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Parametric Modeling of TECET Curves for Highway Geometric Design

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Abstract: This manuscript presents an academic discussion on the use of mathematical tools for the parametric modeling of horizontal road alignments, specifically the Tangent–Spiral–Circular–Spiral–Tangent (TECET) curve. The objective is to show how fundamental concepts of calculus and differential geometry—arc length, curvature, Frenet–Serret frame, and Fresnel integrals—form the analytical basis for geometric design criteria established in road standards. A piecewise smooth parametric model is constructed, using a dimensionless parameter $u \in [0, 1]$ that maps the total curve length, enabling the evaluation of vehicle kinematics at any point of the alignment. The model ensures continuity of position, tangent direction, and curvature (C^2 continuity) at the junctions between spirals and circular arc. An example with data from the SCT (Mexico) is included, implemented in GeoGebra to allow dynamic visualization of tangent and acceleration vectors. This work seeks to highlight the link between mathematics taught in the early semesters of civil engineering and its concrete application in advanced road design projects.
Keywords: Geometric design of roads; TECET curve; Euler spiral; Fresnel integrals; Frenet–Serret frame; Parametric modeling

I. INTRODUCTION

The geometric design of highways constitutes one of the cornerstones of civil engineering, as it determines fundamental aspects such as safety, user comfort, and traffic efficiency. A properly designed highway must not only ensure adequate sight distances and curve radii consistent with operating speed, but also smooth transitions that reduce the dynamic effects on vehicles and, consequently, the risk of accidents. To achieve these objectives, current regulations in Mexico—particularly the Highway Geometric Design Manual [1]—set clear guidelines for the horizontal and vertical alignment of roadways.

In this context, the transition between straight and curved sections is achieved using compound curves, with the Tangent–Spiral–Circular–Spiral–Tangent (TECET) configuration being one of the most widely applied worldwide [2]. This scheme integrates transition spirals and circular arcs, ensuring continuity in position, direction, and curvature—conditions necessary for proper horizontal alignment. In this way, the TECET curve provides a mathematical and geometric solution to the problem of connecting tangents with smooth trajectories suitable for traffic at specific design speeds.

Nevertheless, a deep understanding of these curves requires the use of mathematical tools that civil engineering students typically encounter in their early semesters: differential calculus, vector analysis, and analytic geometry. Concepts such as arc length, parametrization, Fresnel integrals, and the Frenet–Serret frame constitute the theoretical foundation that allows precise description and analysis of the dynamic properties of a roadway trajectory. The aim of this article is, therefore, to highlight the intrinsic relationship between these mathematical foundations and the geometric design of highways, by developing a parametric model of the TECET curve that can be implemented in interactive visualization software such as GeoGebra, for both academic and preliminary design purposes.

II. BASIC CONCEPTS

The horizontal alignment of a highway is composed of straight segments (tangents) and curves that connect them in a continuous manner. To ensure safety and driving comfort, it is not sufficient to link tangents and circular arcs through intersection points; transition curves are required to provide a progressive change in curvature and, consequently, in the lateral acceleration experienced by vehicles [1] and [2].

A. Components of the TECET Curve

The compound Tangent–Spiral–Circular–Spiral–Tangent (TECET) curve is composed of three main segments:

1) Entry spiral (TE→EC): connects the initial tangent with the circular arc, gradually increasing curvature from 0 to ${}^{1}/_{R_c}$.

2) Circular arc (EC→CE): defines a trajectory with constant radius R_c.
3) Exit spiral (CE→ET): progressively reduces curvature from $1/R_c$ to 0 to connect with the final tangent.

This configuration ensures continuity in position, direction, and curvature—a property known as C^2 continuity (twice continuously differentiable)—which is essential for comfortable driving.

B. Key Parameters

The geometric parameters that define a TECET curve as showed in Figure 1 are:
Degree of curvature: $G_c > 0$, which relates to a circular arc radius R_c.
Radius of the circular arc:

$$R_c = \frac{1145.92}{G_c} \tag{II.B.1}$$

Length of the transition spiral: $L_e > 0$, calculated to ensure a gradual change in lateral acceleration.
Length of the circular arc: $L_c > 0$.

