iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH
IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 10 Issue: VII Month of publication: July 2022

DOI: https://doi.org/10.22214/ijraset.2022.45989

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 10 Issue VII July 2022- Available at www.ijraset.com

A Study on Ordering Policy with Time-Varying Demand and Holding Cost for Deteriorating Items with Shortages

Dr. Mamta Sharma[1], Dr. Raveendra Babu A[2]
[1, 2]Department of Mathematics, Prestige Institute of Management and Research, Gwalior – 474 020, M.P., INDIA

***Abstract:** The present paper focuses on time varying demand and holding cost, taking the same into consideration a finite horizon inventory problem for a deteriorating item has been developed for study. Deterioration rate is considered as a linearly time-dependent. The demand rate varies with the time until the shortage occurs, but during the tenure of shortages it becomes static i.e. constant. Shortages are considered to be a partially backlogged in this study. In the study it is also considered that the backlogging rate is inversely proportionate to the length of the waiting time for next recovery. The main objective of the study is to minimize to the total cost of inventory with optimal order quantities of the products in the system. The graphs in the study show the convexity of the cost function. Finally, numerical results are discussed and presented in the section 5 of the study to analyze the degree of sensitivity of the optimal policies with respect to variations in the economic parameters of the model.*
***Keywords:** Inventory model, deteriorating items, time dependent demand, shortages.*

I. INTRODUCTION

There are four forms of demand in the literature on inventory models namely, time-dependent, stock dependent, constant demand, time and stock dependent and probabilistic demand. The majority of inventory models are predicated on the premise that an item's demand remains constant over the planning horizon. In reality, however the product demand cannot be fixed over time. It is dependent on inventory levels, selling prices and in some cases, time.

Many scholars have been working on inventory models for decaying items in recent years. Deterioration of objects has become a typical occurrence in everyday life.

Many things, including fruits, vegetables, pharmaceuticals, volatile liquids, blood banks, high-tech products, and others, degrade with time due to evaporation, spoilage, obsolescence, and other factors. Inventory systems suffer from shortages as a result of deterioration, as well as a loss of goodwill or profit. As a result, degradation must be factored into inventory control models. Deteriorating products are seasonal things such as warm clothing, dairy products, green veggies, and so on. Shortages in the inventory system occur as a result of deterioration, affecting total inventory costs as well as total profit. As a result, another important component in the analysis of declining things inventory is the degrading rate, which indicates the nature of the items' deterioration.

The study of deterministic inventory models is having a finite planning horizon, time-varying demand, and holding cost are worth noting.

The rate of deterioration is considered to be linear in time. It is permissible to have a shortage that is partially backlogged. The main aim is to build an appropriate inventory models to non-instantaneous deteriorating items over a finite planning horizon is the goal of this research.

Shortages and partial backlogs are allowed under the paradigm. The backlog rate varies depending on how long it takes for the next replenishment to arrive. The main goal is to calculate the best total cost and order quantity at the same time. We have demonstrated some numerical examples are presented. The sensitivity analysis is used to investigate the consequences of changing parameters. The rest of our paper is laid out as follows: The literature on time-varying demand, holding cost, deterioration rate, and backorder is reviewed in Section II. The assumptions and notation used throughout the paper are described in Section III.

In Section IV, we established the main mathematical model. In Section V, focused on the solution to obtain total inventory cost and order quantity.

We have illustrated some numerical examples with sensitivity analysis and observations. Finally, we conclude in the section VI with the future work prospects.

II. LITERATURE REVIEW

The first economic order quantity (EOQ) model was initiated by Harris (1915) for constant and known demand. Later on many researchers considered the variable demand such as, constant demand, time-dependent, stock dependent, price dependent and probabilistic demand. Further, some researchers were worked on inventory with deteriorating items. Initially 'A model for exponentially decaying inventory 'was proposed by Ghare. Nahmias (1975) suggested a perishable inventory model with constant demand for optimum ordering. Later Abad P (2001), Chung K.J. and Huwang Y.F. (2009), Shah N.H. AND Shukla (2009) Inventory models with time dependent demand rate were considered by Sivazlian and Stanfel (1976), Chung and Ting (1993) for replenishment of deteriorating items.

Due to shortages, the backlogging happens for which several researchers considered partial backlogging while some assumed fully backlogged inventory models. In practice, it is observed that at the time of the shortages either consumer wait for the arrival of next order (entirely backlogged) or they leave the system (entirely lacked). However, it is sensible to consider that, some consumers can wait to fulfill their demand during the stock- out period (a case of full back order), whereas others don't wish to wait and fulfill their demand from another sources (a case of partial back order).

Backlog shortages that occur will depend on the length of waiting time for the replenishment and is therefore the main factor for determining whether the backlogging will be accepted or not. For commodities with short life cycles like fashionable and high-tech products, the rate of backorder is declining with the length of waiting time. The longer the waiting time is lower the backlogging rate, this results to a greater fraction of loss sales and hence a less profit as a consequence taking into the consideration of partial backlogging is necessary. Customers who have experienced stock-out will be less likely to buy again from the suppliers, they might turn to another store to purchase the goods. The sales for the product might decline due to the introduction of more competitive product or the change in consumers' preferences. The longer the waiting time, the lower the backlogging rate is. This leads to a larger fraction of lost sales and a less profit. As a result, taking the factor of partial backlogging into account is necessary.

