iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 13 **Issue:** III **Month of publication:** March 2025

DOI: https://doi.org/10.22214/ijraset.2025.67413

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 13 Issue III Mar 2025- Available at www.ijraset.com

Pellian Equation Involving Factorian Number

P. Saranya[1], B. Umadevi[2]
[1]*Associate Professor,* [2]*PG Student, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), (Affiliated to Bharathidasan University), Tiruchirapalli-620018*

Abstract: ***The positive Pellian binary quadratic equation*** $145X^2+1=Y^2$ ***is studied for its distinct integer solutions, along with an analysis of several interesting relationships among them. Furthermore, by utilizing the solutions various hyperbolas, parabolas, and special Pythagorean triangles are generated.***
Key Word: Pell equation, Integer solutions, Factorion number.

I. INTRODUCTION

The Pell equation is a Diophantine equation of the form $Dx^2\pm1=y^2$, where D is a given positive square-free integer. As one of the oldest, Diophantine equations has captivated mathematicians across the world since ancient times. J.L Lagrange established a criterion for its solvability, proving that for any positive non-square integer D, the equation always possesses integer solutions.

The Pell equation is a fundamental concept in number theory, playing a crucial role in solving Diophantine equations that require integer solutions. It has enormous applications in areas like cryptography, algebraic number theory, and the study of quadratic forms. This equation demonstrates how ancient mathematical challenges continue to provide meaningful insights into modern mathematics.

A factorion number is a special type of number in which the sum of the factorials of its digits is equal to the number itself. A number n is a factorion number if $n=\sum_{i=1}^{k}$ digit i! Examples of Factorion number are 1,2,145 and 40585.

In this study, we investigate the positive Pell equation and establish the existence of infinitely many integer solutions. Furthermore, we explore various interesting relationships among the solutions.

II. METHOD OF ANALYSIS

The positive Pellian equation representing hyperbola under consideration is,

$$145x^2+1=y^2 \qquad (1)$$

The smallest positive integer solution for (1) is,

$x_0=1, y_0=13$

To obtain the other solutions of (1), consider the Pell equation

$$145x^2+1=y^2$$

Here $(\tilde{x}_0,\tilde{y}_0)=(24,289)$, where D =145,

Applying Brahmagupta lemma between $(\tilde{x}_0,\tilde{y}_0)$ and (x_0,y_0),the other integer solutions of (1) are given by,

$$\tilde{x}_n=\frac{1}{2\sqrt{145}}g_n\ \tilde{y}_n=\frac{1}{2}f_n$$

where

$$f_n=\left(289+24\sqrt{145}\right)^{n+1}+\left(289-24\sqrt{145}\right)^{n+1}$$

$$g_n=\left(289+24\sqrt{145}\right)^{n+1}-\left(289-24\sqrt{145}\right)^{n+1}$$

$$290x_{n+1}=145f_n+13\sqrt{145}g_n$$

$$2y_{n+1}=13f_n+\sqrt{145}g_n$$

©IJRASET: All Rights are Reserved

The recurrence relations satisfied by x and y are given by,

$$x_{n+3} = 578\,x_{n+2} - x_{n+1}$$

$$y_{n+3} = 578\,y_{n+2} - y_{n+1}$$

Some numerical examples of x and y satisfying (1) is given in the following table:

n	x_{n+1}	y_{n+1}
0	1	13
1	601	7237
2	347377	4182973
3	200783305	2417751157

We observe some interesting relations among the solutions which are presented below;

- Each of the following expressions is a perfect square.