Total length of the compound curve:

$$L_T = 2L_e + L_c \tag{II.B.2}$$

Deflection angle of the circular arc:

$$\Delta_c = \frac{L_c}{R_c} \tag{II.B.3}$$

Total deflection angle of the curve (angle between tangents):

$$\Delta_t = 2\theta_e + \Delta_c \tag{II.B.4}$$

with

$$\theta_e = \frac{L_e}{2R_c} \tag{II.B.5}$$

Where:
G_c = degree of curvature
R_c = radius of the circular arc
L_e = length of the transition spiral
L_c = length of the circular arc
L_T = total length of the compound curve
Δ_c = deflection angle of the circular arc
Δ_t = total deflection angle of the curve
θ_e = accumulated rotation angle in each spiral

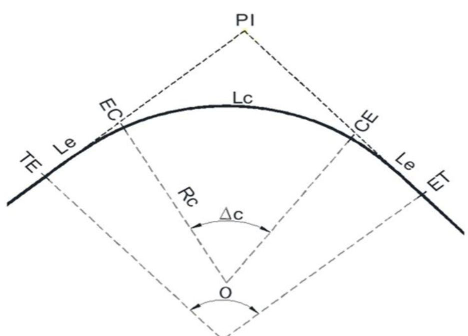

Fig. 1 Curve TECET

C. Euler Spiral Parameter (Clothoid Parameter)

The transition spiral used corresponds to the Clothoid, or Euler spiral, whose fundamental property is that curvature varies linearly with the arc length s:

$$\kappa(s) = \frac{s}{A^2} \tag{II.C.1}$$

Where de parameter A is defined as:

$$A = \sqrt{R_c L_e} \tag{II.C.2}$$

This parameter ensures that over the spiral length L_e , the curvature reaches the value corresponding to the circular arc.

Remark: In highway geometric design, the parameter A is commonly referred to as the Clothoid parameter or Euler spiral parameter. It establishes the relationship between the spiral length and the curvature of the connecting circular arc and is widely adopted in international standards such as [1] and [2].

III. MATHEMATICAL FORMULATION

The alignment of the TECET curve can be represented by a continuous parametrization in terms of a dimensionless variable $u \in [0,1]$, which spans the entire curve length. This formulation allows the trajectory to be treated as a smooth function and enables the dynamic evaluation of kinematic quantities associated with vehicle motion.

A. Global Parametrization

We define the total length of the curve and introduce the dimensionless parameter u:

$$s(u) = u \cdot L_T, 0 \leq u \leq 1 \tag{III.A.1}$$

Here, L_T represents the total length of the compound curve previously defined as equation (II.B.2), which is composed of two transition spirals and a central circular arc. The variable u is a normalized, dimensionless parameter ranging from 0 to 1. Through this parametrization, any point along the TECET curve can be uniquely identified by the value of u, with u = 0 corresponding to the beginning of the entry spiral and u = 1 corresponding to the end of the exit spiral.

This formulation allows the arc length s to be expressed as a linear function of u. Consequently, the trajectory can be studied and computed in a uniform manner, regardless of the absolute dimensions of the curve, which is especially useful for numerical evaluation and visualization in software such as GeoGebra or CAD-based design tools.

