International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 11 **Issue:** III **Month of publication:** March 2023

DOI: https://doi.org/10.22214/ijraset.2023.49694

Proper Colourings in r-Regular Modified Zagreb Index Graph

Dr. E. Litta[1], S. Maragatha Dharshini[2]
[1]Associate Professor in Mathematics, [2]PG Student, PG and Research Department of Mathematics, Cauvery College for Women (Autonomous), Tiruchirappalli - 620018, India.

***Abstract:** In this article, the new concept proper colourings in r - regular Modified Zagreb index graph has been introduced. The first and second Modified Zagreb indices are introduced. In this article, new inequalities on chromatic number related with first and second Modified Zagreb indices are being established.*
***Keywords:** Regular graph, Proper Colouring, Modified Zagreb index, Chromatic number.*

I. INTRODUCTION

In this article, we consider only finite, simple and undirected graphs. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G denoted by p. The cardinality of edge set is called the size of G denoted by q edges is called a (p,q) graph. If G is a r-regular graph, then ${}^{m}M_1(G)=\frac{n}{r^2}$ and ${}^{m}M_2(G)=\frac{m}{r^2}$. Proper colourings in r-regular Modified Zagreb index graph is extended by the result proper colourings in magic and anti-magic graphs[17]. Many results and theorems are proved under Modified Zagreb index[1,8,9,10]. This work can be extended to domination which is related with domatic number and Modified Zagreb index[4,5,6]. Further this work can be extended in the field of automata theory [11,12,13,14,15,16,] which has a wide range of application in automata theory. There are many applications in graph labeling under undirected [21,22,23,24,25,26] and directed graph[18,19,20]

II. MAIN RESULTS

A. Definition 2.1

The first and the second Modified Zagreb indices are respectively defined as ${}^{m}M_1(G)=\sum_{n\in\nabla(G)}\frac{1}{(d(v)^2)}$ and ${}^{m}M_2(G)=\sum_{uv\in E(G)}\frac{1}{d(u)d(v)}$, where $d(v)$ is the degree of the vertex V

Theorem 2.1:
If G is a r-regular Modified Zagreb index graph then the chromatic number satisfies the inequality $\left|\frac{K-1}{r(V+E)}\right|\le\psi(G)\le\frac{1}{2}nr$, $r\ge 2$.

PROOF

Case (i)
Let G be a graph of cycle C_n, 'n' be an odd integer.

Let C_n be r-regular with n vertices and m edges then ${}^{m}M_1(G)=\frac{n}{r^2}$, ${}^{m}M_2(G)=\frac{m}{r^2}$.

Let C_n be a cycle graph with Modified Zagreb index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 3. Therefore, $\psi(G)=3$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G.
The following inequality is obtained.

$$\left|\frac{K-1}{r(V+\in)}\right| \leq \psi(G). \qquad \text{.........} (1)$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \leq \frac{1}{2}nr.$ .... (2)

From the equations (1) and (2) it is easily verify that

$$\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$$

Hence the odd cycle satisfies $\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$ for 2 - regular graph.

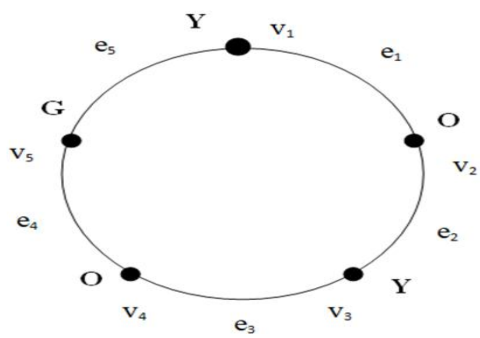


Fig 2.1

B. Modified Zagreb Index Number for ODD Cycle

$$k = {}^{m}M_1(C_5) = 5\left(\frac{1}{2}\right) = 1.2.$$

$$k = {}^{m}M_2(C_5) = 5\left(\frac{1}{2.2}\right) = 1.2.$$

$$\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$$

$$0.01 \leq 3 \leq 5.$$

Case (ii)

Let G be a graph of cycle C_n, $'n'$ be an even integer.

