), "The Phenomenon of the labor turnover", J.R. Statist. Soc. A. 117, No. 9, pp. 429-440.
- [8] VENKATA SUBBARAO, U.; SRINIVASARAO, K. Additive Uniform Exponential Distribution. Res. J. Sci. Manag. 2015, 05, 26–30.
- [9] VENKATA SUBBARAO, U.; NARAYANARAO, T. Order statistics of Additive Uniform Exponential Distribution. Int J. for Research in Applied Sci & Engineering Technology; Volume 9 Issue X Oct 021, 05, 1084-1087.
- [10] VENKATA SUBBARAO, U.; SRINIVASARAO, K. Doubly Truncated Additive Uniform Exponential Distribution. Int.J. of Multidisciplinary .review. 2021, 05, 199–208.











45.98



IMPACT FACTOR: 7.129







INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 🕓 (24*7 Support on Whatsapp)