Several inventory models under the conditions of perishability and partial backordering for constant demand for optimal ordering policy were studied by Abad, P. (2001), Shah and Shukla (2009). Chung and Huang (2009), Hu and Liu (2010) developed optimal replenishment policy for deteriorating items with constant demand under condition of permissible delay in payment. An inventory model with time proportional demand for deteriorating items were developed by Dave & Patel (1981), Chung and Ting (1993), Chang and Dye (1999), Ouyang et al. (2005) with exponential declining demand and partial backlogging. Teng et al. (2007) studied the comparison between two pricing and lot-sizing models with partial backlogging and deteriorating items. Alamri and Balkhi (2007) proposed a model for the effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand.

Some inventory models with ramp type demand rate, partial backlogging and weibull distributed deteriorating items are proposed by Jalan et al. (1996), Skouri et al. (2009), Mandal (2010), Hung (2011), Sana (2010). Kumar et al. (2012) proposed EOQ model with time dependent deterioration rate under fuzzy environment for ramp type demand and partial backlogging. In real-life inventory model, the holding cost is also time dependant. Mishra (2014), Dutta and Kumar (2015), karmakar (2016), Chandra (2017) developed an inventory model for deteriorating items with time dependent demand and the time varying holding cost. Pervin et al. (2018) developed an EOQ model for time-varying holding cost including stochastic deterioration.

Hou (2006) developed an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting Roy et al. (2011a, 2011b) proposed inventory models for imperfect items in a stock-out situation with partial backlogging. Choudhary et al. (2015) developed inventory model for deteriorating items with stock-dependent demand, time-varying holding cost and shortages. Pervin et al. 2017) developed two-echelon inventory model with stock-dependent demand and variable holding cost for deteriorating items.

Abad (1996) developed an EOQ model under conditions of perishability and partial backordering with price dependent demand. Dye (2007) developed an EOQ model with a varying rate of deterioration and exponential partial backlogging with price-sensitive demand. Roy (2008) developed an inventory model for deteriorating items with price dependent demand and time varying holding cost. Sana (2010) proposed an EOQ model with time varying deterioration and partial backlogging. Shah et al. (2012) proposed an inventory model for optimisation and marketing policy for non-instantaneous deteriorating items with generalised type deterioration and holding cost rates. Rastogi et al. (2017) developed an EOQ model with variable holding cost and partial backlogging under credit limit policy and cash discount. Sharma et al. (2018) developed an inventory model for deteriorating items with expiry date and time varying holding.

Some products like fashionable and seasonal products are characterized by unpredictable demand.

The demand of such items is low at the beginning of the season and increases as the season progresses, i.e., it changes with time, especially seasonal products like seasonable fruits, garments, shoes, etc. Most of these products have a lifespan of very short window in which to make the right calls on demand and procurement and achieve profits. There are three fundamental questions to answer such situations. How do sellers plan at firms selling such products for a season? How do they decide how much to order, when to schedule shipments and how much to mark down prices to reduce season-ending inventory as much as possible?

In this paper, we develop a deteriorating inventory model with the time dependent demand rate and deterioration rate. The unsatisfied demands will be partially backlogged. The backlogging rate is a variable and is inversely proportional to the length of the waiting time for next replenishment. So our aim is to minimize the total cost of inventory with optimal order quantity and optimal time of inventory exhausting. This paper is organized as follows: Section 2 describes the notations and assumptions, Section 3 presents the mathematical formulation of the problem, Section 4 provides a numerical example to illustrate the proposed inventory model, Section 5 discuss the sensitivity analysis, and finally Section 6 proposes the conclusions.

A. Notations

c_h Holding cost per unit per unit time
c_p Purchase cost per unit item
c_o Ordering cost per unit item
c_s Shortage cost per unit per unit time
c_l Cost of lost sales per unit
θ Deterioration rate
T Length of the cycle (Decision Variable)
t_1 Time at which shortage starts, i.e., inventory exhausted time (decision variable), $0 \leq t \leq t_1$
$T - t_1$ Length of waiting time
W Maximum inventory level during a cycle of length T
D_B Maximum amount of demand backlogged during a cycle of length T
W $(= Q + D_B)$ order quantity during a cycle of length T
C_H Inventory holding cost per cycle
C_D Deterioration cost per cycle
C_S Shortage cost per cycle
C_L Lost sales cost per cycle
TC Average total cost per unit time per cycle
T* Optimal value of T, where T is any variable
$Q(T)$ Inventory level at time, $0 \leq t \leq t_1$
$Q_1(t)$ Inventory level at time, $t \epsilon [0, t_1]$
$Q_2(t)$ Inventory level at time, $t \epsilon [t_{1,} T]$

B. Assumptions

1) Demand rate, $D(t) = \begin{cases} R_1 + \beta t, & for\ 0 \leq t \leq t_1 \\ R_{2,} & for\ t_1 \leq t \leq T \end{cases}$, where R_1, R_2 and $\beta > 0$ are arbitrary constants.
2) Inventory system contains single item.
3) Considered the time horizon of the inventory system is finite.
4) Lead time to be zero.
5) Replenishment rate is infinite but size is finite.
6) Rate of Deterioration is depending on time and there is no replacement of deteriorated units.
7) For the time- interval $t_1 \leq t \leq T$, the shortage is allowed and is partially backlogged with backordering rate:
$$\boldsymbol{\gamma(t) = \frac{1}{1+\delta(T-t)}}$$
The backlogging rate $\boldsymbol{\delta}$ is a positive constant. $\boldsymbol{\delta = 0}$, $\boldsymbol{\gamma(t) = 1}$ is the special case i.e. the fully backlogged case. In the projected model, we suppose δ < 1 for the approximation by Taylor's Series.
8) Holding cost is linear function of time: $c_h(t) = \propto_0 + \propto_1 t$ where $\propto_0, \propto_1 > 0$ are the scale parameters of holding cost.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 10 Issue VII July 2022- Available at www.ijraset.com

III. FORMULATION OF INVENTORY MODEL

The objective of the model is to determine the optimal order quantity in order to keep the total relevant cost as low as possible. The inventory is replenished at time t = 0, when the inventory level is at its maximum W. Now, because of both the demand and deterioration of the item, the inventory level begins to decrease during the period $[0, t_1]$, and finally becomes zero, when t= t_1 Further, during the period $[t_1, T]$ the shortages are allowed, and the demand is assumed to be partially backlogged. The representation of inventory system at any time is shown in Figure 1.