B. Entry Spiral (TE→EC)

The development of the Euler spiral can be expressed in local coordinates through the Fresnel integrals, as defined by [3] and in accordance with [4] and [5]:

$$x_{E1}(u) = A \cdot \sqrt{\pi} \cdot C\left(\frac{s(u)}{A \cdot \sqrt{\pi}}\right), \qquad y_{E1}(u) = A \cdot \sqrt{\pi} \cdot S\left(\frac{s(u)}{A \cdot \sqrt{\pi}}\right) \tag{III.A.1}$$

where the Fresnel integrals are given by:

$$C(t) = \int_0^t cos\left(\frac{\pi}{2} \cdot \tau^2\right) d\tau, \quad S(t) = \int_0^t sin\left(\frac{\pi}{2} \cdot \tau^2\right) d\tau \tag{III.A.2}$$

These integrals define the Clothoid (Euler spiral) in terms of cumulative oscillatory functions, and they ensure that curvature increases linearly with arc length. In practice, the coordinates (x_{E1}, y_{E1}) describe the trajectory of the entry spiral in its local reference system, with the scaling governed by the parameter A. This representation is particularly convenient because Fresnel integrals are tabulated and implemented in most mathematical software libraries, allowing for efficient numerical evaluation and visualization of the spiral segment.

The importance of using the Fresnel integrals lies in their ability to capture the smooth transition from zero curvature (tangent) to the constant curvature of the circular arc, ensuring both mathematical rigor and geometric continuity in highway alignment design.

C. Circular Arc (EC→CE)

The endpoint of the entry spiral is obtained as:

$$T_e = \frac{L_e}{A\sqrt{\pi}} \quad \text{(III.C.1)}$$

$$(X_E, Y_E) = \left(x_{E1}(T_e), y_{E1}(T_e)\right) \quad \text{(III.C.2)}$$

The center of the circular arc is then determined by:

$$(x_O, y_O) = \left(X_E - R_c \cdot sin(\theta_e), T_E - R_c \cdot cos(\theta_e)\right) \quad \text{(III.C.3)}$$

The parametric equations of the arc are expressed as:

$$\gamma = \theta_e - \frac{\pi}{2} + \frac{s(u) - L_e}{R_c} \quad \text{(III.C.4)}$$

$$x_{C2}(u) = x_O + R_c \cdot \cos(\gamma), \qquad y_{C2}(u) = y_O + R_c \cdot \sin(\gamma) \quad \text{(III.C.5)}$$

The circular arc (EC→CE) provides the central portion of the TECET curve and connects the exit of the entry spiral with the beginning of the exit spiral. Its radius is constant and equal to , ensuring that the curvature remains fixed during this portion of the trajectory.

Equation (III.C.1) defines the normalized arc-length parameter T_e that corresponds to the end of the entry spiral, while (III.C.2) gives the cartesian coordinates of this transition point. Using this point and the known radius R_c, the coordinates of the arc's center (x_O, y_O) can be determined as in (III.C.3).

Finally, equations (III.C.4) to (III.C.5) define the circular arc parametrically in terms of the global arc-length function (u). The trigonometric shift $\theta_e - \pi/2$ ensures the correct angular orientation of the arc with respect to the tangent and spiral coordinates.

In summary, this segment guarantees a smooth continuation of the trajectory with constant curvature, allowing the vehicle to negotiate the curve under steady lateral acceleration before entering the exit spiral, where curvature decreases back to zero.

D. Exit Spiral (CE→ET)

The exit spiral is constructed using Fresnel differences. First, we introduce the reduced variable:

$$t_3(u) = \frac{s(u) - (L_e + L_c)}{A \cdot \sqrt{\pi}} \quad \text{(III.D.1)}$$

The local displacements are then expressed as:

$$\Delta_x(u) = A \cdot \sqrt{\pi} \cdot \left[C(T_e) - C\left(T_e - t_3(u)\right)\right] \quad \text{(III.D.2)}$$

$$\Delta_y(u) = A \cdot \sqrt{\pi} \cdot \left[S(T_e) - S\left(T_e - t_3(u)\right)\right] \quad \text{(III.D.2)}$$

Once the reduced variable $t3(u)$ is defined, auxiliary parameters are introduced to express the differentials of the Fresnel integrals in the exit spiral:

τ_{low}: the lower integration limit in the reduced variable of the exit spiral. It corresponds to the value at the end of the entry spiral and ensures curve continuity at the connection point CE.

