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Proper Colorings in R-Regular Randic Index Graphs

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***Abstract:** The new idea of proper colourings in the r-regular Randic index graph has been proposed in this paper. New Chromatic number inequalities connected to the Randic index are established in this paper.*
***Keywords:** Regular graph; Proper Colouring; Randic index; Chromatic number.*

I. INTRODUCTION

In this article, we simply take into consideration finite, undirected, simple graphs. The vertex set and edge set of a graph G are denoted by the letters V(G) and E(G). The order of G, denoted by p, is the cardinality of the vertex set. A (p,q) graph is defined as the cardinality of the edge set denoted by q edges. If G is a r – regular graph, then R(G) = $\frac{m}{r}$. Proper colourings in r-regular Randic Index graph is expanded by the result of proper colourings in the magic and anti-magic graphs[11,16]. Several results and theorems have been verified by the Randic Index[1,4,5,6]. This research can be expanded to include domination which is based on Domatic numbers andRandicIndex[8,9,10]. This work can also be expanded upon in the context of Automata theory[13,14,15] which has a numerous applications. There are numerous application for graph labeling in both undirected[12,17,18,22,23] and directed graphs[19,20,21].

II. MAIN RESULT

1) Definition 2.1

The Randic indices are respectively defined as$R(G) = \sum_{rs\in E(G)} \frac{1}{\sqrt{d(r)\cdot d(s)}}$, where d(s) is the degree of the vertex s in G.

2) Theorem2.1 :

If G is a r -regular Randic index graph then the chromatic number satisfies the inequality $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2} nr$, r ≥ 2.

Proof :

a) Case (i)

Let G be a graph of cycle C_n, 'n' be an odd integer.

Let C_n be r -regular with n vertices and m edges then $R(G) = \frac{m}{r}$

Let C_n be a cycle graph with Randic index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 3 .Therefore, $\psi(G) = 3$.

The following inequality is satisfied. Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \qquad \text{......... (1)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr$.

Therefore $\psi(G) \le \frac{1}{2} nr.$ .... (2)

From the equations (1) and (2) it is easy to verify that $\left|\frac{k+r}{(v+E)^2}\right| \le \psi(G) \le \frac{1}{2} nr$

Hence the odd cycle satisfies $\left|\frac{k+r}{(v+E)^2}\right| \le \psi(G) \le \frac{1}{2} nr$ for 2 - regular graph.

For example if $n = 5$, the corresponding graph is shown in Fig 2.1

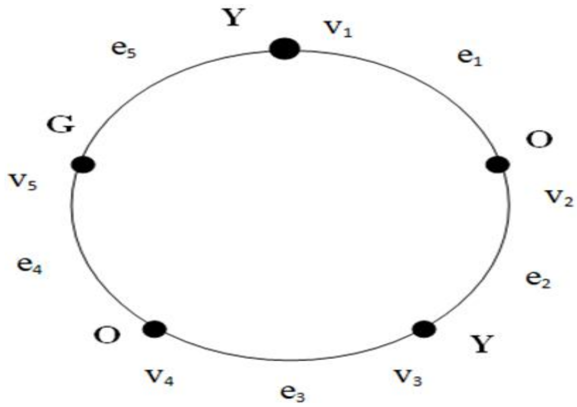

Fig 2.1Randic Index Number For Odd Cycle

K = R (C_5) = 5 ($\frac{1}{\sqrt{2.2}}$) = 2.5

$$\left|\frac{k+r}{(v+E)^2}\right| \le \psi(G) \le \frac{1}{2} nr$$

$0.045 \le 3 \le 5.$

b) Case (ii)

Let G be a graph of cycle C_n, 'n' be an even integer.

Let C_n be r -regular with n vertices and m edges then $R(G) = \frac{m}{r}$

Let C_n be a cycle graph with Randic index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 2 .Therefore, $\psi(G) = 2$.

The following inequality is satisfied. Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained

$$\left|\frac{k+r}{(v+E)^2}\right| \le \psi(G) \quad .... (3)$$

The general condition of r -regular graph is denoted by as $\frac{1}{2} nr.$

Therefore $\psi(G) \le \frac{1}{2} nr.$ .... (4)

From the equations (3) and (4) it is easy to verify $\left|\frac{k+r}{(v+E)^2}\right| \le \psi(G) \le \frac{1}{2} nr$

Hence the even cycle satisfies $\left|\frac{k+r}{(v+E)^2}\right| \le \psi(G) \le \frac{1}{2} nr$ for 2 - regular graph.