1. $\frac{1}{8352}\left[377x_{2n+3} - 209873x_{2n+2} + 16704\right]$
2. $\frac{1}{4827456}\left[377x_{2n+4} - 121306217x_{2n+2} + 9654912\right]$
3. $\frac{1}{12}\left[13y_{2n+2} - 145x_{2n+2} + 24\right]$
4. $\frac{1}{3468}\left[13y_{2n+3} - 87145x_{2n+2} + 6936\right]$
5. $\frac{1}{2004492}\left[13y_{2n+4} - 50369665x_{2n+2} + 4008984\right]$

- Each of the following expression is a nasty number

1. $\frac{6}{8352}\left[377x_{2n+3} - 209873x_{2n+2} + 16704\right]$
2. $\frac{6}{4827456}\left[377x_{2n+4} - 121306217x_{2n+2} + 9654912\right]$
3. $\frac{6}{12}\left[13y_{2n+2} - 145x_{2n+2} + 24\right]$
4. $\frac{6}{3468}\left[13y_{2n+3} - 87145x_{2n+2} + 6936\right]$
5. $\frac{6}{2004492}\left[13y_{2n+4} - 50369665x_{2n+2} + 4008984\right]$

- Each of the following expressions is a cubical integer

1. $\frac{1}{8352}\left[377x_{3n+4} - 209873x_{3n+3} + 1131x_{n+2} - 629619x_{n+1}\right]$
2. $\frac{1}{4827456}\left[377x_{3n+5} - 121306217x_{3n+3} + 1131x_{n+3} - 363918651x_{n+1}\right]$
3. $\frac{1}{12}\left[13y_{3n+3} - 145x_{3n+3} + 39y_{n+1} - 435x_{n+1}\right]$

4. $\frac{1}{3468}\left[13y_{3n+4}-87145x_{3n+3}+39y_{n+2}-261435x_{n+1}\right]$
5. $\frac{1}{2004492}\left[13y_{3n+5}-50369665x_{3n+3}+39y_{n+3}-151108995x_{n+1}\right]$

III. REMARKABLE OBSERVATIONS

Employing linear combinations among the solution of (1), one may generate integer solution for other choices of hyperbola which are presented in the table:

HYPERBOLA

S.No	Hyperbola
1	$x^2-145y^2=2790236160$
2	$x^2-145y^2=9321732572\ 0000$
3	$145x^2-y^2=334080$
4	$145x^2-y^2=2790269568$
5	$145x^2-y^2=9321732572$

PARABOLA

S.No	Parabola
1	$576x-y^2=192430080$
2	$332928x-y^2=6428781084\,00$
3	$3480x-y^2=334080$
4	$1005720x-y^2=2790269568$
5	$5813026799x-y^2=9321732572$

IV. CONCLUSION

This paper examines integer solutions to the positive Pell equation involving Factorian numbers. From the diverse nature of Diophantine equations, future research can build on this approach by investigating various types of Factorian numbers within different mathematical contexts.

REFERENCES

[1] AI - Zaid. H, Brindza .B and Printer .A, "On Positive integer solutions of the Equation $xy+yz+zn=n''$, Canad.Math.Bull, Vol.39,pp.199,1996.

[2] Burton D; Elementary Number Theory. Tata Hill Publishing Company Ltd., New Delhi,2002.

[3] Dickson LE; History of Theory of Numbers. Volume 2,Chelsea Publishing Company, New York, 1952.

[4] Gopalan M.A, Janaki G; Observation on y 2 = 3x 2 + 1.Acta Ciancia Indica, 2008;XXXIVM(2):693-696.

[5] Janaki G, Saranya P, Half companion sequences of Dio 3-tuples from $\frac{CC_n}{Gno_n}$, International Journal for Science and Advance Research in Technology, Vol 4, Issue 4 , April 2018, Pg No: 3013-3015,Impact Factor: 5.388, ISSN No: 2395-1052.

[6] Janaki G, Vidhya S "An integral solution of negative Pell's equation involving two-digit sphenic numbers", International Journal of Computer Science and Engineering, Vol.6,Issue.7,pp.444-445,2018.

[7] Janaki G, Vidhya S "On the integer solutions of Pell equation $x^2 - 79y^2 = 9^k$ ", International Journal of scientific Research in Science, Engineering and Technology, Vol.2, Issue.2,pp.1195-1197,2016L.K.Hua, "Introduction To The Theory Of Numbers", Springer-Verlag, Berlin-New York, 1982.