$$\tau_{low} = -\frac{A}{R_c\sqrt{\pi}} \tag{III.D.3}$$

τ_{high}(u): the variable upper limit depending on the parameter u. It represents the progression of arc length within the exit spiral.

$$\tau_{high}(u) = \frac{s(u) - (L_e + L_c) - \frac{A^2}{R_c}}{A\sqrt{\pi}} \tag{III.D.4}$$

κ_{shift}: the angular shift associated with the accumulated curvature at the CE junction. Its function is to "rotate" the Fresnel integrals so that the exit spiral begins with curvature $1/R_c$ and decays smoothly to zero at ET.

$$\kappa_{shift} = \frac{A^2}{2{R_c}^2} \tag{III.D.5}$$

$J_c(u)$ and $J_s(u)$: represent the local coordinate increments obtained from evaluating the Fresnel differences between τ_{low} and $\tau_{high}(u)$, incorporating the rotation by the angle κ_{shift}. These terms allow the exit spiral to be expressed as an accumulated displacement (Δ_x, Δ_y) starting from the CE point:

$$J_c(u) = A\sqrt{\pi}\cdot\left[cos(\kappa_{shift})\left(C(\tau_{high}) - C(\tau_{low})\right) + \sin(\kappa_{shift})\left(S(\tau_{high}) - S(\tau_{low})\right)\right] \tag{III.D.6}$$

$$J_s(u) = A\sqrt{\pi}\cdot\left[sin(\kappa_{shift})\left(C(\tau_{high}) - C(\tau_{low})\right) - cos(\kappa_{shift})\left(S(\tau_{high}) - S(\tau_{low})\right)\right] \tag{III.D.7}$$

Finally, by rotating and translating the increments from the endpoint of the circular arc (x_{CE}, y_{CE}):

$$\beta = \theta_e - \frac{\pi}{2} + \Delta_c \tag{III.D.8}$$

$$(x_{CE}, y_{CE}) = (x_O + R_c\cdot\cos(\beta), (y_O + R_c\cdot sin(\beta)) \tag{III.D.9}$$

The global coordinates of the exit spiral are obtained as:

$$x_{E3}(u) = x_{CE} + J_c(u)\cdot cos(\theta_e + \Delta_c) - J_s(u)\cdot sin(\theta_e + \Delta_c) \tag{III.D.10}$$

$$y_{E3}(u) = y_{CE} + J_c(u)\cdot sin(\theta_e + \Delta_c) + J_s(u)\cdot cos(\theta_e + \Delta_c) \tag{III.D.11}$$

The exit spiral (CE→ET) is mathematically the symmetric counterpart of the entry spiral, but with decreasing curvature. The introduction of the parameters τ_{low}, $\tau_{high}(u)$, and κ_{shift} ensures that the Fresnel-based formulation remains consistent with the curvature accumulated in the preceding circular arc.
The terms (u) and $J_s(u)$ capture the incremental displacements in local coordinates, while equations (III.D.10) to (III.D.11) rotate and translate these displacements into the global coordinate system. This guarantees that the spiral starts with curvature $1/R_c$ at CE and reduces smoothly to zero at ET, thus ensuring C^2 continuity of the entire TECET alignment.
From the perspective of highway design, this formulation is critical for vehicle dynamics: it ensures that lateral acceleration diminishes progressively as the driver exits the curve, improving safety and comfort.

E. Composite function

The global coordinates of the TECET curve can be expressed as a piecewise function:

$$\left(x(u), y(u)\right) = \begin{cases} (x_{E1}, y_{E1})\,; & 0 \le s \le L_e \\ (x_{C2}, y_{C2})\,; & L_e < s \le L_e + L_c \\ (x_{E3}, y_{E3})\,; & L_e + L_c < s \le L_T \end{cases} \tag{III.E.1}$$

Equation (III.E.1) unifies the three segments of the TECET curve—entry spiral, circular arc, and exit spiral—into a single global representation.

For $0 \leq s \leq L_e$, the coordinates are governed by the Euler spiral equations(x_{E1}, y_{E1}), where curvature increases linearly from zero to $1/R_c$. For $L_e < s \leq L_e + L_c$, the coordinates follow the circular arc equations (x_{C2}, y_{C2}), with constant curvature $1/R_c$. For $L_e + L_c < s \leq L_T$, the coordinates are given by the exit spiral (x_{E3}, y_{E3}), where curvature decreases smoothly back to zero.

By joining these three expressions in a piecewise fashion, the TECET alignment achieves C^2 continuity—that is, continuity in position, direction, and curvature across all junctions. This ensures not only mathematical elegance but also engineering relevance, as vehicle dynamics are governed by smooth variations of curvature and lateral acceleration.

Such a composite formulation is particularly useful for implementation in computational tools (e.g., GeoGebra, CAD, or custom design software), since it allows the entire alignment to be modeled and visualized with a single parametric variable u.

Each segment of the TECET curve corresponds to a specific range of the normalized parameter u:

- Entry spiral: $\left[0, \frac{L_e}{L_T}\right]$
- Circular arc: $\left[\frac{L_e}{L_T}, \frac{L_e+L_c}{L_T}\right]$
- Exit spiral: $\left[\frac{L_e+L_c}{L_T}, 1\right]$

This partition ensures that the entire curve is parameterized continuously over the interval $[0,1]$.

F. Numerical example

For the parameters R_c = 416.698 m, L_e = 88 m, and L_c = 325.764 m, the following results were obtained:

L_T = 501.764 m, A = 191.49 m, T_e = 0.2593, θ_e = 0.1056 rad, Δ_c = 0.7818 rad

The EC point is located at (87.90 m, 3.09 m), the center of the arc at (43.98 m, 417.47 m), and the CE point at (367.09 m,154.35 m). As expected, curvature varies linearly along the spirals and remains constant in the circular arc, confirming the C^2 continuity of the alignment, which guarantees smoothness in position, direction, and curvature.

IV. PRACTICAL APPLICATION: FRENET-SERRET FRAME

Following the classical formulation of differential geometry [4], the Frenet–Serret frame is defined as a fundamental tool for analysing vehicle motion along the TECET curve. Given a point $((u), y(u))$ on the trajectory, the tangent vector T and the normal vector N are defined, allowing the vehicle's velocity and acceleration to be decomposed into geometric components.

A. Tangent and Normal vectors

Starting from the parametrization:

$$\boldsymbol{r}(u) = (x(u), y(u)) \qquad \text{(IV.A.1)}$$

the unit tangent vector $\boldsymbol{T}(u)$ and the unit normal vector $\boldsymbol{N}(u)$ are defined as:

$$\boldsymbol{T}(u) = \frac{\boldsymbol{r}'(u)}{\|\boldsymbol{r}'(u)\|}, \quad \boldsymbol{N}(u) = \langle -T_y(u), T_x(u) \rangle \qquad \text{(IV.A.2)}$$

Here, (u) represents the instantaneous direction of motion along the curve, while (u) is obtained by a 90° rotation of $\boldsymbol{T}(u)$, pointing toward the center of curvature. Together, these vectors form an orthonormal basis for analysing motion along the curve.

B. Curvature

In differential geometry, the curvature (u) of a planar parametric curve is given by the classical determinant formula:

$$\kappa(u) = \frac{x'(u) \cdot y''(u) - y'(u) \cdot x''(u)}{(x'(u)^2 + y'(u)^2)^{\frac{3}{2}}} \qquad \text{(IV.B.1)}$$

This expression measures how sharply the trajectory bends at each point. For a straight tangent, $\kappa = 0$; for a circular arc of radius , $\kappa = 1 / R_c$. Along the TECET curve, curvature varies linearly in the spirals and remains constant in the circular arc, reflecting the desired smoothness (C^2 continuity) of the alignment.

C. Velocity and Acceleration

Let V denote the scalar speed of the vehicle, assumed constant in geometric design. The velocity vector and acceleration vector are expressed as:

$$\boldsymbol{V}(u) = V \cdot \boldsymbol{T}(u) \tag{IV.C.1}$$

$$\boldsymbol{a}(u) = at \cdot \boldsymbol{T}(u) + V^2 \cdot \kappa(u) \cdot \boldsymbol{N}(u) \tag{IV.C.2}$$

where:

at : is the tangential acceleration (associated with changes in speed magnitude),

$V^2 \cdot \kappa(u)$: is the normal acceleration (centripetal), directly proportional to the curvature.

In road design, it is common to assume $at \approx 0$, since vehicles are expected to maintain nearly constant speed along the designed alignment. Under this assumption, the acceleration is purely normal and depends only on the square of the velocity and the curvature of the path. This highlights why controlling curvature through proper spiral transitions is essential for ensuring both safety (by limiting lateral acceleration) and comfort (by avoiding abrupt changes in direction).

Figure 2 shows a schematic of the Frenet–Serret frame in the plane, illustrating the vectors involved, where in this case our focus is on (u) and (u).

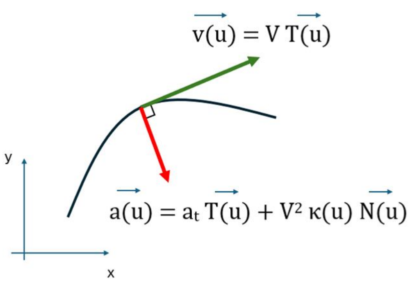

Figure 2. Frenet-Serret fame in the plane.

Figures 3, 4, and 5 respectively show the results of the formulation presented in this article implemented in GeoGebra. These figures illustrate the C^2 piecewise smooth continuity of the TECET curve, as well as how the lateral acceleration vector decreases when the vehicle transitions from the circular arc to the spiral curve and subsequently reduces to zero upon entering the exit tangent.

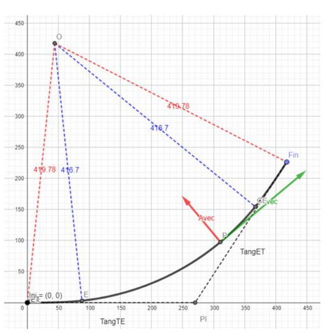

Fig. 2 Vehicle traveling along the circular arc.

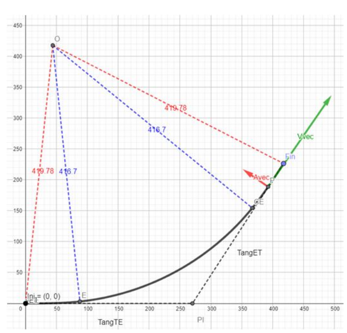

Fig. 3 Vehicle traveling along the exit spiral.

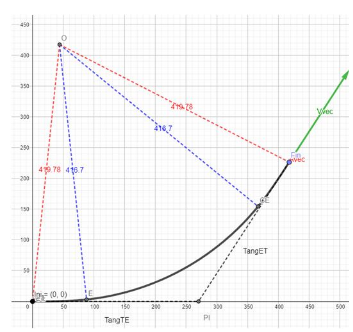

Fig. 4 Vehicle at the end of the spiral.

D. Dynamic example

Consider a vehicle traveling at V=110 km/h = 30.56 m/s. On the circular arc of radius R_c = 416.698 m, the normal (centripetal) acceleration is:

$$a_n = V^2 R_c \approx 2.24 \text{ m/s}^2$$

In the transition spiral, curvature increases linearly with the arc length. For the given speed, let the spiral length be L_e = 150 m

According to equation (II.C.2) :

$$A^2 = R_c \cdot L_e = 416.698 \times 150 \approx 62{,}505$$

The normal acceleration as a function of arc length s along the spiral is:

$$a_n(s) = \frac{V^2}{R_c} \cdot \frac{s}{L_e} = \frac{30.56^2}{62{,}505} s \approx 0.01493\, s$$

TABLE I

RESULTS AT SELECTED POINTS

s (m)	$a_n(s)$ (m/s²)
0.00	0.00
37.50	0.56
75.00	1.12
112.50	1.68
150.00	2.24

Thus, along the entry spiral, the normal acceleration increases linearly from zero to V^2/R_c. Conversely, along the exit spiral, the normal acceleration decreases from V^2/R_c back to zero.

This progressive behavior illustrates the essential role of transition spirals in highway design:

- They avoid abrupt changes in lateral acceleration.
- They improve driver's comfort by ensuring smooth motion.
- They enhance safety by reducing the risk of vehicle instability when entering or leaving a curve.

In practice, this confirms why highway design standards [1] and [2] prescribe the use of spiral transitions: they provide not only mathematical C^2 continuity but also dynamic continuity, linked to the physical experience of road users.

V. DISCUSSION AND DESIGN RECOMMENDATIONS

The parametric analysis of the TECET curve is not only an academic tool but also provides practical criteria for geometric design. The selection of parameters L_e, R_c, and e (superelevation) must satisfy the requirements of safety, comfort, and economy.

A. Minimum Spiral Length

One of the most important conditions is that the spiral length must be sufficient to ensure a gradual variation in lateral acceleration. Various design manuals, including [1] and [2], propose expressions that relate L_e to the design speed V_p and the curve radius R_c. A general form is:

$$L_e \geq \frac{V_p^3}{C \cdot R_c} \qquad \text{(V.A.1)}$$

where C is an empirical constant, whose value depends on the adopted comfort criteria. In practice, this requirement tends to be more restrictive than others, such as those based on superelevation.

B. Superelevation

Superelevation e allows compensation of the centrifugal force experienced by vehicles when traveling along horizontal curves. The balance of lateral forces express as:

$$a_{lat} = \frac{V^2}{R_c} - g \cdot e \quad \text{(V.B.1)}$$

where a_{lat} is the uncompensated lateral acceleration, V is the operating speed, R_c is the curve radius, and g is gravitational acceleration. The design objective is to keep a_{lat} within acceptable limits, which implies an adequate combination of R_c and e.

C. Practical Recommendations

The use of transition spirals in horizontal curves significantly reduces the effects of sudden lateral acceleration. In highway projects, the following recommendations are suggested:

1) Adopt spiral lengths that simultaneously satisfy both superelevation and comfort criteria.
2) Avoid excessively small radios that compromise safety.
3) Consider coordination between horizontal and vertical alignment to improve sight distance.
4) Implement parametric models in dynamic environments (e.g., GeoGebra, CAD) to validate design behavior prior to construction.

VI. CONCLUSIONS

The compound Tangent–Spiral–Circular–Spiral–Tangent (TECET) curve represents a paradigmatic example of the connection between fundamental mathematics and applied civil engineering. The analysis developed in this work allows us to draw several conclusions:

1) The use of the Euler spiral ensures that curvature varies linearly with arc length, providing a progressive increase or decrease in lateral acceleration. This results in safer and more comfortable trajectories for road users.
2) Parametrization by means of a dimensionless parameter $u \in [0, 1]$ facilitates the mathematical description of the TECET curve as a piecewise smooth function, enabling dynamic analysis of position, orientation, and curvature at any point along the trajectory.
3) The presented formulation is compatible with symbolic and numerical computation tools such as GeoGebra, which makes it possible to visualize the behaviour of the curve as well as the velocity and acceleration vectors interactively. This opens didactic opportunities for early semesters of civil engineering education.
4) The Frenet–Serret framework, applied to the parametrization of the curve, provides a solid basis for evaluating the kinematic components of a vehicle, showing how abstract notions of vector calculus have a direct application in the professional practice of highway design.
5) Finally, the proposed methodology highlights the importance of mathematical training in civil engineering: courses such as calculus, analytic geometry, and vector analysis are not isolated subjects, but essential tools for understanding and developing solutions in real roadway infrastructure projects.

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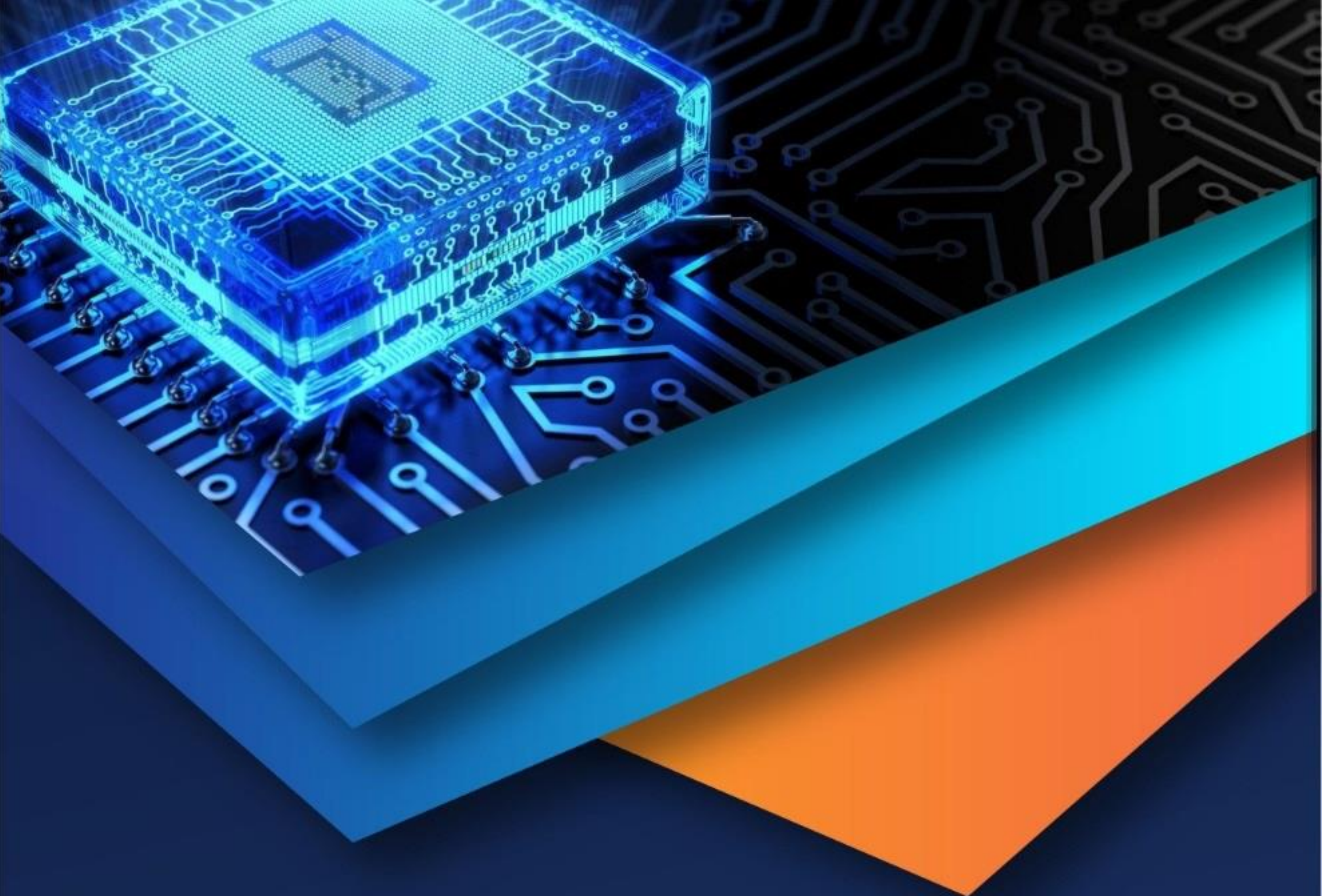

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