# Proper Colorings in R-Regular Randic Index Graphs 

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Abstract: The new idea of proper colourings in the r-regular Randic index graph has been proposed in this paper. New Chromatic number inequalities connected to the Randic index are established in this paper.<br>Keywords: Regular graph; Proper Colouring; Randic index; Chromatic number.

## I. INTRODUCTION

In this article, we simply take into consideration finite, undirected, simple graphs. The vertex set and edge set of a graph $G$ are denoted by the letters $V(G)$ and $E(G)$. The order of $G$, denoted by $p$, is the cardinality of the vertex set. A ( $\mathrm{p}, \mathrm{q}$ ) graph is defined as the cardinality of the edge set denoted by $q$ edges. If G is a $r$-regular graph, then $\mathrm{R}(\mathrm{G})=\frac{m}{r}$. Proper colourings in r -regular Randic Index graph is expanded by the result of proper colourings in the magic and anti-magic graphs[11,16]. Several results and theorems have been verified by the Randic Index[1,4,5,6]. This research can be expanded to include domination which is based on Domatic numbers andRandicIndex $[8,9,10]$. This work can also be expanded upon in the context of Automata theory[13,14,15] which has a numerous applications. There are numerous application for graph labeling in both undirected $[12,17,18,22,23]$ and directed graphs[19,20,21].

## II. MAIN RESULT

## 1) Definition 2.1

The Randic indices are respectively defined as $R(G)=\sum_{r s \in E(G)} \frac{1}{\sqrt{d(r) \cdot d(s)}}$, where $\mathrm{d}(\mathrm{s})$ is the degree of the vertex s in G .
2) Theorem2.1:

If $G$ is a $r$-regular Randic index graph then the chromatic number satisfies the inequality $\left|\frac{k+r}{(v+E)}\right| \leq \psi(G) \leq \frac{1}{2} n r, \mathrm{r} \geq 2$.
Proof:
a) Case (i)

Let $G$ be a graph of cycle $C_{n}$, ' $n$ ' be an odd integer.
Let $C_{n}$ be $r$-regular with $n$ vertices and $m$ edges then $R(G)=\frac{m}{r}$
Let $C_{n}$ be a cycle graph with Randic index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 3 . Therefore, $\psi(G)=3$.
The following inequality is satisfied. Since K is the index number, $r$ is the regular graph, $V$ is the number of vertices and $E$ is the number of edges in graph $G$.
The following inequality is obtained.
$\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G)$
The general condition of $r$-regular graph is denoted by as $\frac{1}{2} n r$.

Therefore $\psi(G) \leq \frac{1}{2} n r$.
From the equations (1) and (2) it is easy to verify that $\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$
Hence the odd cycle satisfies $\left|\frac{k+r}{(v+E)^{2}}\right| \leq \varphi(G) \leq \frac{1}{2} n r$ for 2 - regular graph.
For example if $n=5$, the corresponding graph is shown in Fig 2.1


Fig 2.1Randic Index Number For Odd Cycle
$\mathrm{K}=\mathrm{R}\left(\boldsymbol{C}_{5}\right)=5\left(\frac{1}{\sqrt{2.2}}\right)=2.5$
$\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$
$0.045 \leq 3 \leq 5$.
b) Case (ii)

Let $G$ be a graph of cycle $C_{n}$, ' $n$ ' be an even integer.
Let $C_{n}$ be $r$-regular with $n$ vertices and $m$ edges then $R(G)=\frac{m}{r}$
Let $C_{n}$ be a cycle graph with Randic index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 2 . Therefore, $\psi(G)=2$.
The following inequality is satisfied. Since K is the index number, $r$ is the regular graph, $V$ is the number of vertices and $E$ is the number of edges in graph $G$.
The following inequality is obtained
$\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G)$
The general condition of $r$-regular graph is denoted by as $\frac{1}{2} n r$.
Therefore $\psi(G) \leq \frac{1}{2} n r$.
From the equations (3) and (4) it is easy to verify $\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$
Hence the even cycle satisfies $\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$ for 2 - regular graph.
For example if $n=4$, the corresponding graph is shown in Fig 2.2


Fig 2.2 Randic Index Number ForEven Cycle
$\mathrm{K}=\mathrm{R}\left(\mathcal{C}_{4}\right)=4\left(\frac{1}{\sqrt{2 \cdot 2}}\right)=2$
$\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$
$0.063 \leq 2 \leq 4$.
c) Case (iii)

Let the graph $G$ be Generalized Petersen Graph, here ' $n$ ' is an even integer.
Let $V(p)=\left\{v_{1}, v_{2}, \ldots \ldots, v_{10}\right\}$ be the vertices and $E(p)=\left\{e_{1}, e_{2}, \ldots \ldots, e_{15}\right\}$ be the edges of $P(n, m)$ then $R(G)=\frac{m}{r}$
Let $P(n, m)$ be a Generalized Petersen Graph with Randic index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is 3 . Therefore, $\psi(P)=3$.
The following inequality is satisfied. Since K is the index number, $r$ is the regular graph, $V$ is the number of vertices and $E$ is the number of edges in graph $G$.

The following inequality is obtained.
$\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G)$
The general condition of $r$-regular graph is denoted by as $\frac{1}{2} n r$.
Therefore $\psi(G) \leq \frac{1}{2} n r$.
From the equations (5) and (6) it is easy to verify $\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$
Hence the Generalized Petersen Graph satisfies $\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$ for 3 - regular graphs.
The Generalized Petersen Graph is shown in Fig 2.3


Fig 2.3Randic Index Number For Generalized Petersen Graph
$\mathrm{K}=\mathrm{R}\left(\boldsymbol{P}_{15}\right)=15\left(\frac{\mathbf{1}}{\sqrt{\mathbf{3 . 3}}}\right)=5$
$\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$
The inequality of Randic index for generalized Petersen graph is $0.0632 \leq 3 \leq 15$.
d) Case (iv)

Let $G$ be a complete graph, ' $n$ ' be an any integer.
Let $V=\left\{v_{1}, v_{2}, \ldots \ldots \ldots, v_{n}\right\}$ be the vertices and $E=\left\{e_{1}, e_{2}, \ldots \ldots ., e_{n}\right\}$ be the edges of $k_{n}$, then $R(G)=\frac{m}{r}$
Let $\mathrm{k}_{\mathrm{n}}$ be a complete Graph with Randic index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is $n$.Therefore, $\psi\left(k_{n}\right)=n$.
The following inequality is satisfied. Since K is the index number, r is the regular graph, $V$ is the number of vertices and $E$ is the number of edges in graph $G$.

The following inequality is obtained.
$\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G)$
The general condition of $r$-regular graph is denoted by as $\frac{1}{2} n r$.
Therefore $\psi(G) \leq \frac{1}{2} n r$.
From the equations (7) and (8) it is easy to verify $\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$
Hence the complete Graph satisfies $\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$ for $n$-regular graphs.
For example if $n=5$, the corresponding graph is shown in Fig 2.4


Fig 2.4 Randic Index Number For Complete Graph
$\mathrm{K}=\mathrm{R}\left(K_{5}\right)=10\left(\frac{1}{\sqrt{4.4}}\right)=2.5$
$\left|\frac{k+r}{(v+E)^{2}}\right| \leq \psi(G) \leq \frac{1}{2} n r$
The inequality of Randic index for complete graph is $0.029 \leq 5 \leq 10$.

## III. CONCLUSION

In this paper, new inequality has been established. Further, it has been verified for Randic Index. Finally, we conclude that new inequalities in chromatic numbers are related to RandicIndex.

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