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Proper Colourings in r-Regular Modified Zagreb Index Graph

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Abstract: ***In this article, the new concept proper colourings in r - regular Modified Zagreb index graph has been introduced. The first and second Modified Zagreb indices are introduced. In this article, new inequalities on chromatic number related with first and second Modified Zagreb indices are being established.***
Keywords: Regular graph, Proper Colouring, Modified Zagreb index, Chromatic number.

I. INTRODUCTION

In this article, we consider only finite, simple and undirected graphs. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The cardinality of the vertex set is called the order of G denoted by p. The cardinality of edge set is called the size of G denoted by q edges is called a (p,q) graph. If G is a r-regular graph, then ${}^{m}M_1(G)=\frac{n}{r^2}$ and ${}^{m}M_2(G)=\frac{m}{r^2}$. Proper colourings in r-regular Modified Zagreb index graph is extended by the result proper colourings in magic and anti-magic graphs[17]. Many results and theorems are proved under Modified Zagreb index[1,8,9,10]. This work can be extended to domination which is related with domatic number and Modified Zagreb index[4,5,6]. Further this work can be extended in the field of automata theory [11,12,13,14,15,16,] which has a wide range of application in automata theory. There are many applications in graph labeling under undirected [21,22,23,24,25,26] and directed graph[18,19,20]

II. MAIN RESULTS

A. *Definition 2.1*

The first and the second Modified Zagreb indices are respectively defined as ${}^{m}M_1(G)=\sum_{n\in\nabla(G)}\frac{1}{(d(v)^2)}$ and ${}^{m}M_2(G)=\sum_{uv\in E(G)}\frac{1}{d(u)d(v)}$, where $d(v)$ is the degree of the vertex V

Theorem 2.1:

If G is a r-regular Modified Zagreb index graph then the chromatic number satisfies the inequality $\left|\frac{K-1}{r(V+E)}\right|\le\psi(G)\le\frac{1}{2}nr$, r ≥ 2.

PROOF

Case (i)

Let G be a graph of cycle C_n, 'n' be an odd integer.

Let C_n be r-regular with n vertices and m edges then ${}^{m}M_1(G)=\frac{n}{r^2}$, ${}^{m}M_2(G)=\frac{m}{r^2}$.

Let C_n be a cycle graph with Modified Zagreb index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 3. Therefore, $\psi(G)=3$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G.
The following inequality is obtained.

$$\left|\frac{K-1}{r(V+\in)}\right| \le \psi(G). \qquad \text{..... (1)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \le \frac{1}{2}nr.$ ..... (2)

From the equations (1) and (2) it is easily verify that

$$\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$$

Hence the odd cycle satisfies $\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$ for 2 - regular graph.

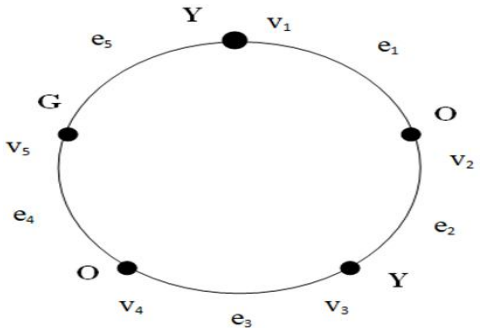

Fig 2.1

B. Modified Zagreb Index Number for ODD Cycle

$$k = {}^{m}M_1(C_5) = 5\left(\frac{1}{2}\right) = 1.2.$$

$$k = {}^{m}M_2(C_5) = 5\left(\frac{1}{2.2}\right) = 1.2.$$

$$\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$$

$$0.01 \le 3 \le 5.$$

Case (ii)

Let G be a graph of cycle C_n, 'n' be an even integer.

Let C_n be r -regular with n vertices and m edges then ${}^{m}M_1(G) = \frac{n}{r^2}, {}^{m}M_2(G) = \frac{m}{r^2}.$

Let C_n be a cycle graph with Modified Zagreb index, then the vertices in the cycle graphs are coloured with different colours, by proper colouring and the number used for colouring the cycle graph is 2 .Therefore, $\psi(G) = 2$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G.
The following inequality is obtained.

$$\left|\frac{K-1}{r(V+\in)}\right| \le \psi(G). \qquad \text{..... (3)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \le \frac{1}{2}nr.$ ..... (4)

From the equations (3) and (4) it is easily verify that

$$\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$$

Hence the even cycle satisfies $\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$ for 2 - regular graph.