Let C_n be r -regular with n vertices and m edges then ${}^{m}M_1(G) = \frac{n}{r^2}, {}^{m}M_2(G) = \frac{m}{r^2}.$

Let C_n be a cycle graph with Modified Zagreb index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 2 .Therefore, $\psi(G) = 2$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G.
The following inequality is obtained.

$$\left|\frac{K-1}{r(V+\in)}\right| \leq \psi(G). \qquad \text{........ (3)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr$.

Therefore $\psi(G) \leq \frac{1}{2}nr$. .... (4)

From the equations (3) and (4) it is easily verify that

$$\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$$

Hence the even cycle satisfies $\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$ for 2 - regular graph.

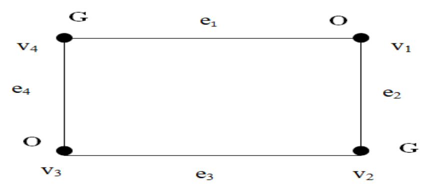


Fig 2.2

C. Modified Zagreb Index Number for Even Cycle

$$K = {}^{m}M_1(C_4) = 4\left(\frac{1}{2^2}\right) = 1.$$

$$K = {}^{m}M_2(C_4) = 4\left(\frac{1}{2.2}\right) = 1.$$

$$\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$$

$$0 \leq 2 \leq 4.$$

Case (iii)

Let the graph G be Generalized Petersen Graph, here $'n'$ is an even integer.

Let $V(p) = \{v_1, v_2, \ldots\ldots, v_{10}\}$ be the vertices and $E(p) = \{e_1, e_2, \ldots\ldots, e_{15}\}$ be the edges of $P(n,m)$ then ${}^{m}M_1(G) = \frac{n}{r^2}, {}^{m}M_2(G) = \frac{m}{r^2}$.

Let $P(n,m)$ be a Generalized Petersen Graph with Modified Zagreb index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is 3 .Therefore, $\psi(P) = 3$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\left|\frac{K-1}{r(V+\in)}\right| \leq \psi(G). \qquad \text{......... (5)}$$

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 11 Issue III Mar 2023- Available at www.ijraset.com

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr$.

Therefore $\psi(G) \leq \frac{1}{2}nr$. .... (6)

From the equations (5) and (6) it is easily verify that

$$\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr.$$

Hence the Generalized Petersen Graph satisfies $\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$ for 3 - regular graphs.

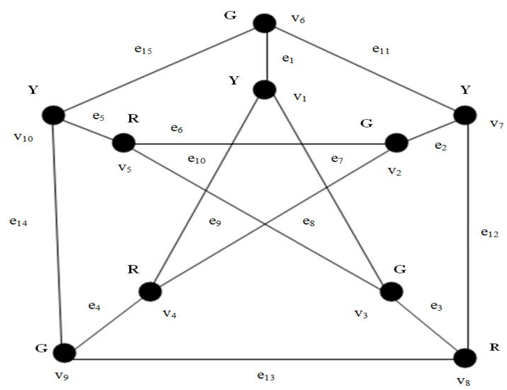


Fig 2.3

D. Modified Zagreb Index Number for Generalised Petersen Graph

$$K = {}^{m}M_1(P) = 10\left(\frac{1}{3^2}\right) = 1.11$$

$$K = {}^{m}M_2(P) = 15\left(\frac{1}{3^2}\right) = 1.67$$

$$\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$$

The inequality of first Modified Zagreb index for generalized Petersen graph is $0.0015 \leq 3 \leq 15$.

The inequality of second Modified Zagreb index for generalized Petersen graph is $0.0089 \leq 3 \leq 15$.

Case (iv)

Let G be a complete graph, $'n'$ be an any integer.

Let $V = \{v_1, v_2, ..., v_n\}$ be the vertices and $E = \{e_1, e_2, ..., e_n\}$ be the edges of k_n , then

$${}^{m}M_1(G) = \frac{n}{r^2}, {}^{m}M_2(G) = \frac{m}{r^2}.$$

Let K_n be a complete Graph with Modified Zagreb index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is n.