[8] Janaki G, Saranya P, On the Ternary Quadratic Diophantine Equation $5(x^2 + y^2) - 6xy = 4z^2$, Imperial Journal of Interdisciplinary Research, Vol 2, Issue 3 , Jan 2016 , Pg No: 396-397,Impact Factor: 3.75, ISSN No: 2454-1362.

[9] Janaki G, Saranya P, Connections between Cylinder Frustum of a Cone with Truncated Octahedral Number and other Special Numbers, Journal of Mathematics and Informatics, Vol 10, Special Issue , Dec 2017, Pg No: 95-101,Impact Factor: 1.627, ISSN No: 2349-0632.

[10] Janaki G, Saranya P, On the Ternary Cubic Diophantine Equation $5(x^2 + y^2) - 6xy + 4(x + y) + 4 = 40\ z^3$, International Journal of Science and Research-Online, Vol 5, Issue 3 , March 2016, Pg No: 227-229,Impact Factor: 5.611, ISSN No: 2319-7064.

[11] Janaki G, Saranya P, Special Pairs of Pythagorean Triangles and Narcissistic Number, International Journal of Multidisciplinary Research and Development, Vol 3, Issue 4 , April 2016 , Pg No: 106-108,Impact Factor: 5.72, ISSN No: 2349-4182.

[12] Mordel L.J, Diophantine Equations. Academic Press, New York 1969. [16] Telang S, Number Theory. Tata Mcgraw Hill Publishing Company Ltd., New Delhi, 2000.

[13] Saranya, C., Janani M., and Salini R. "Integer Solutions of Some Exponential Diophantine Equations." The Mathematics Education, vol. 58, no. 1, Mar. 2024.

[14] Saranya P, Janaki G, titled On the Quintic Non-Homogeneous Diophantine Equation $x^4 - y^4 = 40(z^2 - w^2)p^3$, International Journal of Engineering Science and Computing, Vol 7, Issue 2 , February 2017, Pg No: 4685-4687,Impact Factor: 5.611, ISSN No: 2321-3361.

[15] Saranya P & Janaki G, "On The Exponential Diophantine Equation $36^x + 3^y = z^2$ ", International Research Journal Of Engineering & Technology, Vol 4, Issue 11, 1042-1044, Nov 2017.

[16] Saranya P, Janaki G, Allegory Of Lateral Surface Area Of A Cube With A Special Number, International Journal Of Scientific Research And Reviews, Vol 7, Issue 4 , Oct-Dec 2018, Pg No: 2102-2110,Impact Factor: 6.946, ISSN No: 2279–0543.