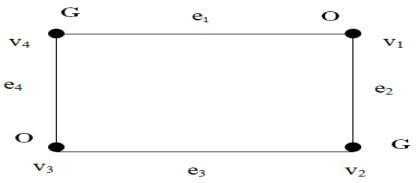

Fig 2.2

C. Modified Zagreb Index Number for Even Cycle

$$K = {}^{m}M_1(C_4) = 4\left(\frac{1}{2^2}\right) = 1.$$

$$K = {}^{m}M_2(C_4) = 4\left(\frac{1}{2.2}\right) = 1.$$

$$\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$$

$$0 \le 2 \le 4.$$

Case (iii)

Let the graph G be Generalized Petersen Graph, here $'n'$ is an even integer.

Let $V(p) = \{v_1, v_2, ..., v_{10}\}$ be the vertices and $E(p) = \{e_1, e_2, ..., e_{15}\}$ be the edges of $P(n,m)$ then

$${}^{m}M_1(G) = \frac{n}{r^2}, {}^{m}M_2(G) = \frac{m}{r^2}.$$

Let $P(n,m)$ be a Generalized Petersen Graph with Modified Zagreb index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is 3 .Therefore, $\psi(P) = 3$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the number of edges in graph G .

The following inequality is obtained.

$$\left|\frac{K-1}{r(V+\in)}\right| \le \psi(G). \qquad \text{..... (5)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \le \frac{1}{2}nr.$ ..... (6)

From the equations (5) and (6) it is easily verify that

$$\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr\ .$$

Hence the Generalized Petersen Graph satisfies $\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$ for 3 - regular graphs.

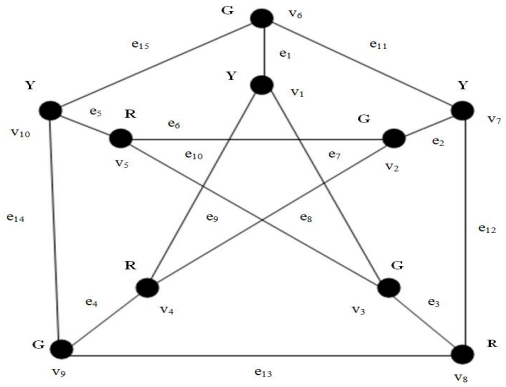

Fig 2.3

D. Modified Zagreb Index Number for Generalised Petersen Graph

$$K = {}^{m}M_1(P) = 10\left(\frac{1}{3^2}\right) = 1.11$$

$$K = {}^{m}M_2(P) = 15\left(\frac{1}{3^2}\right) = 1.67$$

$$\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$$

The inequality of first Modified Zagreb index for generalized Petersen graph is $0.0015 \le 3 \le 15.$

The inequality of second Modified Zagreb index for generalized Petersen graph is $0.0089 \le 3 \le 15.$

Case (iv)

Let G be a complete graph, 'n' be an any integer.

Let $V = \{v_1, v_2, ..., v_n\}$ be the vertices and $E = \{e_1, e_2, ..., e_n\}$ be the edges of k_n , then

$${}^{m}M_1(G) = \frac{n}{r^2}, {}^{m}M_2(G) = \frac{m}{r^2}.$$

Let K_n be a complete Graph with Modified Zagreb index, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is n .

Therefore, $\psi(k_n) = n$.

Since K is the index number, r is the regular graph, V is the number of vertices and E is the

The following inequality is obtained.

$$\left|\frac{K-1}{r(V+\in)}\right| \le \psi(G). \qquad \text{..... (7)}$$

The general condition of r -regular graph is denoted by as $\frac{1}{2}nr.$

Therefore $\psi(G) \le \frac{1}{2}nr.$ ..... (8)

From the equations (7) and (8) it is easily verify that

$$\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr\ .$$

Hence the complete Graph satisfies $\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$ for n - regular graphs.

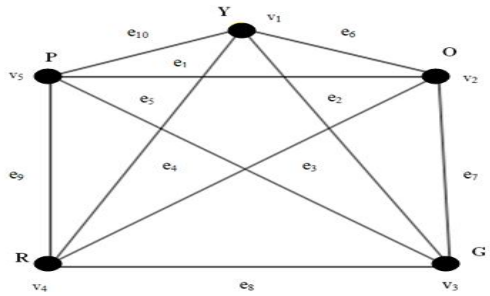

Fig 2.4

E. Modified Zagreb Index Number For Complete Graph

$$K = {}^{m}M_1(k_5) = 5\left(\frac{1}{4^2}\right) = 0.3125.$$

$$K = {}^{m}M_2(k_5) = 5\left(\frac{1}{4.4}\right) = 0.625$$

$$\left|\frac{K-1}{r(V+E)}\right| \le \psi(G) \le \frac{1}{2}nr$$

The inequality of first Modified Zagreb index for complete graph is $0.01 \le 5 \le 10.$

The inequality of second Modified Zagreb index for complete graph is $0.006 \le 5 \le 10.$

III. CONCLUSION

In this article, new inequality has been established. Further, it has been verified for first and second Modified Zagreb indices. Finally, we conclude that new inequality on chromatic number related with Modified Zagreb indices.

