iJRASET
International Journal For Research in Applied Science and Engineering Technology



INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 12 **Issue:** II **Month of publication:** February 2024

DOI: https://doi.org/10.22214/ijraset.2024.58636

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 12 Issue II Feb 2024- Available at www.ijraset.com

Proper Colorings in R-Regular Randic Index Graphs

Dr. E. Litta[1], N. Jayavarshini[2]
[1]*Associate Professor in Mathematics,* [2]*PG Student, PG and Research Department of Mathematics, Cauvery College For Women (Autonomous), Tiruchirappalli – 620018, India*

Abstract: ***The new idea of proper colourings in the r-regular Randic index graph has been proposed in this paper. New Chromatic number inequalities connected to the Randic index are established in this paper.***
Keywords: Regular graph; Proper Colouring; Randic index; Chromatic number.

I. INTRODUCTION

In this article, we simply take into consideration finite, undirected, simple graphs. The vertex set and edge set of a graph G are denoted by the letters V(G) and E(G). The order of G, denoted by p, is the cardinality of the vertex set. A (p,q) graph is defined as the cardinality of the edge set denoted by q edges. If G is a r – regular graph, then R(G) = $\frac{m}{r}$. Proper colourings in r-regular Randic Index graph is expanded by the result of proper colourings in the magic and anti-magic graphs[11,16]. Several results and theorems have been verified by the Randic Index[1,4,5,6]. This research can be expanded to include domination which is based on Domatic numbers andRandicIndex[8,9,10]. This work can also be expanded upon in the context of Automata theory[13,14,15] which has a numerous applications. There are numerous application for graph labeling in both undirected[12,17,18,22,23] and directed graphs[19,20,21].

II. MAIN RESULT

1) Definition 2.1

The Randic indices are respectively defined as $R(G) = \sum_{rs \in E(G)} \frac{1}{\sqrt{d(r) \cdot d(s)}}$, where d(s) is the degree of the vertex s in G.

2) Theorem2.1 :

If G is a r -regular Randic index graph then the chromatic number satisfies the inequality $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2} nr$, r ≥ 2.

Proof :

a) Case (i)

Let G be a graph of cycle C_n, 'n' be an odd integer.

Let C_n be r -regular with n vertices and m edges then $R(G) = \frac{m}{r}$

Let C_n be a cycle graph with Randic index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 3 .Therefore, $\psi(G) = 3$.

The following inequality is satisfied. Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \quad \text{..... (1)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2} nr.$

©IJRASET: All Rights are Reserved

Therefore $\psi(G) \leq \frac{1}{2} nr.$ ..... (2)

From the equations (1) and (2) it is easy to verify that $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\ nr$

Hence the odd cycle satisfies $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\ nr$ for 2 - regular graph.

For example if $n = 5$, the corresponding graph is shown in Fig 2.1

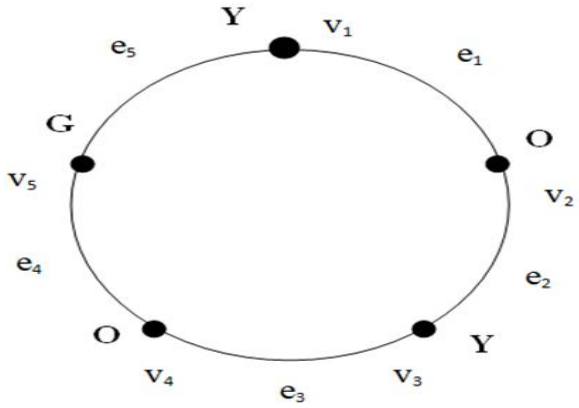


Fig 2.1Randic Index Number For Odd Cycle

K = R (C_5) = 5 ($\frac{1}{\sqrt{2.2}}$) = 2.5

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\ nr$$

$0.045 \leq 3 \leq 5.$

b) Case (ii)

Let G be a graph of cycle C_n, 'n' be an even integer.

Let C_n be r -regular with n vertices and m edges then *R (G)* = $\frac{m}{r}$

Let C_n be a cycle graph with Randic index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 2 .Therefore, $\psi(G) = 2$.

The following inequality is satisfied. Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G)$$ ..... (3)

The general condition of r -regular graph is denoted by as $\frac{1}{2} nr.$

Therefore $\psi(G) \leq \frac{1}{2} nr.$ ..... (4)

From the equations (3) and (4) it is easy to verify $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\ nr$

Hence the even cycle satisfies $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\ nr$ for 2 - regular graph.

For example if $n = 4$, the corresponding graph is shown in Fig 2.2

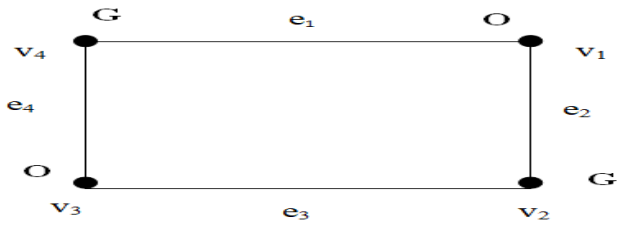


Fig 2.2 Randic Index Number ForEven Cycle

K = R (C_4) = 4 ($\frac{1}{\sqrt{2.2}}$) = 2

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}nr$$

$0.063 \leq 2 \leq 4$.

c) Case (iii)

Let the graph G be Generalized Petersen Graph, here 'n' is an even integer.

Let $V(p)=\{v_1,v_2,.....,v_{10}\}$ be the vertices and $E(p)=\{e_1,e_2,.....,e_{15}\}$ be the edges of $P(n,m)$ then *R (G)* = $\frac{m}{r}$

Let $P(n,m)$ be a Generalized Petersen Graph with Randic index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is 3 .Therefore, $\psi(P)=3$.

The following inequality is satisfied. Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \qquad \text{..... (5)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \leq \frac{1}{2}nr.$ ..... (6)

From the equations (5) and (6) it is easy to verify $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}nr$

Hence the Generalized Petersen Graph satisfies $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}nr$ for 3 - regular graphs.

The Generalized Petersen Graph is shown in Fig 2.3

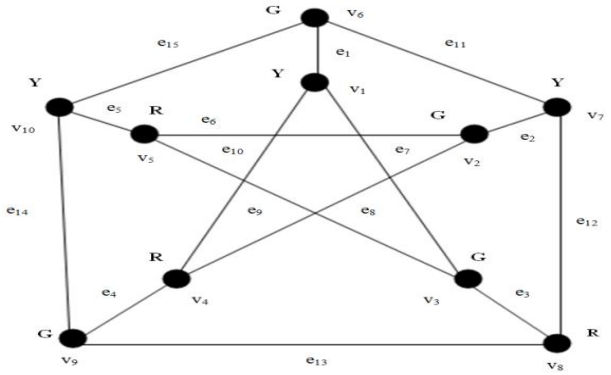


Fig 2.3Randic Index Number For Generalized Petersen Graph

K = R (P_{15}) = 15 ($\frac{1}{\sqrt{3.3}}$) = 5

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}nr$$

The inequality of Randic index for generalized Petersen graph is $0.0632 \leq 3 \leq 15$.

d) Case (iv)

Let G be a complete graph, 'n' be an any integer.

Let $V = \{v_1, v_2, \ldots\ldots, v_n\}$ be the vertices and $E = \{e_1, e_2, \ldots\ldots, e_n\}$ be the edges of k_n, then $R(G) = \frac{m}{r}$

Let k_n be a complete Graph with Randic index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is n .Therefore, $\psi(k_n) = n$.

The following inequality is satisfied. Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \qquad \ldots\ldots\ldots (7)$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \leq \frac{1}{2}nr.$ ..... (8)

From the equations (7) and (8) it is easy to verify $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}nr$

Hence the complete Graph satisfies $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}nr$ for n - regular graphs.

For example if $n = 5$, the corresponding graph is shown in Fig 2.4

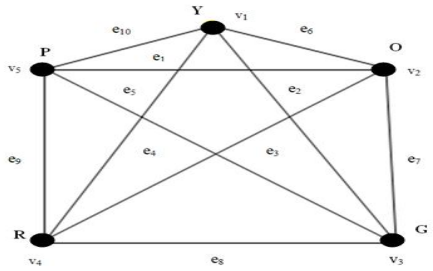


Fig 2.4 Randic Index Number For Complete Graph

K = R (K_5) = 10 ($\frac{1}{\sqrt{4.4}}$) = 2.5

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}nr$$

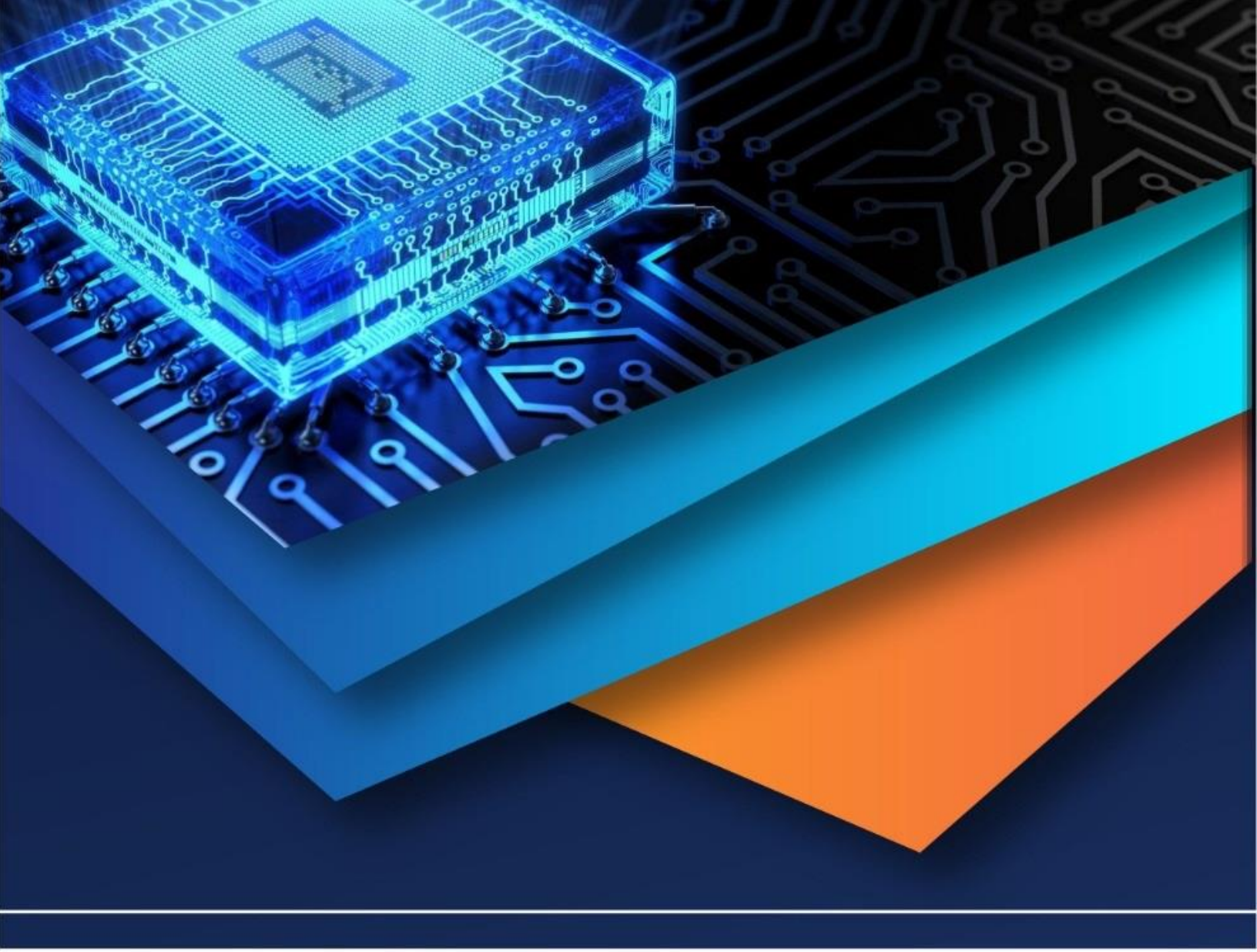
The inequality of Randic index for complete graph is $0.029 \leq 5 \leq 10$.

III. CONCLUSION

In this paper, new inequality has been established. Further, it has been verified for Randic Index. Finally, we conclude that new inequalities in chromatic numbers are related to RandicIndex.

REFERENCES

[1] AmalorpavaJerline J., Litta E., Dhanalakshmi K., Benedict Michael Raj L.,F-Index of Generalized Mycielskian Graphs, International Journal of Recent Scientific Research, Volume 10, Issue 6, June 2019,P.No.:32366-32371.
[2] Bodendick, R. and Walther, G., On number theoretical methods in graph labelingsRes.Exp.Maths (2,/1995) 3-25.
[3] Bloom, D.F. Hsu, On graceful directed graphs, SIAMJ, Alg. Discrete Math.,6(1985),519-536.
[4] Felix J., Litta E., Benedict Micheal Raj L., Changing and Unchanging properties of Single Chromatic Transversal Domination Number of Graphs, International Journal of Mathematics Trends and Technology, Volume 52, Issue 4, Dec 2017,P.No.:262-266.
[5] Felix J., Litta E., Benedict Micheal Raj L., Single Chromatic Transversal Dominating Irredundant Number for odd cycles, Peterson graph and Mycielski graph, Infokara Research, Volume 8, Issue 10, Oct 2019,P.No.:139-145.
[6] Felix J., Litta E., Benedict Micheal Raj L., Single Chromatic Transversal Dominating Irredundant Number of graphs, Adalya Journal, Volume 10, Issue 8, Oct 2019,P.No.:264-272.
[7] Harary, F., Graph Theory, New Delhi: Narosa Publishing House, 2001.
[8] Litta E., AmalorpavaJerline J.,DhanalakshmiK.,BenedictMicheal Raj L.,andModified Zagreb Indices of Bridge Graphs, International Journal of Mathematical Archive, Volume 8, Issue 3, Mar 2017,P.No.: 86-91.
[9] Litta E., AmalorpavaJerline J., Rasika R., F- Co-index of Generalized Mycielskian Graphs, International Journal of Research in Advent Technology, Volume 7, Issue 4, Apr 2019,P.No.:243-249.
[10] Litta E., AmalorpavaJerline J., Felix J., Benedict Micheal Raj L.,First and Second Modified Zagreb Indices of Product Graphs, InfokaraResearch,Volume 9, Issue 1, Jan 2020,P.No.: 279-293.
[11] Litta, E., Datchini, S, Proper Colourings in r – Regular Zagreb Index Graph", Aryabhatta Journal of Mathematics and Informatics, Volume 15, Issue 1, Jan - June 2023. Pg. No. 149-154, Impact Factor: 5.856, ISSN No: 0975-7139(P) 2394-9309.
[12] Prakash, V., Gopi, R., Shalini, P., Anti Skolem Mean Labeling of Quadrilateral Snake Related Graphs, TuijinJishu/Journal of Propulsion Technology, Vol. 44, No. 5(2023), 495 – 500.
[13] Saridha, S. and HaridhaBanu. S, A New Direction Towards Plus weighted Grammar, International Journal for Research in Applied Science and Engineering Technology (IJRASET), ISSN: 2321 – 9653, Vol. 11, Issue II, February 2023.
[14] Saridha, S. and Jothika, T, An Introduction to Plus Weighted Dendrolanguage and its properties, Aryabhatta Journal of Mathematics and Informatics, Volume 15, Issue 1, Jan -June 2023. Pg. No. 93-100, Impact Factor: 5.856, ISSN No: 0975-7139(P) 2394-9309.
[15] Saridha, S. and HaridhaBanu. S, An Innovative ideas on plus weighted linear grammer, Aryabhatta Journal of Mathematics and Informatics, Volume 15, Issue 2, December 2023. Pg. No. 173 – 178.
[16] Shalini. P, Paul Dhayabaran. D., Proper Colourings in Magic and Anti-magic Graph, International Journal of Engineering and Research Technology, Vol. 3, Issue. 2, pages 815-818. February 2014.
[17] Shalini, P., Paul Dhayabaran, D., An Absolute Differences of Cubic and Square Difference Labeling, International Journal of Advanced Scientific and Technical Research, May-June 2015, Issue-5, Volume-3, pages 1-8.
[18] Shalini, P., Paul Dhayabaran, D., A Study on Root Mean Square Labelings in Graphs, International Journal of Engineering Science and Innovative Technology, May 2015, Volume-4, Issue-3, pages 305-309.
[19] Shalini, P., Gowri, R., Paul Dhayabaran, D., An Absolute Differences of Cubic and Square Difference Labeling For Some Families Of Graphs, The International journal of analytical and experimental modal analysis, Volume XI, Issue 10, October 2019, Page no: 538-544.
[20] Shalini, P., Meena, S.A., Lehmer-4 Mean Labeling for Some Path Related Graphs, Aryabhatta Journal of Mathematics and Informatics, Volume 15, Issue 1, Jan -June 2023. Pg. No. 105-110.
[21] Shalini, P., Skolem Odd Vertex Graceful Signed Graphs for Star Graphs, International Journal of Mathematics Trends and Technology, Volume 69, Issue 8, August 2023, Pg. No. 30 – 35.
[22] Shalini, P., Paul Dhayabaran, D., Maximization of Multiplicative Labeling, International journal of Research in Advent Technology(IJRAT), Special Issue January 2019, Page no: 209-214.
[23] Shalini, P., Priyadarshini, K., SP Mean Difference Labeling for Some Families of Graphs", Research and Applications Towards Mathematics and Computer Science, Volume 9, February 2024, Pg. No. 1-12, B P International, Print ISBN: 978-81-970187-8-7, eBook ISBN: 978-81-970187-9-4.



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089 (24*7 Support on Whatsapp)