Therefore, $\psi(k_n) = n$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the
The following inequality is obtained.

$$\left|\frac{K-1}{r(V+\in)}\right| \leq \psi(G). \qquad \text{......... (7)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr$.

Therefore $\psi(G) \leq \frac{1}{2}nr. \qquad$ .... (8)

From the equations (7) and (8) it is easily verify that

$$\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr .$$

Hence the complete Graph satisfies $\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$ for n - regular graphs.

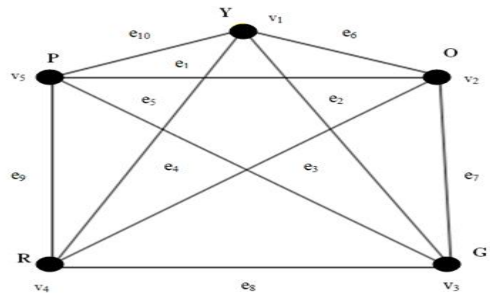


Fig 2.4

E. Modified Zagreb Index Number For Complete Graph

$$K = {}^{m}M_{1}(k_{5}) = 5\left(\frac{1}{4^{2}}\right) = 0.3125.$$

$$K = {}^{m}M_{2}(k_{5}) = 5\left(\frac{1}{4.4}\right) = 0.625$$

$$\left|\frac{K-1}{r(V+E)}\right| \leq \psi(G) \leq \frac{1}{2}nr$$

The inequality of first Modified Zagreb index for complete graph is $0.01 \leq 5 \leq 10.$

The inequality of second Modified Zagreb index for complete graph is $0.006 \leq 5 \leq 10.$

III. CONCLUSION

In this article, new inequality has been established. Further, it has been verified for first and second Modified Zagreb indices. Finally, we conclude that new inequality on chromatic number related with Modified Zagreb indices.

REFERENCES

[1] Amalorpava Jerline J., Litta E., Dhanalakshmi K., Benedict Michael Raj L.,F-Index of Generalized Mycielskian Graphs, International Journal of Recent Scientific Research, Volume 10, Issue 6, June 2019, P.No.: 32366-32371.