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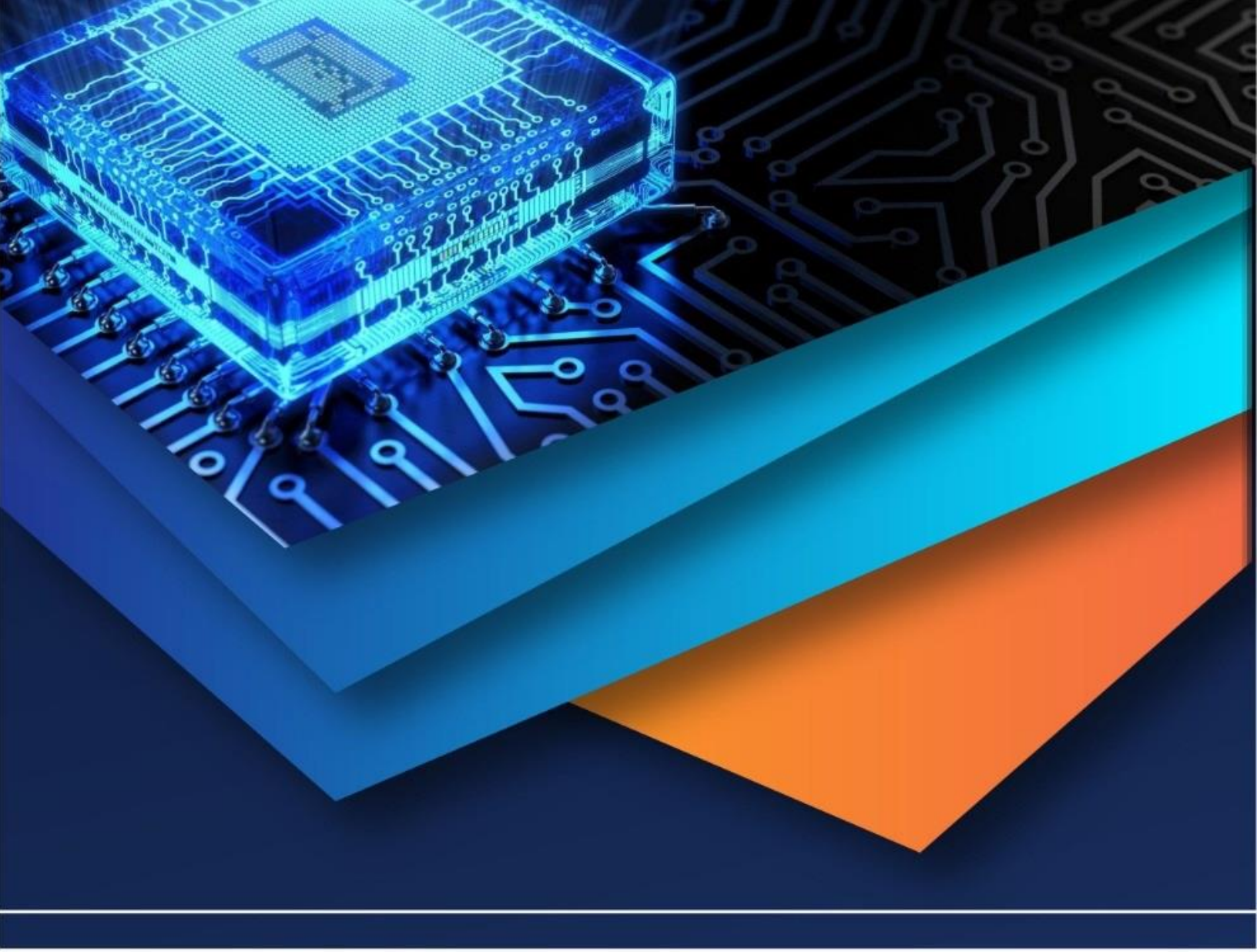

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