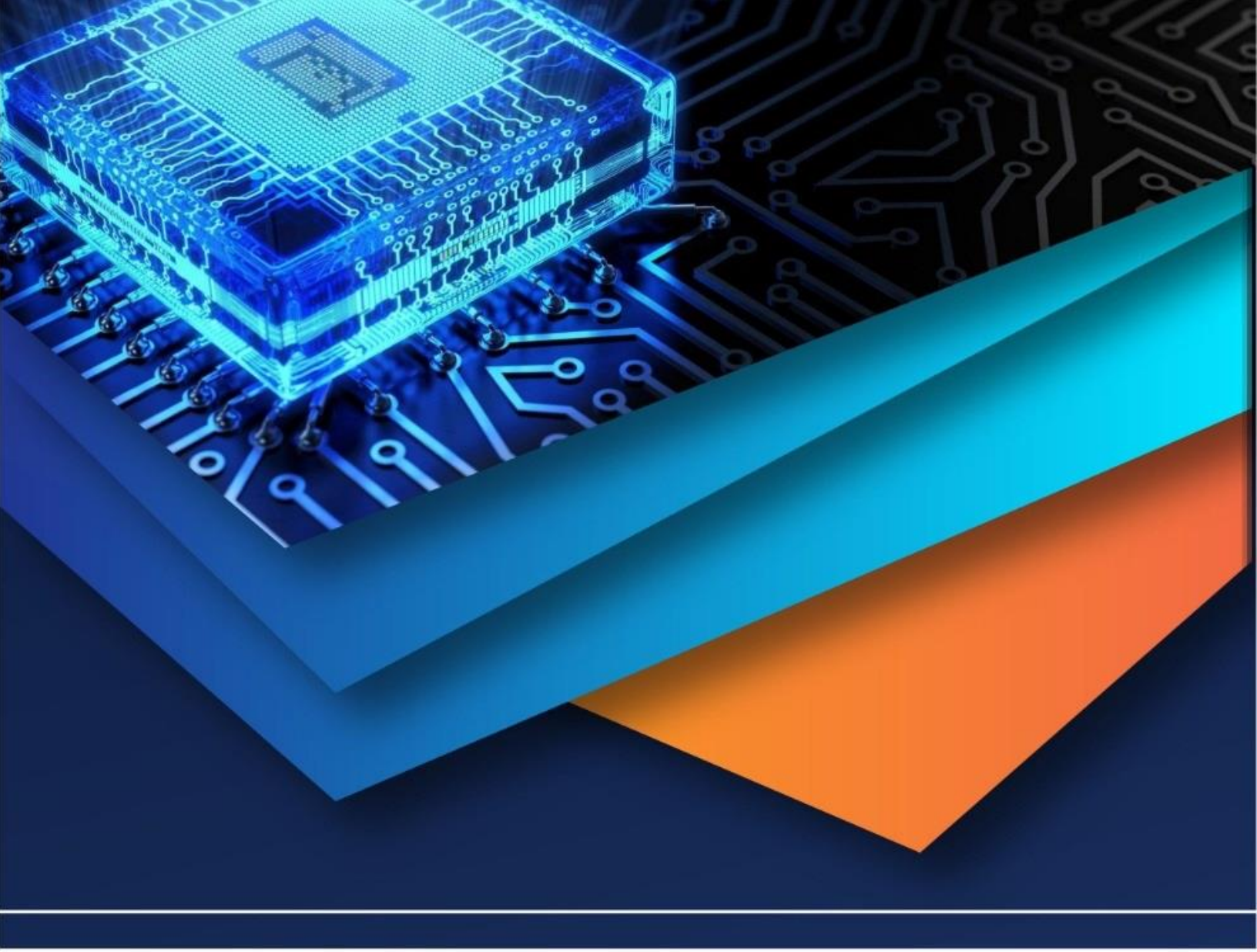
[17] Saranya P, Janaki G, Ascertainment on The Exponential Equation $p^{3c} - a(p^{2c} - p^b) = p^{b+c}$,Compliance Engineering Journal, Vol 10 , Issue 8 , August 2019, Pg No:225-229,Impact Factor: 6.1 , ISSN No: 0898-3577.(UGC Approved).

[18] Saranya P, Janaki G, Explication of The Transcendental Equation $p + \sqrt{p^3 + q^3 - pq} + \sqrt[3]{r^2 + s^2} = (m^2 + 1)t^3$, Adalya Journal, Vol 8, Issue 9, September 2019, Pg No: 70-73Impact Factor:5.3, ISSN No:1301-2746.

[19] Saranya P, Janaki G, Ascertainment on the integral solutions of the Biquadratic Diophantine Equation $m^4 - n^4 = 4(u^2 + 1)(r - s)t^3$, Parishodh Journal , Vol IX Issue III, March 2020, Pg No: 792-800, Impact Factor: 6.3, ISSN No: 2347-6648.

[20] Saranya P, Janaki G, On some Non-Extendable Gaussian Triples involving Mersenne and Gnomonic number, Aryabhatta Journal of Mathematics and Informatics , Vol 12 Issue 1 ,Jan - June 2020, Pg No: 81 - 84 , Impact Factor: 5.856, ISSN No: 0975-7139(P).

©IJRASET: All Rights are Reserved | SJ Impact Factor 7.538 | ISRA Journal Impact Factor 7.894 |



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 (24*7 Support on Whatsapp)