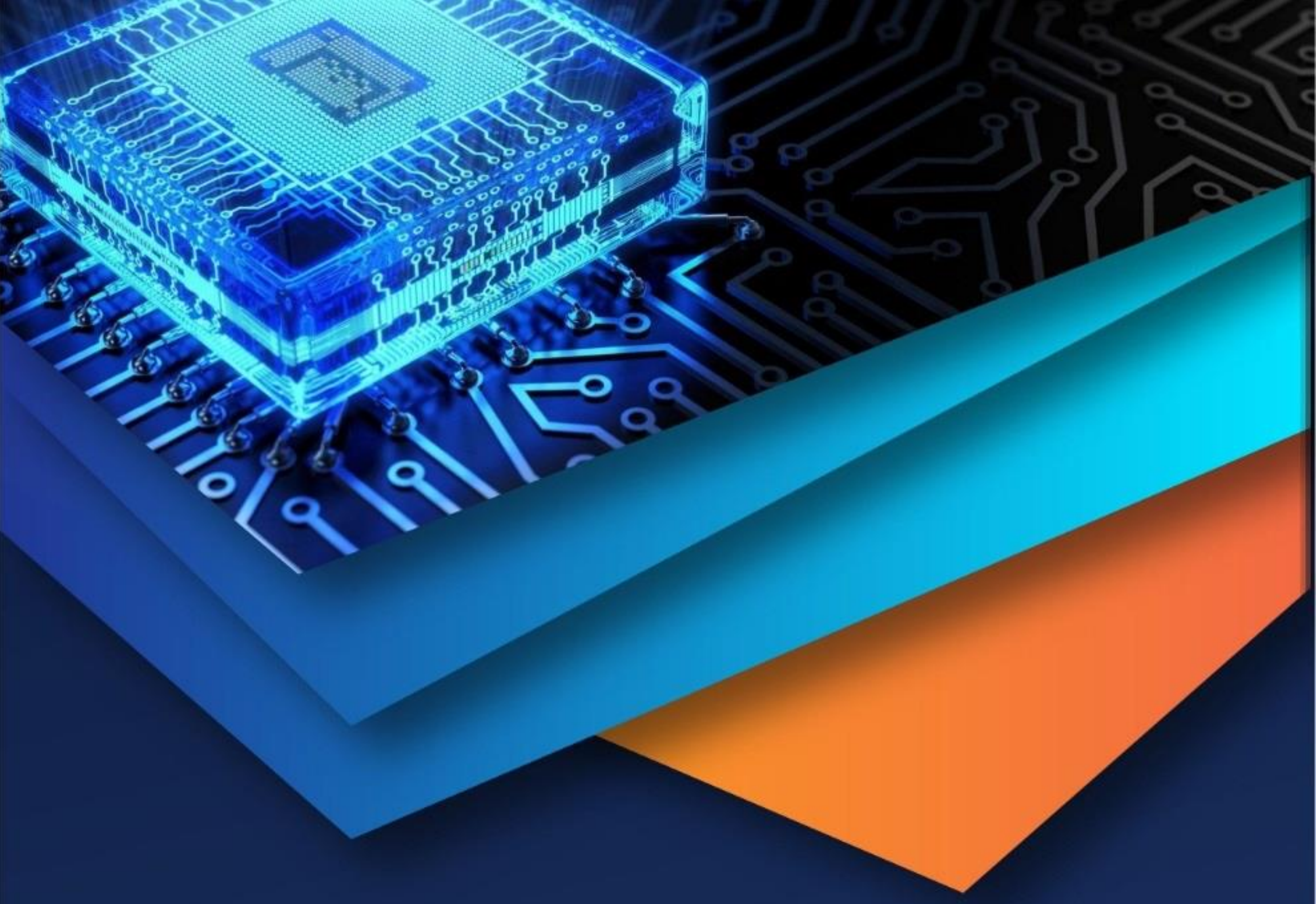
[2] Bodendick, R. and Walther, G., On number theoretical methods in graph labelings Res.Exp.Maths (2,/1995) 3-25.

[3] Bloom, D.F. Hsu, On graceful directed graphs, SIAMJ, Alg. Discrete Math.,6(1985),519-536.

[4] Felix J., Litta E., Benedict Micheal Raj L., Changing and Unchanging properties of Single Chromatic Transversal Domination Number of Graphs, International Journal of Mathematics Trends and Technology, Volume 52, Issue 4, Dec 2017, P.No.: 262-266.

[5] Felix J., Litta E., Benedict Micheal Raj L., Single Chromatic Transversal Dominating Irredundant Number for odd cycles, Peterson graph and Mycielski graph, Infokara Research, Volume 8, Issue 10, Oct 2019, P.No.: 139-145.