For example if $n = 4$, the corresponding graph is shown in Fig 2.2

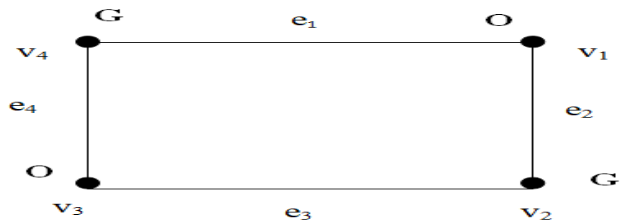

Fig 2.2 Randic Index Number ForEven Cycle

K = R (C_4) = 4 ($\frac{1}{\sqrt{2.2}}$) = 2

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\, nr$$

$0.063 \leq 2 \leq 4.$

c) Case (iii)

Let the graph G be Generalized Petersen Graph, here $'n'$ is an even integer.

Let $V(p)=\{v_1,v_2,.....,v_{10}\}$ be the vertices and $E(p)=\{e_1,e_2,.....,e_{15}\}$ be the edges of $P(n,m)$ then R (G) = $\frac{m}{r}$

Let $P(n,m)$ be a Generalized Petersen Graph with Randic index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is 3 .Therefore, $\psi(P)=3$.

The following inequality is satisfied. Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \qquad \text{......... (5)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G)\leq\frac{1}{2}nr.$ .... (6)

From the equations (5) and (6) it is easy to verify $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\, nr$

Hence the Generalized Petersen Graph satisfies $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\, nr$ for 3 - regular graphs.

The Generalized Petersen Graph is shown in Fig 2.3

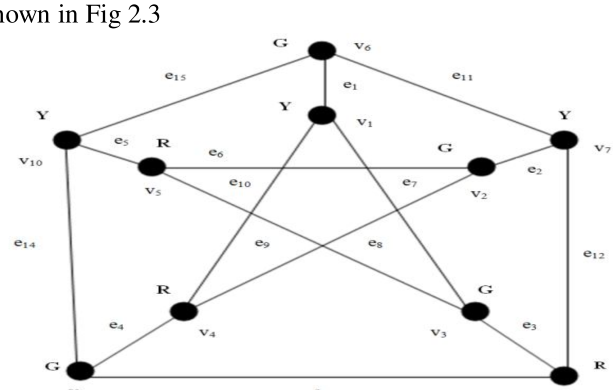

Fig 2.3Randic Index Number For Generalized Petersen Graph

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K = R (P_{15}) = 15 ($\frac{1}{\sqrt{3.3}}$) = 5

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\,nr$$

The inequality of Randic index for generalized Petersen graph is $0.0632 \leq 3 \leq 15$.

d) Case (iv)

Let G be a complete graph, $'n'$ be an any integer.

Let $V = \{v_1, v_2, ..., v_n\}$ be the vertices and $E = \{e_1, e_2, ..., e_n\}$ be the edges of k_n, then $R\,(G) = \frac{m}{r}$

Let k_n be a complete Graph with Randic index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is n .Therefore, $\psi(k_n) = n$.

The following inequality is satisfied. Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \qquad .... (7)$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \leq \frac{1}{2}nr.$ .... (8)

From the equations (7) and (8) it is easy to verify $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\,nr$

Hence the complete Graph satisfies $\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\,nr$ for n - regular graphs.

For example if $n = 5$, the corresponding graph is shown in Fig 2.4

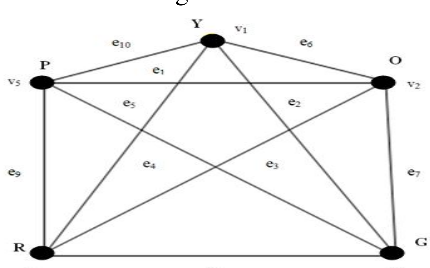

Fig 2.4 Randic Index Number For Complete Graph

K = R (K_5) = 10 ($\frac{1}{\sqrt{4.4}}$) = 2.5

$$\left|\frac{k+r}{(v+E)^2}\right| \leq \psi(G) \leq \frac{1}{2}\,nr$$

The inequality of Randic index for complete graph is $0.029 \leq 5 \leq 10$.

III. CONCLUSION

In this paper, new inequality has been established. Further, it has been verified for Randic Index. Finally, we conclude that new inequalities in chromatic numbers are related to RandicIndex.

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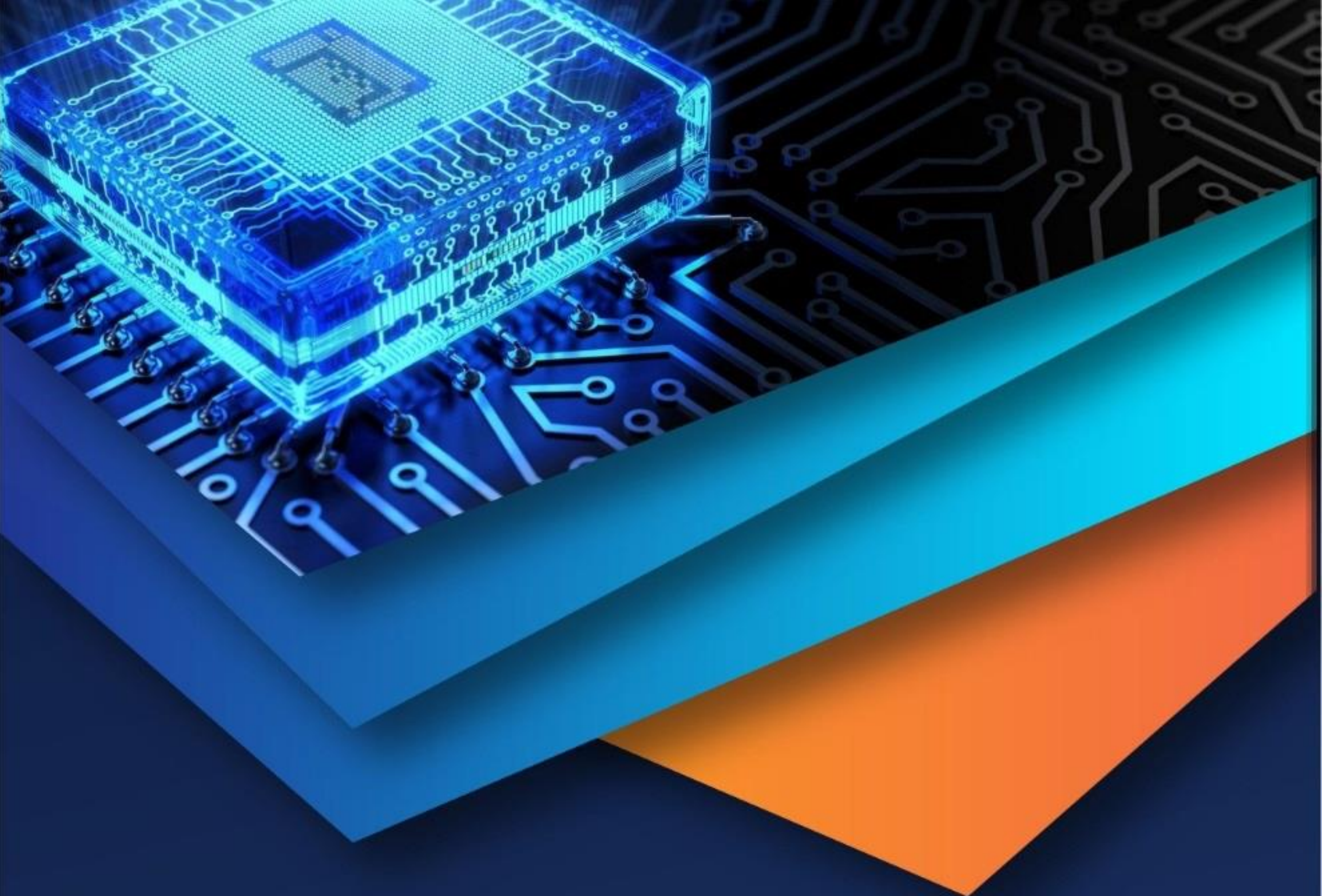

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