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Pythagorean Triangle with Area/Perimeter as a Disarium Number of Order 2 to 4

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Abstract: We present patterns of Pythagorean triangles, in each of which the ratio Area/Perimeter is represented by the Disarium number. A few interesting relations among the sides are also given.
Keywords: Pythagorean triangles, Disarium numbers.

I. INTRODUCTION

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. Number theory is one of the largest and oldest branches of mathematics. The main goal of number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In number theory, Pythagorean triangles have been a matter of interest to various mathematicians, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For an extensive variety of fascinating problems, one may refer [1-5]. Apart from the polygonal numbers, we have some more fascinating patterns of numbers namely Jarasandha numbers, nasty numbers and dhuruva numbers. These numbers have been presented in[6-9]. In [10], special pythagorean triangles connected with nasty numbers are obtained. In [11], special pythagorean triangles connected with Jarasandha numbers are obtained. In [12], special pairs of pythagorean triangles and Jarasandha numbers are presented. Recently in [13] &[14], rectangles in connection with Jarasandha numbers are obtained. In this communication, we search for patterns of Pythagorean triangles, in each of which the ratio Area/Perimeter is represented by the Disarium number of order 2 to 4. Also, a few interesting relations among the sides are given.

II. BASIC DEFINITIONS

A. Definition

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation, where x, y and z are natural numbers. The above equations are also referred to as Pythagorean triangle and denote it by $T(x, y, z)$

Also, in Pythagorean triangle $T(x, y, z): x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

B. Definition

Most cited solution of the Pythagorean equation is $x = m^2 - n^2$, $y = 2mn$, $z = m^2 + n^2$, where $m > n > 0$. This solution is called **primitive**, if m, n are of opposite party and $\gcd(m, n) = 1$.

C. Definition

A number will be called "DISARIUM" if sum of its digits powered with their respective position is equal to the original number.

III. METHOD OF ANALYSIS

Denoting the Area and Perimeter of the triangle by A and P respectively, the assumption

$$\frac{A}{P} = \text{Disarium number.}$$

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The above relation leads to the equation

$$\frac{n(m-n)}{2} = \text{Disarium number} \qquad \text{……….. (1)}$$

1) Case 1

$$\text{When } \frac{n(m-n)}{2} = 89 \text{ (2- digit Disarium number)} \qquad \text{……….. (2)}$$

$$\Rightarrow n(m-n) = 178 \qquad \text{……….. (3)}$$

On evaluation, the values of the generators m, n satisfying (3) are given in the following table1:

Table 1

S.NO	n	$m-n$	m	x	y	z	A	P	$\frac{A}{P}$
1	1	178	179	32040	358	32042	5735160	64440	89
2	2	89	91	8277	364	8285	1506414	16926	89
3	89	2	91	360	16198	16202	2915640	32760	89
4	178	1	179	357	63724	63725	11374734	127806	89

Thus it is seen that there are 4 Pythagorean triangles. Of these 4 Pythagorean triangles, 2 triangles are Primitive and remaining 2 triangles are non-primitive triangles.

2) Case 2

Consider the 3-digit Disarium number 135,

In this case $n(m-n) = 270$ ……….. (4)

Following the same procedure as in case1, we have 16 distinct values for m, n satisfying (4) are Presented below:

Table 2

S.NO	n	$m-n$	m	x	y	z	A	P	$\frac{A}{P}$
1	1	270	271	73440	542	73442	19902240	147424	135
2	2	135	137	18765	548	18773	5141610	38086	135
3	3	90	93	8640	558	8658	2410560	17856	135
4	5	54	59	3456	590	3506	1019520	7552	135
5	6	45	51	2565	612	2637	784890	5814	135
6	9	30	39	1440	702	1602	505440	3744	135
7	10	27	37	1269	740	1469	469530	3478	135
8	15	18	33	864	990	1314	427680	3168	135
9	18	15	33	765	1188	1413	454410	3366	135
10	27	10	37	640	1998	2098	639360	4736	135
11	30	9	39	621	2340	2421	726570	5382	135
12	45	6	51	576	4590	4626	1321920	9792	135
13	54	5	59	565	6372	6397	1800090	13334	135
14	90	3	93	549	16740	16749	4595130	34038	135
15	135	2	137	544	36990	36994	10061280	74528	135
16	270	1	271	541	146340	146341	39584970	293222	135

Thus it is seen that there are 16 Pythagorean triangles. Of these 16 Pythagorean triangles, 4 triangles are Primitive and remaining 12 triangles are non-primitive triangles.