[6] Felix J., Litta E., Benedict Micheal Raj L., Single Chromatic Transversal Dominating Irredundant Number of graphs, Adalya Journal, Volume 10, Issue 8, Oct 2019, P.No.: 264-272.
[7] Harary, F., Graph Theory, New Delhi: Narosa Publishing House, 2001.
[8] Litta E., Amalorpava Jerline J., Dhanalakshmi K.,Benedict Micheal Raj L.,and Modified Zagreb Indices of Bridge Graphs, International Journal of Mathematical Archive, Volume 8, Issue 3, Mar 2017, P.No.: 86-91.
[9] Litta E., Amalorpava Jerline J., Rasika R., F- Co-index of Generalized Mycielskian Graphs, International Journal of Research in Advent Technology, Volume 7, Issue 4, Apr 2019, P.No.: 243-249.
[10] Litta E., Amalorpava Jerline J., Felix J., Benedict Micheal Raj L.,First and Second Modified Zagreb Indices of Product Graphs, Infokara Research,Volume 9, Issue 1, Jan 2020, P.No.: 279-293.
[11] Saridha.S. and Rajaretnam, T., "Algebraic Properties of Plus Weighted Finite State Machine", International Journal Of Applied Engineering Research, e-ISSN:0973-9769, p-ISSN:0973-4652, Vol.13, Number 21, 2018, 14974-14982.
[12] Saridha, S. and Rajaretnam, T., "A Study On Plus Weighted Multiset Transformation Semigroups", International Journal Of Information And Computing Science, e-ISSN:0972-1347, Vol.6, Issue I, January 2019, 84-98.
[13] Saridha, S. and Rajaretnam, T., "On Regular Properties Of Plus Weighted Multiset Finite State Automaton", Journal Of Applied Science And Computations, e-ISSN:1076-5131, Vol.5, Issue XII, December 2018, 87-95.
[14] Saridha, S., Rajaretnam, T., Plus weighted finite state automaton, in Journal Of Computer And Mathematical Sciences (JCMS 2017), Vol.8, Issue 11, ISSN 0976-5727, pp 674-690.
[15] Saridha, S. and Rajaretnam, T., "Some properties of plus weighted multiset grammars", International Journal Of Information And Computing Science, e-ISSN:0972-1347, Vol.6, Issue 5, May 2019, 24-37
[16] Saridha, S. and Haridha Banu . S, “A New Direction Towards Plus weighted Grammar”, International Journal for Research in Applied Science and Engineering Technology(IJRASET), ISSN: 2321 – 9653, Vol. 11, Issue II, February 2023.
[17] Shalini. P, Paul Dhayabaran. D, “Proper Colourings in Magic and Anti-magic Graphs”, International Journal of Engineering and Research Technology , Vol. 3, Issue. 2, pages 815-818. February 2014.
[18] Shalini. P, Paul Dhayabaran. D, “Generalization of Skolem Even Graceful Digraphs for Various Graphs”, International Journal of Mathematical Archive, 5(4), 2014, pages 65-69.
[19] Shalini. P, Paul Dhayabaran. D, “Skolem Graceful Signed Graphs on Directed Graphs”, Asian Journal of Current Engineering and Maths, 3:2 March-April(2014),pages33-34.
[20] Shalini. P, Paul Dhayabaran. D, “Generalization of Skolem Odd Graceful Digraphs for Various Graphs”, International Journal of Scientific and Research Technology,(2015) Volume-3, Pages 1-3.
[21] Shalini. P, Paul Dhayabaran. D, An Absolute Differences of Cubic and Square Difference Labeling, International Journal of Advanced Scientific and Technical Research, May-June 2015, Issue-5, Volume-3, pages 1-8.
[22] Shalini. P, Paul Dhayabaran. D, A Study on Root Mean Square Labelings in Graphs, International Journal of Engineering Science and Innovative Technology, May 2015, Volume-4, Issue-3, pages 305-309.
[23] Shalini. P, Paul Dhayabaran. D, Minimization of Multiplicative Graphs, International Journal of Current Research, Volume 7, Issue-08,pages 19511-19518,August 2015.
[24] Shalini. P, Gowri. R, Paul Dhayabaran. D, An absolute Differences of Cubic and Square Difference Labeling For Some Families of Graphs, International Journal of Analytical and Experimental Modal Analysis, Vol.11, Issue 10, October 2019, Pages 538 - 544, Impact Factor: 6.3. ISSN No: 0886 - 9367.
[25] Shalini. P, S.A.Meena , Lehmer -4 mean labeling of graphs, International journal for research in Applied Science and Engineering Technology (IJRASET) , Volume 10, Issue XII, December 2022,Page no: 1348-1351,ISSN : 2321-9653.
[26] Shalini.P, Tamizharasi.S, Power-3 Heronian Odd Mean Labeling of Graphs, International Journal for Research in Applied Science and Engineering Technology (IJRASET), Volume 10 Issue XII , December 2022, Page no: 1605-1608,ISSN:2321-9653.



10.22214/IJRASET

INDEX COPERNICUS
45.98

ISRA JIF
IMPACT FACTOR:
7.129

TOGETHER WE REACH THE GOAL
IMPACT FACTOR:
7.429

ISI

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY