# Pythagorean Triangle with Area/Perimeter as a Disarium Number of Order 2 to 4 

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#### Abstract

We present patterns of Pythagorean triangles, in each of which the ratio Area/Perimeter is represented by the Disarium number. A few interesting relations among the sides are also given. Keywords: Pythagorean triangles, Disarium numbers.


## I. INTRODUCTION

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. Number theory is one of the largest and oldest branches of mathematics. The main goal of number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In number theory, Pythagorean triangles have been a matter of interest to various mathematicians, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For an extensive variety of fascinating problems, one may refer [1-5]. Apart from the polygonal numbers, we have some more fascinating patterns of numbers namely Jarasandha numbers, nasty numbers and dhuruva numbers. These numbers have been presented in[6-9]. In [10], special pythagorean triangles connected with nasty numbers are obtained. In [11], special pythagorean triangles connected with Jarasandha numbers are obtained. In [12], special pairs of pythagorean triangles and Jarasandha numbers are presented. Recently in [13] \& [14], rectangles in connection with Jarasandha numbers are obtained. In this communication, we search for patterns of Pythagorean triangles, in each of which the ratio Area/Perimeter is represented by the Disarium number of order 2 to 4 . Also, a few interesting relations among the sides are given.

## II. BASIC DEFINITIONS

## A. Definition

The ternary quadratic Diophantine equation given by $x^{2}+y^{2}=z^{2}$ is known as Pythagorean equation, where $x, y$ and $z$ are natural numbers. The above equations are also referred to as Pythagorean triangle and denote it by $T(x, y, z)$
Also, in Pythagorean triangle $T(x, y, z): x^{2}+y^{2}=z^{2}, x$ and $y$ are called its legs and $z$ its hypotenuse.

## B. Definition

Most cited solution of the Pythagorean equation is $x=m^{2}-n^{2}, y=2 m n, z=m^{2}+n^{2}$, where $m>n>0$. This solution is called primitive, if $m, n$ are of opposite party and $\operatorname{gcd}(m, n)=1$.

## C. Definition

A number will be called "DISARIUM" if sum of its digits powered with their respective position is equal to the original number.

## III. METHOD OF ANALYSIS

Denoting the Area and Perimeter of the triangle by $A$ and $P$ respectively, the assumption

$$
\frac{A}{P}=\text { Disarium number. }
$$

The above relation leads to the equation

$$
\begin{equation*}
\frac{n(m-n)}{2}=\text { Disarium number } \tag{1}
\end{equation*}
$$

1) Case 1

$$
\begin{align*}
& \text { When } \frac{n(m-n)}{2}=89 \text { (2-digit Disarium number) }  \tag{2}\\
& \Rightarrow n(m-n)=178 \tag{3}
\end{align*}
$$

On evaluation, the values of the generators $m, n$ satisfying (3) are given in the following table1:
Table 1

| S.NO | $n$ | $m-n$ | $m$ | $x$ | $y$ | $z$ | A | $P$ | $\frac{A}{P}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 178 | 179 | 32040 | 358 | 32042 | 5735160 | 64440 | 89 |
| 2 | 2 | 89 | 91 | 8277 | 364 | 8285 | 1506414 | 16926 | 89 |
| 3 | 89 | 2 | 91 | 360 | 16198 | 16202 | 2915640 | 32760 | 89 |
| 4 | 178 | 1 | 179 | 357 | 63724 | 63725 | 11374734 | 127806 | 89 |

Thus it is seen that there are 4 Pythagorean triangles. Of these 4 Pythagorean triangles, 2 triangles are Primitive and remaining 2 triangles are non-primitive triangles.

## 2) Case 2

Consider the 3-digit Disarium number 135,
In this case $\quad n(m-n)=270$
Following the same procedure as in case1, we have 16 distinct values for $\mathrm{m}, \mathrm{n}$ satisfying (4) are Presented below:

Table 2

| S.NO | $n$ | $m-n$ | $m$ | $x$ | $y$ | $z$ | A | $P$ | $\frac{A}{P}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 270 | 271 | 73440 | 542 | 73442 | 19902240 | 147424 | 135 |
| 2 | 2 | 135 | 137 | 18765 | 548 | 18773 | 5141610 | 38086 | 135 |
| 3 | 3 | 90 | 93 | 8640 | 558 | 8658 | 2410560 | 17856 | 135 |
| 4 | 5 | 54 | 59 | 3456 | 590 | 3506 | 1019520 | 7552 | 135 |
| 5 | 6 | 45 | 51 | 2565 | 612 | 2637 | 784890 | 5814 | 135 |
| 6 | 9 | 30 | 39 | 1440 | 702 | 1602 | 505440 | 3744 | 135 |
| 7 | 10 | 27 | 37 | 1269 | 740 | 1469 | 469530 | 3478 | 135 |
| 8 | 15 | 18 | 33 | 864 | 990 | 1314 | 427680 | 3168 | 135 |
| 9 | 18 | 15 | 33 | 765 | 1188 | 1413 | 454410 | 3366 | 135 |
| 10 | 27 | 10 | 37 | 640 | 1998 | 2098 | 639360 | 4736 | 135 |
| 11 | 30 | 9 | 39 | 621 | 2340 | 2421 | 726570 | 5382 | 135 |
| 12 | 45 | 6 | 51 | 576 | 4590 | 4626 | 1321920 | 9792 | 135 |
| 13 | 54 | 5 | 59 | 565 | 6372 | 6397 | 1800090 | 13334 | 135 |
| 14 | 90 | 3 | 93 | 549 | 16740 | 16749 | 4595130 | 34038 | 135 |
| 15 | 135 | 2 | 137 | 544 | 36990 | 36994 | 10061280 | 74528 | 135 |
| 16 | 270 | 1 | 271 | 541 | 146340 | 146341 | 39584970 | 293222 | 135 |

Thus it is seen that there are 16 Pythagorean triangles. Of these 16 Pythagorean triangles, 4 triangles are Primitive and remaining 12 triangles are non-primitive triangles.
3) Case 3

A Similar observation, regarding 3, 4 - digit Disarium numbers are exhibited in the table 3 below:

Table 3

| Disarium <br> Number | Pairs of Pythagorean <br> Triangles | Pairs of primitive Pythagorean <br> triangles | Pairs of non-primitive Pythagorean <br> triangles |  |
| :--- | :--- | :--- | :--- | :--- |
| 175 | 12 | 4 |  | 8 |
| 518 | 12 | 4 | 8 |  |
| 598 | 12 | 4 | 8 |  |
| 1306 | 6 | 2 | 4 |  |
| 1676 | 8 | 2 | 6 |  |
| 2427 | 8 | 4 | 4 |  |

## IV. REMARKABLE OBSERVATIONS

1) $y+z$ is a Perfect square.
2) $3(x+z)$ is a nasty number.
3) $3\left(\frac{x+y-z}{12}\right)$ is a Disarium number.
4) For the Disarium number $89, x+y-z+14=$ Narcissistic number of 370 .
5) For the Disarium number 518, $x+y-z-47=$ Jarasandha number of 2025.

## V. CONCLUSION

To conclude, One may search for the connections between the Pythagorean triangles and other Disarium numbers of higher order and other number patterns.

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