3) Case 3

A Similar observation, regarding 3, 4 – digit Disarium numbers are exhibited in the table 3 below:

Table 3

Disarium Number	Pairs of Pythagorean Triangles	Pairs of primitive Pythagorean triangles	Pairs of non-primitive Pythagorean triangles
175	12	4	8
518	12	4	8
598	12	4	8
1306	6	2	4
1676	8	2	6
2427	8	4	4

IV. REMARKABLE OBSERVATIONS

1) $y+z$ is a Perfect square.

2) $3(x+z)$ is a nasty number.

3) $3\left(\frac{x+y-z}{12}\right)$ is a Disarium number.

4) For the Disarium number 89, $x+y-z+14=$ Narcissistic number of 370.

5) For the Disarium number 518, $x+y-z-47=$ Jarasandha number of 2025.

V. CONCLUSION

To conclude, One may search for the connections between the Pythagorean triangles and other Disarium numbers of higher order and other number patterns.

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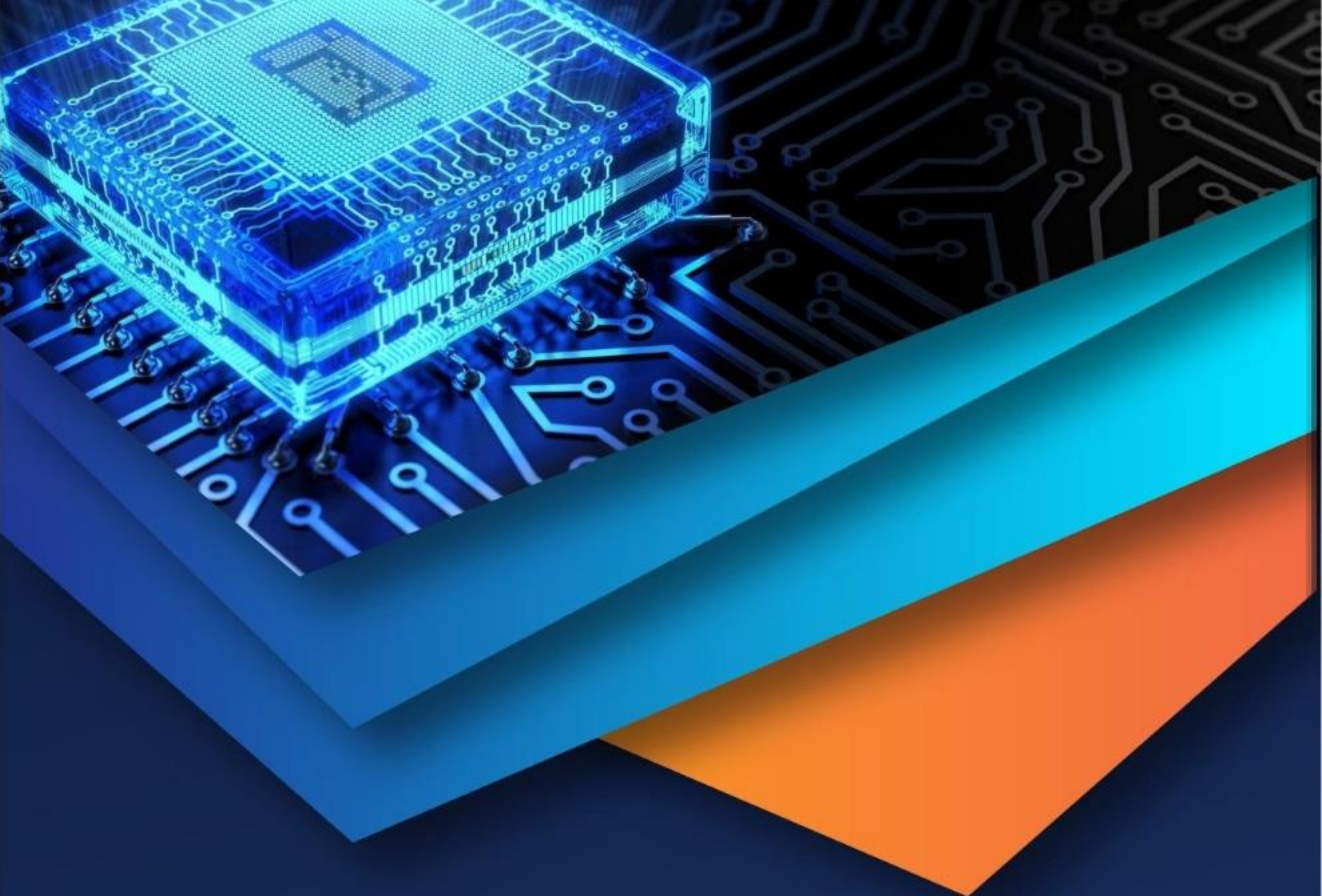

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