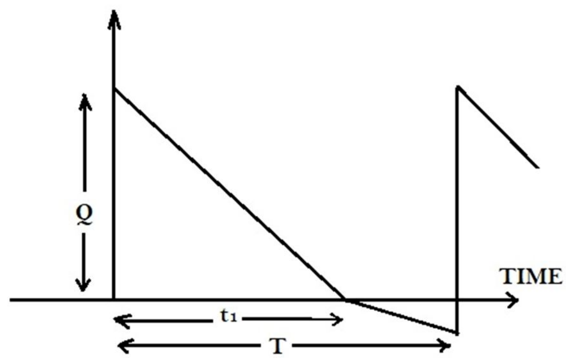


Figure 1. Graphical presentation of inventory system

The changes in the inventory at any time t are governed by the differential equation during the periods $[0, t_1]$ and $[t_1, T]$ are respectively given by:

$$\frac{dQ_1(t)}{dt} + \theta t . Q_1(t) = -(R_1 + \beta t), \text{for } 0 \leq t \leq t_1 \tag{1}$$

And

$$\frac{dQ_2(t)}{dt} = -\frac{R_2}{1+\delta(T-t)}, \text{for } t_1 \leq t \leq T \tag{2}$$

With boundary conditions

$$\left.\begin{matrix} Q_1(t) = Q_2(t) = 0 \text{ at t } = t_1, \\ and \\ Q_1(t) = Q \ at \ t = 0 \end{matrix}\right\} \tag{3}$$

The objective of this inventory problem is to determine the order quantity and length of ordering cycle so as to keep the total relevant costs as low as possible. That is, to determine Q* and T* so that the total cost is minimized. Now there are two cases:

Case I: $0 \leq t \leq t_1$

In this case, the inventory level decreases due to the demand as well as deterioration, and the inventory level is governed by (1). Using the boundary conditions (3), the solution of (1) is given by

$$Q_1(t) = R_1\left[(t_1 - t) + \frac{\theta}{6}(t_1^3 - t^3 - 3t_1 t^2 + 3t^3)\right] + \frac{\beta}{2}[(t_1^2 - t^2) + \frac{\theta}{8}(t_1^4 - 4t_1^2 t^2 + 3t^4)] \tag{4}$$

Therefore, the maximum inventory level for each cycle

$$Q = Q_1(0) = R_1\left[t_1 + \frac{\theta}{6} t_1^3\right] + \frac{\beta}{2}[t_1^2 + \frac{\theta}{8} t_1^4] \tag{5}$$

Case II: $t_1 \leq t \leq T$

In this case, the inventory level depends due on demand. But a fraction of demand is backlogged. The inventory level is governed by (2). Using the boundary conditions (3), the solution of (2) is given by

$$Q_2(t) = \frac{R_2}{\delta}[\log\{1 + \delta(T - t)\} - \log\{1 + \delta(T - t_1)\}], \quad t_1 \leq t \leq T \tag{6}$$

Put t=T in (6), we obtain the maximum amount of demand backlogged per cycle as follows:

$$D_B = -Q_2(T) = \frac{R_2}{\delta}[\log\{1 + \delta(T - t_1)\}] \tag{7}$$

So, the order quantity per cycle is given by

$$W = Q + D_B = R_1\left[t_1 + \frac{\theta}{6} t_1^3\right] + \frac{\beta}{2}\left[t_1^2 + \frac{\theta}{8} t_1^4\right] + \frac{R_2}{\delta}\log\{1 + \delta(T - t_1)\} \tag{8}$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 10 Issue VII July 2022- Available at www.ijraset.com

For $\boldsymbol{\delta} < 1$, the Taylor's series expansion yields the following second degree approximations:

$$\log\{1+\boldsymbol{\delta}(T-t_1)\} \approx \boldsymbol{\delta}(T-t_1) - \frac{\boldsymbol{\delta}^2 (T-t_1)^2}{2} \tag{9}$$

From equations (8) and (9), we get

$$W = R_1\left[t_1 + \frac{\theta}{6}\, t_1^3\right] + \frac{\beta}{2}\left[t_1^2 + \frac{\theta}{8} t_1^4\right] + \frac{R_2}{\delta}\left[\boldsymbol{\delta}(\boldsymbol{T}-t_1) - \frac{\boldsymbol{\delta}^2(\boldsymbol{T}-t_1)^2}{2}\right]$$

$$W = R_1\left[t_1 + \frac{\theta}{6}\, t_1^3\right] + \frac{\beta}{2}\left[t_1^2 + \frac{\theta}{8} t_1^4\right] + R_2\left[(\boldsymbol{T}-t_1) - \frac{\boldsymbol{\delta}\,(\boldsymbol{T}-t_1)^2}{2}\right] \tag{10}$$

Inventory holding cost per cycle is

$$C_H = \int_0^{t_1} c_h(t)\; Q_1(t)\, dt$$

$$=\int_0^{t_1}(\alpha_0+\alpha_1\, t)\; R_1\left[(t_1-t) + \frac{\theta}{6}\,(t_1^3 - t^3 - 3t_1\, t^2 + 3t^3)\right] + \frac{\beta}{2}\left[(t_1^2-t^2) + \frac{\theta}{8}(t_1^4 - 4t_1^2\, t^2 + 3t^4)\right] dt$$

$$= \alpha_0(R_1(\frac{t_1^2}{2} - \frac{\theta t_1^4}{6}) + \frac{\beta}{2}(\frac{2t_1^3}{3} + \frac{\theta t_1^4}{30})) + \alpha_1(R_1(\frac{t_1^3}{6} + \frac{\theta t_1^5}{40}) + \frac{\beta t_1^4}{8}) \tag{11}$$

Deterioration cost per cycle is

$$C_D = c_2[Q - \int_0^{t_1} D(t)\, dt$$

$$= c_2\left[R_1\left[t_1 + \frac{\theta}{6}\, t_1^3\right] + \frac{\beta}{2}\left[t_1^2 + \frac{\theta}{8} t_1^4\right] - \int_0^{t_1}(R_1 + \beta\, t)\, dt\right]$$

$$= \frac{c_2\theta}{2}\left[\frac{R_1\, t_1^3}{3} + \frac{\beta\, t_1^4}{8}\right] \tag{12}$$

Shortage cost per cycle is given by

$$C_S = c_4[-\int_{t_1}^{T} Q_2(t)\, dt\,]$$

$$= c_4[-\int_{t_1}^{T}\frac{R_2}{\boldsymbol{\delta}}[\log\{1+\boldsymbol{\delta}(T-t)\} - \log\{1+\boldsymbol{\delta}(T-t_1)\}]\, dt$$

$$= c_4 R_2[\frac{(T-t_1)^2}{2} - \frac{\boldsymbol{\delta}\,(T-t_1)^3}{3}] \tag{13}$$

Lost sale cost per cycle is given by

$$C_L = c_5[\int_{t_1}^{T}\left[1 - \frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{\delta}(\boldsymbol{T}-\boldsymbol{t})}\right] R_2 dt\,]$$

$$= c_5 R_2\left[(T-t_1) - \frac{1}{\boldsymbol{\delta}}\log\{1+\boldsymbol{\delta}(\boldsymbol{T}-t_1)\}\right]$$

$$= c_5 R_2\, \frac{\boldsymbol{\delta}\,(T-t_1)^2}{2} \tag{14}$$

So, the average total cost per unit time per cycle is

$$\mathrm{TC} = \frac{1}{\boldsymbol{T}}\,[C_H + C_D + c_O + C_S + C_L]$$

$$\Rightarrow TC = \frac{1}{T}(c_O + c_4R_2(\frac{1}{2}(T-t_1)^2 - \frac{1}{3}\delta(T-t_1)^3) + \frac{1}{2}c_5R_2(T-t_1)^2 + \frac{1}{2}\theta c_2(\frac{1}{3}R_1t_1^3 + \frac{\beta t_1^4}{8}) + (R_1(\frac{t_1^2}{2} - \frac{\theta t_1^4}{6}) + \frac{1}{2}\beta(\frac{2t_1^3}{3} + \frac{\theta t_1^4}{30}))\alpha_0 + (\frac{\beta t_1^4}{8} + R_1(\frac{t_1^3}{6} + \frac{\theta t_1^5}{40}))\alpha_1) \tag{15}$$

So, the developed inventory model can be written as Minimize

$$TC = \frac{1}{T}(c_O + c_4R_2(\frac{1}{2}(T-t_1)^2 - \frac{1}{3}\delta(T-t_1)^3) + \frac{1}{2}c_5R_2(T-t_1)^2 + \frac{1}{2}\theta c_2(\frac{1}{3}R_1t_1^3 + \frac{\beta t_1^4}{8}) + (R_1(\frac{t_1^2}{2} - \frac{\theta t_1^4}{6}) + \frac{1}{2}\beta(\frac{2t_1^3}{3} + \frac{\theta t_1^4}{30}))\alpha_0 + (\frac{\beta t_1^4}{8} + R_1(\frac{t_1^3}{6} + \frac{\theta t_1^5}{40}))\alpha_1) \tag{16}$$

Subject to

$$(T - t_1) \geq 0,$$
$$and\ t_1 \geq 0, T \geq 0$$

The problem is a single-objective non-linear optimization problem. The objective function TC is a function of t_1 and T. To find the optimality of t_1 and T, the partial derivative of TC with respect to t_1 and T are equated to zero and after solving these equations simultaneously we get the optimal values of t_1 and T.

$$\frac{\partial TC}{\partial t_1} = \frac{1}{T}(-c_5R_2(T-t_1) + c_4R_2(-T + \delta(T-t_1)^2 + t_1) + \frac{1}{2}\theta c_2(R_1t_1^2 + \frac{\beta t_1^3}{2}) + (R_1(t_1 - \frac{2\theta t_1^3}{3}) + \frac{1}{2}\beta(2t_1^2 + \frac{2\theta t_1^3}{15}))\alpha_0 + (\frac{\beta t_1^3}{2} + R_1(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8}))\alpha_1) = 0 \tag{17}$$

$$\frac{\partial TC}{\partial \mathrm{T}} = \frac{c_5R_2(T-t_1) + c_4R_2(T - \delta(T-t_1)^2 - t_1)}{T} - \frac{1}{T^2}(c_O + c_4R_2(\frac{1}{2}(T-t_1)^2 -$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 10 Issue VII July 2022- Available at www.ijraset.com

$$\frac{1}{3}\delta(T-t_1)^3) + \frac{1}{2}c_5R_2(T-t_1)^2 + \frac{1}{2}\theta c_2(\frac{1}{3}R_1t_1^3 + \frac{\beta t_1^4}{8}) + (R_1(\frac{t_1^2}{2} - \frac{\theta t_1^4}{6}) + \frac{1}{2}\beta(\frac{2t_1^3}{3} + \frac{\theta t_1^4}{30}))\alpha_0 + (\frac{\beta t_1^4}{8} + R_1(\frac{t_1^3}{6} + \frac{\theta t_1^5}{40}))\alpha_1) = 0 \quad (18)$$

After solving equations (17) and (18) simultaneously, we obtain optimal values of t_1 and T and then substitute these values in equation (16) we get minimum average total cost per unit time of the inventory model. But the behavior of the cost function TC is highly non-linear, so the convexity of cost function TC is shown graphically in the next section. Hence, the optimality of TC would be globally minimum.

Numerical Problem:

Let $D(t) = \begin{cases} 20 + 40\,t, & for\ 0 \le t \le t_1 \\ 25 & for\ t_1 \le t \le T \end{cases}$, with $\beta = 40$, $R_1 = 20$, $R_{2,} = 25$. Also $c_h(t) = 0.5 + 0.011\,t$ where $\alpha_0 = 0.5$, $\alpha_1 = 0.011 > 0$. By using Mathematica software, we obtain the optimal solution of the problem $t_1^* = 1.08772$, $T^* = 1.1412$, shortage period = 1.1412 – 1.08772= 0.05348 unit, W* = 46.7644units and TC* = 66.3502 units.

IV. CONVEXITY OF COST FUNCTION

To show the convexity of cost function, we generate the graphs of total cost function C based on the parameter values taken in above examples. If we plot the total cost function with some values of t_1 and T, then we get the strictly convex graph of total cost function given by Figures 2, 3, 4, respectively.

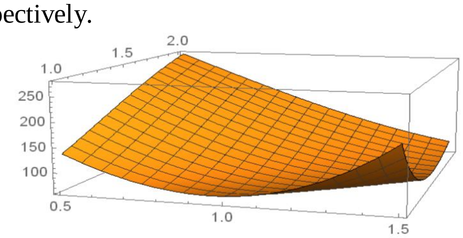


Figure 2: Convexity of cost function

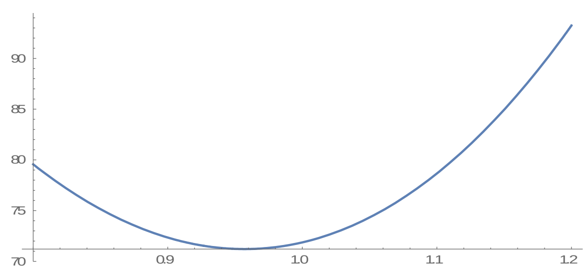


Figure 3: Total cost v/s t_1 at $T = 1.1412$

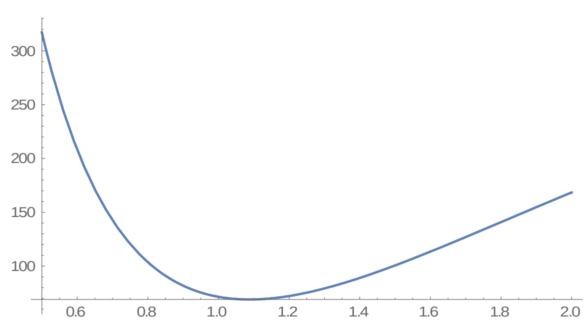


Figure 4: Total cost v/s T at $t_1 = 0.9816$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 10 Issue VII July 2022- Available at www.ijraset.com

V. SENSITIVITY ANALYSIS

Sensitivity analysis of various system parameters is required to observe whether the current solutions remain unchanged, then the current solutions become infeasible. Therefore, we analyze the effect of changes in various inventory system parameters: t_1^*, T*, W*, TC*. The sensitivity analysis is carried out by changing the value of each of the parameters by -20%, - 10%, 20% and 10%, taking one parameter at time and keeping the remaining parameters unchanged.

The results are displayed in Table 1.

Parameter	Value of parameter	t_1^*	T*	W*	TC*
α_0	0.40	1.09762	1.1724	47.9016	63.7815
	0.45	1.09264	1.1573	47.3443	65.0734
	0.55	1.08285	1.1254	46.1923	67.6122
	0.60	1.07804	1.1189	45.8468	68.8597
α_1	0.0088	1.1219	1.1783	49.0327	65.3827
	0.0099	1.1056	1.1605	47.9426	65.8326
	0.0121	1.0715	1.1237	45.707	66.8494
	0.0132	1.0599	1.1112	44.9576	67.2264
c_p	3.2	1.08814	1.1415	46.7716	66.3148
	3.6	1.08761	1.1410	46.7553	66.342
	4.4	1.0871	1.1406	46.7255	66.3801
	4.8	1.08657	1.1401	46.7135	66.4073
c_o	48	1.00542	1.0521	41.5007	57.7538
	54	1.03244	1.0813	43.1976	62.599
	66	1.11818	1.1743	48.7845	70.6117
	72	1.16911	1.2298	52.249	74.0456
c_s	9.6	1.09295	1.1521	47.2332	65.979
	10.8	1.08903	1.1451	46.9097	66.2068
	13.2	1.08852	1.1398	46.7627	66.4233
	14.4	1.08795	1.1371	46.6753	66.5292
c_l	12	1.09619	1.1575	47.4916	65.8117
	13.8	1.08938	1.1456	46.9356	66.1922
	16.5	1.08755	1.1381	46.6834	66.479
	18	1.08291	1.1305	46.3175	66.7287
R_1	16	1.09478	1.1454	42.7627	65.1723
	18	1.09245	1.1446	44.847	65.7273
	22	1.07629	1.1305	48.2119	67.1691
	24	1.07343	1.1291	50.249	67.7293
R_2	20	1.11179	1.1816	48.3566	65.1053
	22.5	1.09395	1.1541	47.1741	65.9135
	27.5	1.08405	1.1323	46.524	66.6702
	30	1.07722	1.1209	46.0776	67.0548
β	32	1.15763	1.2088	45.8912	62.9931
	36	1.11609	1.1681	46.0575	64.85
	44	1.06388	1.1189	47.561	67.7112
	48	1.03527	1.0911	47.8285	69.2199
θ	0.0040	1.054	1.1045	44.5633	67.3853
	0.0045	1.07691	1.1294	46.0515	66.6641
	0.0055	1.08738	1.1409	46.7477	66.3712
	0.0060	1.11151	1.1672	48.3509	65.703
δ	0.64	1.10348	1.1581	47.8055	65.9073
	0.72	1.09574	1.1498	47.2929	66.1216
	0.88	1.08477	1.1381	46.5705	66.4336
	0.96	1.07601	1.1287	45.9976	66.6943

VI. OBSERVATIONS

The following observations are made from sensitivity analysis:

1) Average total cost TC increases any of one among α_0 , $\alpha_1, c_p, c_o, c_s, c_l, R_1, R_2, \beta$ *and* δ increases; however it decreases as θ increases.
2) Order quantity W decreases as any one among α_0, $\alpha_1, c_p, c_s, c_l, R_2$ *and* δ increases; however it increases as θ, c_o, β, R_1 increase.
3) Average total cost is more sensitive to the changes in ordering cost c_o than other parameters.
4) Average total cost is relatively less sensitive to the changes in $c_s, c_l,$ *and* δ than other parameters.
5) When backlogging parameter δ increases, TC increases while W decreases and vice-versa. Hence for minimum values of total cost TC, δ should be minimum that is back ordered rate $\boldsymbol{\gamma(t) = \frac{1}{1+\delta(T-t)}}$ should be as high as possible.

The above observations indicate that, in order to minimizing total cost, the policy should be to stock more to reduce the lost sale cost. Further, for avoiding high holding cost, low inventory level should be maintained strategically.

VII. APPENDIX

Here we have given derivation and proof of convexity of total coat TC (t_1, T) with respect to T and t_1 . For this objective we have to prove that

$$\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1 \partial T}\right)^2 > 0$$

Where

$$\frac{\partial^2 TC}{\partial t_1^2}=\frac{1}{T}(c_5R_2+c_4R_2(1-2\delta(T-t_1))+\frac{1}{2}\theta c_2(2R_1t_1+\frac{3\beta t_1^2}{2})+(R_1(1-2\theta t_1^2)+\frac{1}{2}\beta(4t_1+\frac{2\theta t_1^2}{5}))\alpha_0+(\frac{3\beta t_1^2}{2}+R_1(t_1+\frac{\theta t_1^3}{2}))\alpha_1)$$

$$\frac{\partial^2 TC}{\partial T^2}=\frac{c_5R_2+c_4R_2(1-2\delta(T-t_1))}{T}-\frac{2(c_5R_2(T-t_1)+c_4R_2(T-\delta(T-t_1)^2-t_1))}{T^2}+\frac{1}{T^3}2(c_O+c_4R_2(\frac{1}{2}(T-t_1)^2-\frac{1}{3}\delta(T-t_1)^3)+\frac{1}{2}c_5R_2(T-t_1)^2+\frac{1}{2}\theta c_2(\frac{1}{3}R_1t_1^3+\frac{\beta t_1^4}{8})+(R_1(\frac{t_1^2}{2}-\frac{\theta t_1^4}{6})+\frac{1}{2}\beta(\frac{2t_1^3}{3}+\frac{\theta t_1^4}{30}))\alpha_0+(\frac{\beta t_1^4}{8}+R_1(\frac{t_1^3}{6}+\frac{\theta t_1^5}{40}))\alpha_1)$$

$$\frac{\partial^2 TC(t_1,T)}{\partial t_1\partial T}=\frac{-c_5R_2+c_4R_2(-1+2\delta(T-t_1))}{T}-\frac{1}{T^2}(-c_5R_2(T-t_1)+c_4R_2(-T+\delta(T-t_1)^2+t_1)+\frac{1}{2}\theta c_2(R_1t_1^2+\frac{\beta t_1^3}{2})+(R_1(t_1-\frac{2\theta t_1^3}{3})+\frac{1}{2}\beta(2t_1^2+\frac{2\theta t_1^3}{15}))\alpha_0+(\frac{\beta t_1^3}{2}+R_1(\frac{t_1^2}{2}+\frac{\theta t_1^4}{8}))\alpha_1)$$

Subsequently the above found cost function are enormously non-linear therefore it is tough to develop the convexity mathematically. Therefore, some numerical values are taken in the above example and graphs to develop the convexity (Figures 2, 3 and 4).

By using numerical problem, we get

$\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right) = 569.05195 > 0$, $\left(\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right) = 475.396 > 0$ and

$$\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1 \partial T}\right)^2 = 50398.34 > 0$$

which prove the convexity of total cost function.

VIII. CONCLUSIONS

We proposed a deteriorating inventory model with time-dependent demand rate and varying holding cost under partial backlogging along with time dependent deterioration rate. Shortages are allowed and partially backlogged. The classical optimisation technique is used to derive the optimal order quantity and optimal average total cost.

For the practical use of this model, we consider a numerical example with sensitivity analysis. Both the numerical example and sensitivity analysis are implemented the help of MATHEMATICA-8.0. The proposed model can assist the manufacturer and retailer in accurately determining the optimal order quantity, cycle time and total cost. Moreover, in market there are certain items where during the season period, the demand increases with time, and when the season is off, the demand sharply decreases and then becomes constant. Thereby, the proposed model can also be used in inventory control of seasonal items. This paper can be further extended by considering ramp type demand function. Further, the fuzzy or stochastic uncertainty in inventory parameters may be considered.

REFERENCES

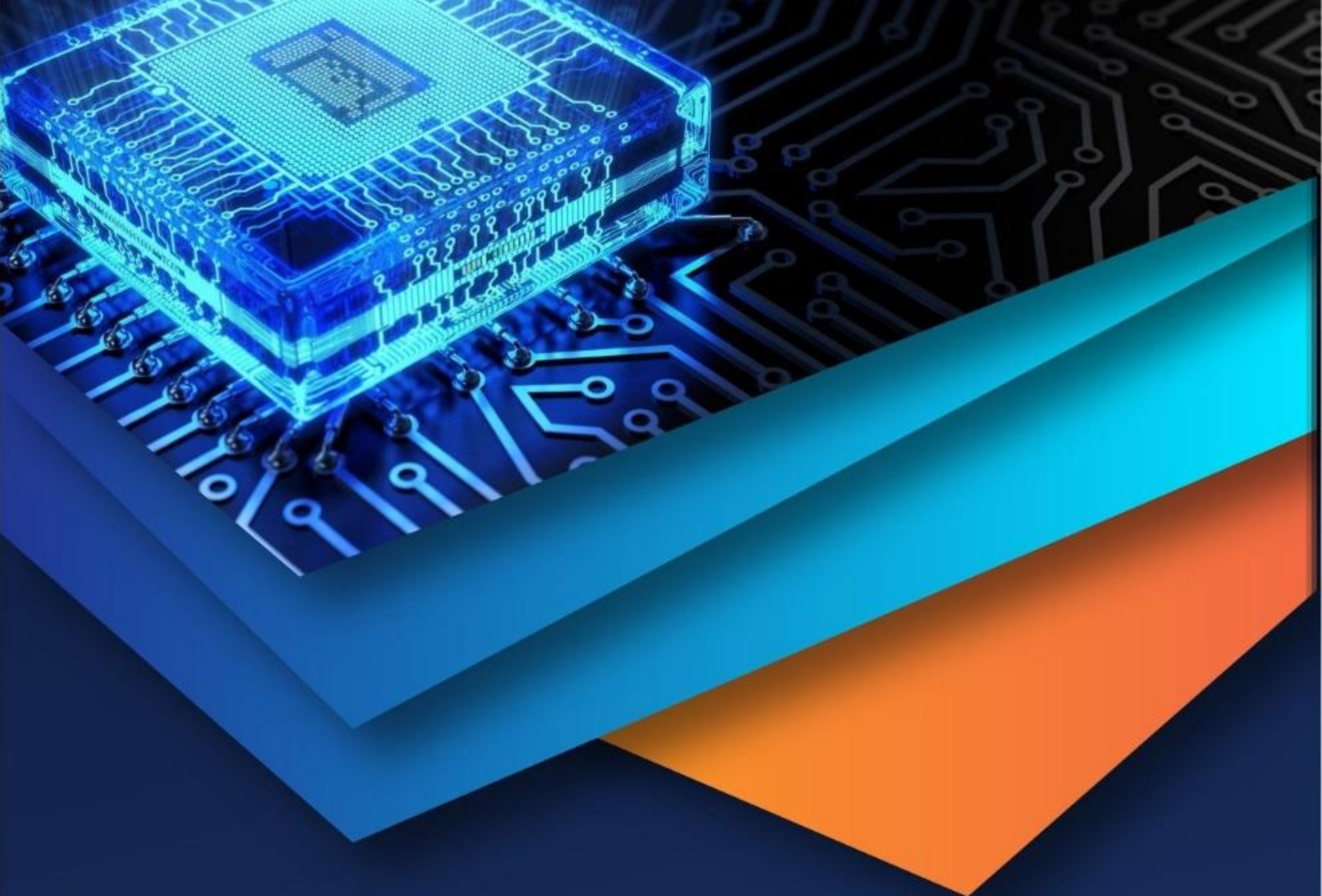
[1] Abad, P. 'Optimal pricing and lot-sizing under conditions of perishability and partial backordering', Management Science, Vol. 42, No. 8, pp.1093–1104 (1996)
[2] Abad, P. 'Optimal price and order-size for a reseller under partial backlogging', Computers and Operation Research, Vol. 28, No. 1, pp.53–65 (2001)
[3] Chang, H. and Dye, C. 'An EOQ model for deteriorating items with time varying demand and partial backlogging', Journal of the Operational Research Society, 50, 11, 1176–1182 (1999)
[4] Chung, K.J. and Ting, P.S. 'A heuristic for replenishment of deteriorating items with a linear trend in demand', Journal of Operational Research Society, Vol. 44, No. 12, pp.1235–1241(1993)
[5] Dave, U. and Patel, L. '(T, si)-policy inventory model for deteriorating items with time proportional demand', Journal of Operational Research Society, 32, 2, 137–142 (1981)
[6] Dye, C. 'Determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging', European Journal of Operational Research, Vol. 181, No. 2, pp.668–678 (2007)
[7] Ghare, P.M. 'A model for exponentially decaying inventory', The Journal of Industrial Engineering, Vol. 5, No. 14, pp.238–243 (1963)
[8] Goyal, S. and Giri, B. 'Recent trends in modeling of deteriorating inventory', European Journal of Operational Research, Vol. 134, No. 1, pp.1–16 (2001)
[9] Hung, K. 'An inventory model with generalized type demand, deterioration and backorder rates', European Journal of Operational Research, Vol. 208, No. 3, pp.239–242 (2011)
[10] Jalan, A., Giri, R. and Chaudhary, K. 'EOQ model for items with weibull distribution deterioration shortages and trended demand', International Journal of System Science, Vol. 27, No. 9, pp.851–855 (1996)
[11] Kumar, R.S., De, S.K. and Goswami, A. 'Fuzzy EOQ models with ramp type demand, partial backlogging and time dependent deterioration rate', International Journal of Mathematics in Operational Research, Vol. 4, No. 5, pp.473–502 (2012)
[12] Roy, A. 'An inventory model for deteriorating items with price dependent demand and time varying holding cost', Advanced Modelling and Optimization, Vol. 10, No. 1, pp.25–37 (2008)
[13] Ouyang, L.Y., Wu, K.H. and Cheng, M.C. 'An inventory model for deteriorating items with exponential declining demand and partial backlogging', Yugoslav Journal of Operations Research, Vol. 15, No. 2, pp.277–288 (2005)
[14] Mandal, B. 'An EOQ inventory model for Weibull distributed deteriorating items under ramp type demand and shortages', OPSEARCH, Vol. 47, No. 2, pp.158–165 (2010)
[15] Mishra, V.K.: Controllable deterioration rate for time-dependent demand and time-varying holding cost. Yugosl. J. Oper. Res. 24, 87–98 (2014)
[16] Palanivel, M.; Uthayakumar, R.: An EPQ model for deteriorating items with variable production cost, time dependent holding cost and partial backlogging under inflation. OPSEARCH 52, 1–17 (2015)
[17] Choudhury, K.D.; Karmakar, B.; Das, M.; Datta, T.K.: An inventory model for deteriorating items with stock-dependent demand, time-varying holding cost and shortages. OPSEARCH 52, 55–74 (2015)
[18] Rangarajan, K.; Karthikeyan, K.: Analysis of an EOQ Inventory Model for Deteriorating Items with Different Demand Rates. Appl. Math. Sci. 9, 2255–2264 (2015)
[19] Karmakar, B.: Inventory models with ramp-type demand for deteriorating items with partial backlogging and time-varing holding cost. Yugosl. J. Oper. Res. 24, 249–266 (2016)
[20] Alfares, H.K.; Ghaithan, A.M.: Inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts. Comput. Ind. Eng. 94, 170–177 (2016)
[21] Giri, B.C.; Bardhan, S.: Coordinating a two-echelon supply chain with price and inventory level dependent demand, time dependent holding cost, and partial backlogging. Int. J. Math. Oper. Res. 8, 406–423 (2016)
[22] Pervin, M.; Roy, S.K.; Weber, G.W.: A Two-echelon inventory model with stock-dependent demand and variable holding cost for deteriorating items. Numer. Algebra Control Optim. 7, 21–50 (2017)
[23] Chandra, S.: An inventory model with ramp type demand, time varying holding cost and price discount on backorders. Uncertain Supply Chain Manag. 5, 51–58 (2017)
[24] Rastogi, M.; Singh, S.; Kushwah, P.; Tayal, S.: An EOQ model with variable holding cost and partial backlogging under credit limit policy and cash discount. Uncertain Supply Chain Manag. 5, 27–42 (2017)
[25] Pervin, M.; Roy, S.K.; Weber, G.-W.: Analysis of inventory control model with shortage under time-dependent demand and time-varying holding cost including stochastic deterioration. Ann. Oper. Res. 260, 437–460 (2018a)
[26] Pervin, M.; Roy, S.K.; Weber, G.W.: An integrated inventory model with variable holding cost under two levels of trade-credit policy. Numer. Algebra Control Optim. 8, 169–191 (2018b)
[27] Sen, N.; Saha, S.: An inventory model for deteriorating items with time dependent holding cost and shortages under permissible delay in payment. Int. J. Procure. Manag. 11, 518–531 (2018)

[28] Sharma, S.; Singh, S.; Singh, S.R.: An inventory model for deteriorating items with expiry date and time varying holding cost. Int. J. Procure. Manag. 11, 650–666 (2018)

[29] Geetha, K.V.; Naidu S.R.; Economic ordering policy for deteriorating items with inflation induced time dependent demand under infinite time horizon. Int. Journal of Operational Research. 39(1):69-94 (2020)

[30] Khatri, P.D.; Gothi, U.B.; An EPQ Model for Non-Instantaneous Weibully Decaying Items with Ramp Type Demand and Partially Backlogged Shortages. Int. J. of Math. Trends and Tech. (IJMTT) (66)3 (2020)

[31] Adak, S.; Mahapatra, G.S.; Effect of reliability on varying demand and holding cost on inventory system incorporating probabilistic deterioration. American Institute of Mathematical Sciences 18(1): 173-193 (2022)



10.22214/IJRASET

INDEX COPERNICUS

45.98

IMPACT FACTOR: 7.129

TOGETHER WE REACH THE GOAL

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY