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Properties of the Ternary Cubic Equation $5x^2 - 3y^2 = z^3 \sum$

G. Janaki[1], A. Gowri Shankari[2]
[1]Associate Professor, [2]Assistant Professor, PG & Research Department of Mathematics, Cauvery College for Women (Autonomous), Affiliated to Bharathidasan University, Trichy – 18

Abstract:** To identify its different integral non-zero solutions, the ternary cubic equation* $5x^2 - 3y^2 = z^3$ ***is taken into consideration. Different integral solution patterns to the ternary cubic equation under consideration are obtained in each pattern by using the linear transformation and the method of factorization; interesting relationships between the solutions and some polygonal numbers, such as pyramidal and central pyramidal numbers, are also displayed.
***Keywords:** Diophantine equations, Ternary equation, Cubic Equation with Three Unknowns, Integral Solutions*

I. INTRODUCTION

Number theory, which is used to explain anything that can be quantified, is the language of patterns and relationships. A polynomial equation called a Diophantine equation can only have integers as solutions. Theories of numbers were featured in [1–3]. A unique Pythagorean triangle problem and its integral solutions are featured in [4, 5]. Higher order equations are taken into account for integral solutions in [6–10].

The non-homogeneous cubic equation with three unknowns represented by the equation is discussed in this communication, and in particular, a few intriguing relationships between the solutions are highlighted.

A. Notations

$T_{m,a}$: Polygonal Number of rank *a* with side *m*
Gno_a: Gnomonic Number of rank *a*
$Star_a$: Star Number of rank *a*
O_a: Octahedral Number of rank *a*
P_a^m: Pyramidal Number of rank *a* with sides *m*
SO_a: Stella Octangula Number of rank *a*
CC_a: Centered Cube Number of rank *a*
CS_a: Centered Square Number of rank *a*
RD_a: Rhombic Dodecagonal Number of rank *a*
TO_a: Truncated Octahedral Number of rank *a*

II. METHOD OF ANALYSIS

A non-zero integral solution to the Cubic equation can be found by

$$5x^2 - 3y^2 = z^3 \tag{1}$$

Upon switching the transformations,

$$x = X + 3T,\ y = X + 5T,\ z = 2Z \tag{2}$$

in (1) leads to, $X^2 - 15T^2 = 4Z^3$ (3)

Below, we present illustration of distinct integer non-zero patterns (1)

A. Pattern: 1

Assume $Z = z(a,b) = a^2 - 15b^2$ (4)

Where a and b are positive integers.

And write $4=(8+2\sqrt{15})(8-2\sqrt{15})$ (5)

Using the factorization approach, replacing (3) with (4) and (5),

$$(X+\sqrt{15}T)(X-\sqrt{15}T)=(8+2\sqrt{15})(8-2\sqrt{15})(a+\sqrt{15}b)^3(a-\sqrt{15}b)^3$$

Comparing real and imaginary elements while equating similar phrases, $X=8a^3+360ab^2+90a^2b+450b^3$

$T=2a^3+90ab^2+24a^2b+120b^3$

The appropriate integer solutions of equation (1) are provided by substituting the above mentioned values of X and T into equation

$x=x(a,b)=14a^3+630ab^2+162a^2b+810b^3$

(2) $y=y(a,b)=18a^3+810ab^2+210a^2b+1050b^3$

$z=z(a,b)=2a^2-30b^2$

Properties:

1. $-7z(a,a)$ and $-28z(a,a)$ are Perfect Squares
2. $-x(1,1)+y(1,1)-z(1,1)$ is a Harshad Number
3. $y(a,1)-x(a,1)-6P_a^6-star_a-2T_{29,a}-T_{26,a}\equiv 239(\mathrm{mod}123)$
4. $y(a,1)-x(a,1)-124z(a,1)-2SO_a-91Gno_a\equiv 0(\mathrm{mod}\,4051)$
5. $-y(a,1)-z(a,1)+18P_a^3+4Star_a\equiv 1076(\mathrm{mod}852)$
6. $x(a,1)+y(a,1)-16z(a,1)-2TO_a-29T_{30,a}\equiv 2352(\mathrm{mod}1769)$
7. $-x(a,1)+y(a,1)-23z(a,1)+12P_a^5+T_{18,a}\equiv 930(\mathrm{mod}187)$

B. Pattern: 2

The modified form of equation (3) is

$X^2-15T^2=4Z^3*1$ (6)

Put 4 in as, $4=(8+2\sqrt{15})(8-2\sqrt{15})$ (7)

and $1=(4+\sqrt{15})(4-\sqrt{15})$ (8)

When (7) and (8) are substituted in equation (6) and the process of factorization is used, as described in Pattern 1, the corresponding

$$x=x(a,b)=110a^3+4950ab^2+1278a^2b+6390b^3$$

integer solutions of (1) are represented by $y=y(a,b)=142a^3+6390ab^2+1650a^2b+8250b^3$

$$z=z(a,b)=2a^2-30b^2$$

Properties:

1. $y(1,1)-x(1,1)+z(1,1)-22T_{3,4}$ is a nasty number
2. $x(1,1)+y(1,1)-26T_{3,4}$ is a perfect square number
3. $y(1,1)-x(1,1)+11z(1,1)-T_{8,3}$ Is a cubic number
4. $y(a,1)-x(a,1)-8RD_a-72T_{15,a}-886Gno_a\equiv 0(\mathrm{mod}2762)$
5. $y(a,1)-x(a,1)-186z(a,1)-48O_a-712Gno_a\equiv 0(\mathrm{mod}\,8152)$

6. $y(a,1)-x(a,1)-2TO_a-47T_{22,a}\equiv 1392(\mathrm{mod}\,1815)$
7. $y(a,1)-825x(a,1)-213O_a\equiv 33000(\mathrm{mod}\,6319)$
8. $2x(a,1)-426z(a,1)-110SO_a-284T_{26,a}-6567Gno_a\equiv 0(\mathrm{mod}\,6567)$

C. Pattern: 3

Write $1=(31+8\sqrt{15})(31-8\sqrt{15})$ (9)

By replacing (7) and (9) in equation (6) and factorising the result using the steps in Pattern 1, the corresponding integer solutions of (1) are represented by

$$x=x(a,b)=866a^3+38970ab^2+10062a^2b+50310b^3$$
$$y=y(a,b)=1118a^3+50310ab^2+12990a^2b+64950b^3$$
$$z=z(a,b)=2a^2-30b^2$$

Properties:

1. $y(a,1)-16z(a,1)-2236P_a^5-1184T_{22,a}-30483Gno_a\equiv 0(\mathrm{mod}\,95913)$
2. $y(a,1)-x(a,1)+z(a,1)-189HO_a-827T_{10,a}-5796Gno_a\equiv 0(\mathrm{mod}\,23056)$
3. $\frac{1}{10}(x(1,1)-y(1,1))$ is a square number
4. $x(1,b)+y(1,b)+23z(1,b)-138312P_b^7-3239Star_b-44295Gno_b\equiv 0(\mathrm{mod}\,42086)$
5. $y(1,1)-x(1,1)+24T_{3,4}$ is a nasty number
6. $x(a,1)-16z(a,1)-4336CC_a-11329CS_a+22658T_{3,a}\equiv 39894(\mathrm{mod}\,71658)$
7. $-x(a,1)+y(a,1)-23z(a,1)-63RD_a-239T_{30,a}\equiv 14013(\mathrm{mod}\,14195)$
8. $y(a,1)-x(a,1)+19z(a,1)-126SO_a-1483T_{6,a}\equiv 14070(\mathrm{mod}\,12949)$

Note:

Additionally, 4 and 1 might be written as

$$4=(62+16\sqrt{15})(62-16\sqrt{15}$$

$$1=\frac{(8+\sqrt{15})(8-\sqrt{15})}{49}$$

In relation to these options, one may find many patterns of solutions of (1)

III. CONCLUSION

Three unique patterns of non-zero distinct integer solutions to the given non-homogeneous problem $5x^2-3y^2=z^3$ are shown in this paper.

For various options of cubic Diophantine equations, additional patterns of non-zero integer unique solutions and their corresponding characteristics may be found.

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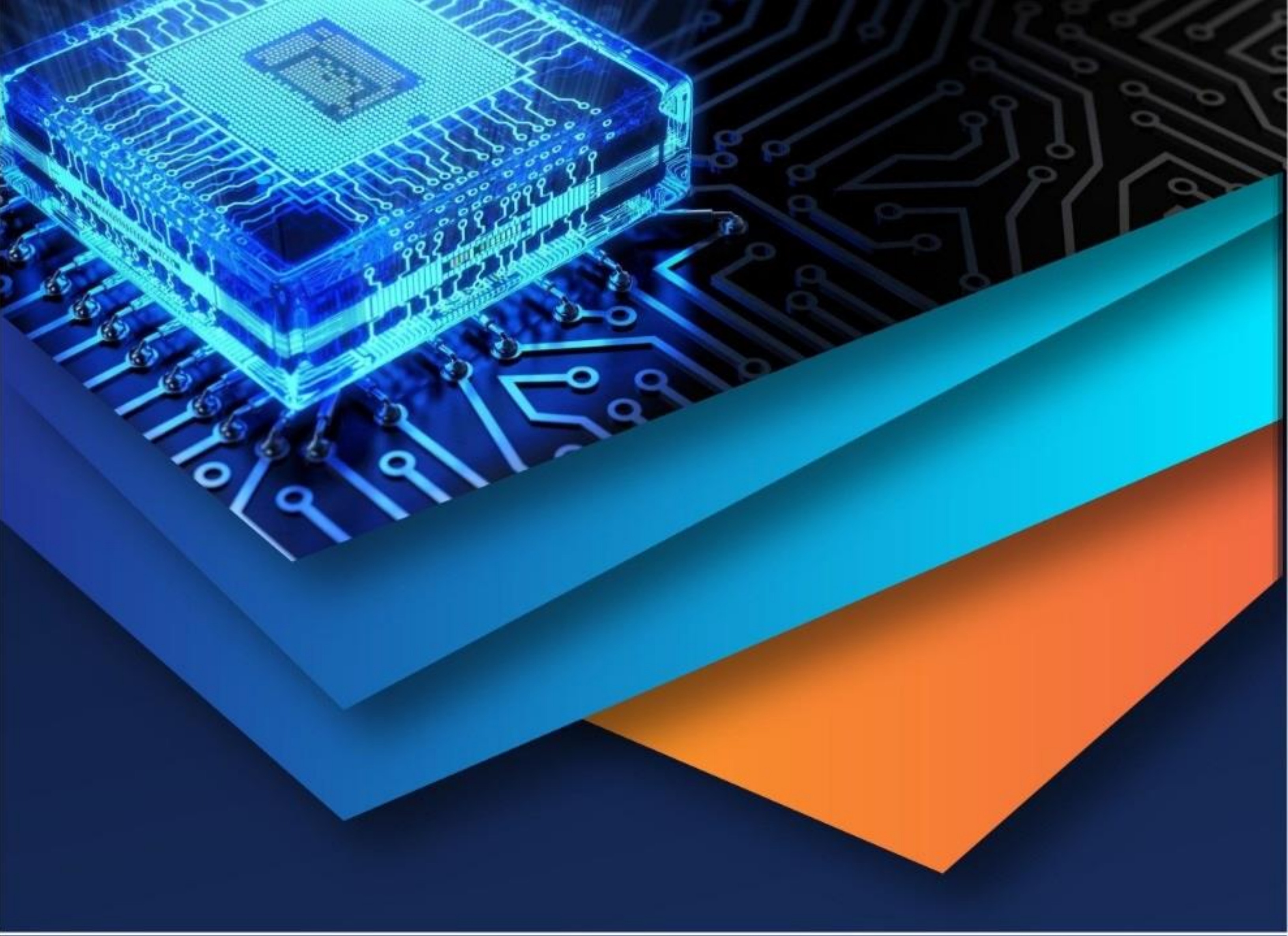

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