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Python Programming on Fibonacci Graceful Anti - Magic Labeling for Star Graphs

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***Abstract:** A graph $G = (V, E, \phi)$ with p vertices and q edges is called Fibonacci Graceful Anti magic Graph $[FGAMG]$ if a function ϕ is defined as $\phi: V \rightarrow \{0, 1, 2, \ldots, F_{q+1}\}$ and the induced edge labeling $\phi^*: E \rightarrow \{F_2, F_3, F_4, \ldots, F_{q+1}\}$ is defined by $\phi^*(uv) = |\phi(u) - \phi(v)|$ is bijective. In addition all the vertex sums are pairwise distinct and all the edges are unique. In this article the concept of Fibonacci Graceful Anti- Magic Labeling $[FGAML]$ is introduce and investigate some star related graphs which are Fibonacci Graceful Anti magic Graph $[FGAMG]$.*

AMS Classification : 05C78
***Keywords:** Crown graph, Bi-star, Double star, Bi-shell, $FGAML$, $FGAMG$.*

I. INTRODUCTION

Fibonacci graceful labeling idea was introduced by E.Barkaukas et.all [3]. Hartsfield and Ringel introduced the concept of Anti-Magic labeling. The concept of Fibonacci mean Anti-magic labeling was introduced by Ameenalbibi and T.Ranjani[1]. Now this article is based on connected, undirected and Star related Graphs. Python is the most popular language in world wide. It was introduced by GuioVan Rossum on Feburary 20,1991.Fibonacci Graceful Anti- Magic Labeling $[FGAML]$concept is introduce here. While investigating some star based graphs it is found that they are Fibonacci Graceful Anti magic Graph $[FGAMG]$.Here Python coding is generated for the Fibonacci Graceful Anti-Magic Labeling.

II. DEFINITIONS

1) Definition 2.1: The Fibonacci numbers $F_1, F_2, F_3, \ldots$ are defined by $F_0 = 0, F_1 = 1, F_2 = 1, \ldots$ and $F_{n+1} = F_n + F_{n-1}$, n≥1. The Fibonacci sequence is $1,1,2,3,5 \ldots$
2) Definition 2.2: The function $\phi: V(G) \longrightarrow \{0,1,2, \ldots, F_{q+1}\}$ (here F_q is the q^{th} Fibonacci number) is called $FGAML$ if the induced edge labeling $\phi^* : E(G) \longrightarrow \{F_2, F_3, F_4, \ldots, F_{q+1}\}$ defined by $\phi * (uv) = |\phi(u) - \phi(v)|$ is bijective. In addition all the vertex sums are pairwise distinct and all the edges are unique.
3) Definition 2.3 : Crown graph is obtained by joining a single pendant edge to each vertex of the cycle C_n. It is denoted by$C_n \odot K_1$.
4) Definition 2.4: Double star $K_{1,n,n}$ is a tree obtained from the star $K_{1,n}$ by adding a new pendant edge to each of the existing n pendant vertices.
5) Definition2.5 : A Bi-star $B_{n,n}$ graph is obtained by adding the central vertices of two copies of Star.
6) Definition 2.6 : A Bi-Shell $BS_{n,n}$graph isobtained by adding the central vertices of two copies of Shell.
7) Definition 2.7 : The Vertex sum at one vertex is the sum of labels of all edges incident to such a vertex.

III. RESULTS

1) *Theorem*

Crown graph $C_n \odot K_1$ is $FGAMG$ for all n.

Proof:

Let $C_n \odot K_1$ be the crown. The order of the crown is $p = 2n$ and the sizeis $q = 2n$. Let $V(G) = \{a_1, a_2, \ldots a_n, b_1, b_2, \ldots, b_n\}$ here $a_1, a_2, \ldots, a_n$ are the vertices of the cycle C_n and $b_i's$ are the pendant vertices adjacent to $a_i's$ of the cycle C_n. The edge set $E(G) = \{e_{ii}, e_{ij}\}$ where $e_{ii} = (a_i b_i)$ and $e_{ij} = (a_i a_j)$.

Define $\phi: V \longrightarrow \{0,1,2, \ldots, F_{q+1}\}$

$\phi(a_1) = 0$
$\phi(a_{i+1}) = F_{2i+2} \quad for\ i = 1,2,3,\dots,n-1$
$\phi(b_{i+1}) = F_{2i+1} \quad for\ i = 1,3\dots..n-1$
$\phi(b_3) = 6$
$\phi(b_1) = F_{2n+1}$
The edge labels are
$e_{ii} = \phi^*(a_i b_i) = |\phi(a_i) - \phi(b_i)| \qquad for\ \ i = 1,2,3,\dots,n$
$e_{ij} = \phi^*(a_i a_j) = |\phi(a_i) - \phi(a_j)| \qquad for\ \ i = 1,2,3,\dots,n\ and\ for\ \ j = i+1$
Here all the vertex sums are pairwise distinct and all the edges are unique.
Thus ϕ admits$FGAML$.
Hence Crown $C_n \odot K_1$ is $FGAMG$.
Python coding and output is enclosed in the Appendix .
Illustration 3.1

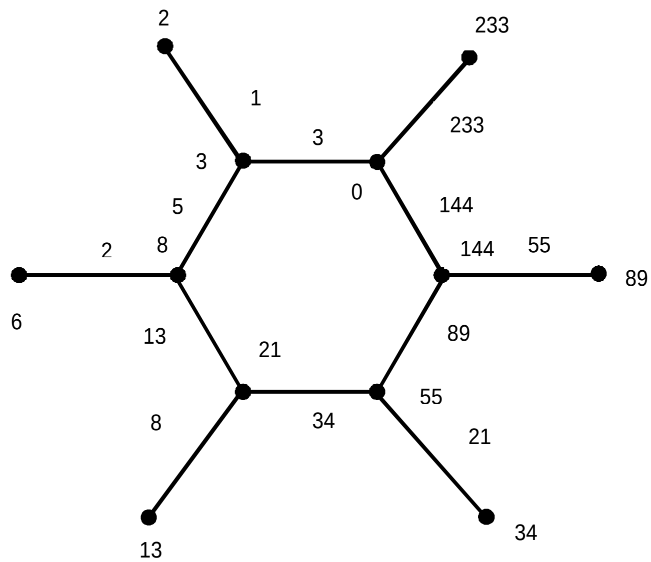


Figure 1. The Crown $C_6 \odot K_1$ is $FGAMG$

2) Theorem
Double star graph is $FGAMG$ for all n.
Proof :
Let $K_{1,n,n}$ be the double star. The order of $K_{1,n,n}$is $p = 2n+1$ and the size $q = 2n$. By the definition of $K_{1,n,n}$ the vertex set $V(G) = \{a, b_1, b_2, \dots, b_n, \ c_1, c_2, \dots, c_n\}$.Let a be the central vertex, and $b_1, b_2, \dots, b_n$ be the vertices of the star and $c_1, c_2, \dots, c_n$ be the vertices adding new pendent edge to each existing of the n pendent edges.The edge set $E(G) = \{e_i, e_{ii}\}$here $e_i = (ab_i)$ and $e_{ii} = (b_i c_i)$.
Define $\phi: V \longrightarrow \{0,1,2\dots..F_{q+1}\}$ as follows
$\phi\ (a) = 0,$
$\phi\ (b_i) = F_{2n+2-i} \qquad i = 1,2,\dots,n.$
$\phi(c_i) = \phi\ (b_i) - F_{n+2-i} \qquad i = 1,2,\dots,n.$
The edge labels are
$e_{ii} = \phi^*(b_i c_i) = |\phi(b_i) - \phi(c_i)| \qquad for\ \ i = 1,2,3,\dots,n$
$e_i = \phi^*(ab_i) = |\phi(a) - \phi(b_i)| \qquad for\ \ i = 1,2,3,\dots,n$
Here all the vertex sums are pairwise distinct and all the edges are unique.
Thus ϕ admits AML .
Hence the double star $K_{1,n,n}$ is$FGAMG$.
Python coding and output is enclosed in the Appendix .

Illustration 3.2

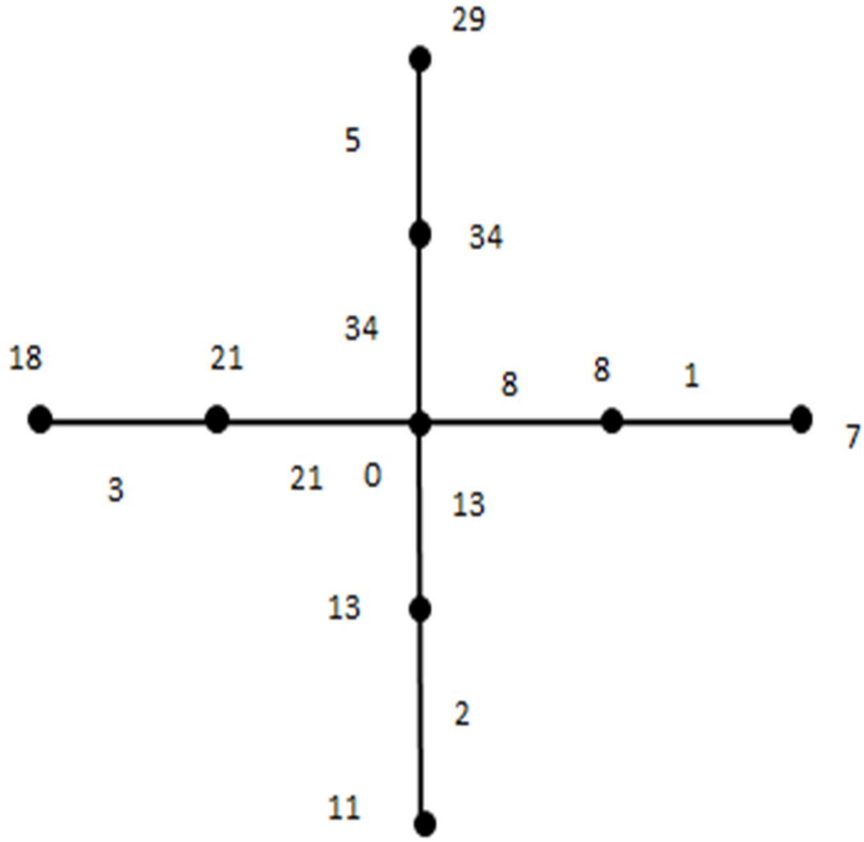


Fig 2. Double star $K_{1,4,4}$ is $FGAMG$

3) Theorem

Bi-star $B_{n,n}$ is $FGAMG$ for all n.

Proof:

Let $B_{n,n}$ be Bi-star, By the definition of$B_{n,n}$, The order of the Bi-star $B_{n,n}$ $is\ p = 2n + 2$ and size is $q = 2n + 1$.The vertex set $V(G) = \{a_0, a_1, a_2, \dots, a_n, b_{0,} b_1, b_2, \dots, b_n\}$. Let $a_1, a_2, \dots, a_n$ be the adjacent vertices of a_0 and $b_1, b_2, \dots, b_n$ be the adjacent vertices of b_0.

$E(G) = \{e_1^*, e_j, e_j\}$here$e_i = (b_0 b_i), g_i = (a_0 a_i), e_1^* = (a_0 b_0)$.

Define $\phi: V \longrightarrow \{0,1,2, \dots, F_{q+1}\}$ as follows

$\phi(a_0) = 0$

$\phi(a_{i+1}) = F_{n+2+i} \quad for\ i = 1,2,3, \dots, n-1$

$\phi(b_{i+1}) = F_{i+2} + F_2 \quad for\ i = 1,2,3, \dots, n-1$

$\phi(b_0) = F_2$

The edge labels are

$e_i = \phi^*(b_0 b_i) = |\phi(b_0) - \phi(b_i)| \quad for\ i = 1,2,3, \dots, n$

$g_i = \phi^*(a_0 a_i) = |\phi(a_0) - \phi(a_i)| \quad for\ i = 1,2,3, \dots, n$

$e_1^* = \phi^*(a_0 b_0) = |\phi(a_0) - \phi(b_0)|$

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus ϕ admits $FGAML$.

Hence the graph $B_{n,n}$ $is\ FGAMG$.

Python coding and output is enclosed in the Appendix .

Illustration 3.3

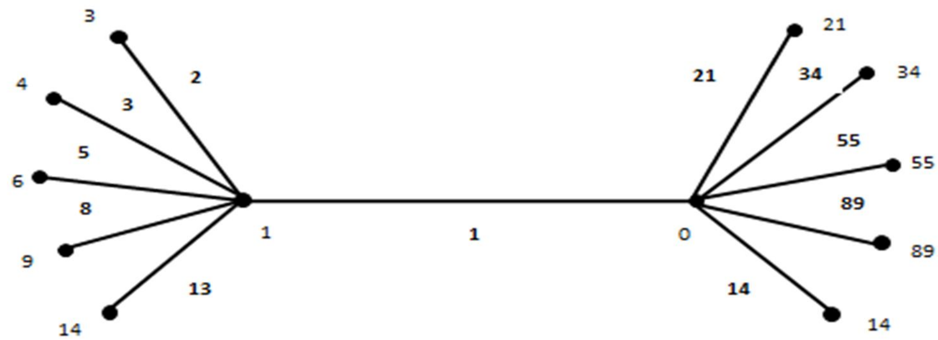


Fig 3. Bi-star$B_{5,5}$ is $FGAMG$

4) Theorem

Bi-shell $BS_{n,n}$ is $FGAMG$ for all n.

Proof:

Let $BS_{n,n}$ be the Bi-shell graph, The order of $B_{n,n}$ is $p = 2n + 2$ and $the\ size\ is\ q = 4n - 1$. The vertex set $V(G) = \{a_0, a_1, a_2, \dots, a_n, b_{0,} b_1, b_2, \dots, b_n\}$.Let $a_1, a_2, \dots, a_n$be the adjacent vertices of a_0 and $b_1, b_2, \dots, b_n$ be the adjacent vertices of b_0.

$E(G) = \{e_1^*, e_i, e_{ij,}g_i, g_{ij}\}$here $e_i = (b_0b_i), g_i = (a_0a_i), e_1^* = (a_0b_0), e_{ij} = (b_ib_j), g_{ij} = (a_ia_j)$.

Define $\phi: V \longrightarrow \{0,1,2, \dots, F_{q+1}\}$

$\phi(a_0) = F_2$

$\phi(a_i) = F_{2i+1} + F_2 \quad for\ i = 1,2,3, \dots, n$

$\phi(b_i) = F_{4n-2i+2} \quad for\ i = 1, 2,3, \dots, n$

$\phi(b_0) = 0$

The edge labels are

$e_i = \phi^*(b_0b_i) = |\phi(b_0) - \phi(b_i)| \quad for\ i = 1,2,3, \dots, n$

$g_i = \phi^*(a_0a_i) = |\phi(a_0) - \phi(a_i)| \quad for\ i = 1,2,3, \dots, n$

$e_1^* = \phi^*(a_0b_0) = |\phi(a_0) - \phi(b_0)|$

$e_{ij} = \phi^*(b_ib_j) = |\phi(b_i) - \phi(b_j)| \quad for\ i = 1,2,3, \dots, n\ and\ for\ j = i + 1$

$g_{ij} = \phi^*(a_ia_j) = |\phi(a_i) - \phi(a_j)| \quad for\ i = 1,2,3, \dots, n\ and\ for\ j = i + 1$

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus ϕ admits$FGAML$.

Hence the graph $BS_{n,n}\ is\ FGAMG$.

Python coding and output is enclosed in the Appendix .

Illustration 3.4

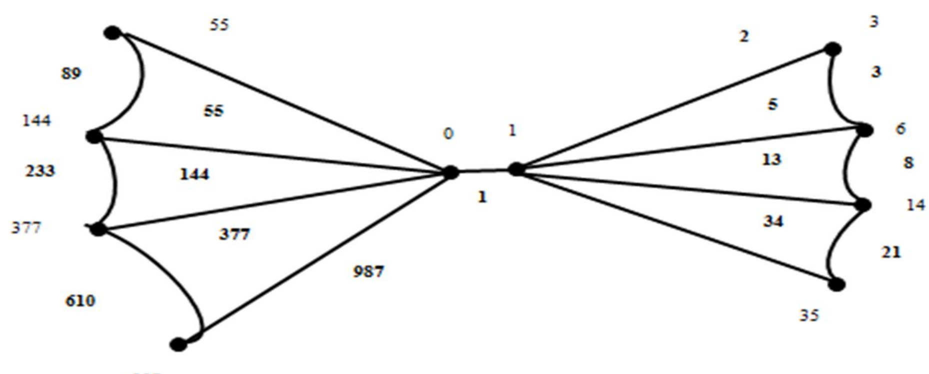


Fig.4.Bi-Shell $BS_{4,4}$ is $FGAMG$

IV. CONCLUSION

In this article the concept of Fibonacci Graceful Anti-Magic Labeling and demonstrated some Star related graphs are Fibonacci Graceful Anti – Magic Graphs. Python codings are generated for the Fibonacci Graceful Anti-Magic Labeling for the Star related Graphs. In future different concept of labeling will be developed.

V. APPENDIX

1) Python Coding and output for Crown graph $C_n \odot K_1$.

```
n=int(input("Enter the n Value: "))

subscript = str.maketrans("0123456789", "₀₁₂₃₄₅₆₇₈₉")

print(f"φ(a{1}) = 0".translate(subscript))
print(f"φ(b{3}) = 6".translate(subscript))
d = 2*n+1
print(f"φ(b{1}) = F{d}".translate(subscript))


for i in range (n):
    i+=1
    b=(2*i)+2
    print(f"φ(a{i+1}) = F{b}".translate(subscript))
    if i!=2:
      c=(2*i)+1
      print(f"φ(b{i+1}) = F{c}".translate(subscript))
    else:
       None

    print("")
```

```
Enter the n Value: 5
φ(a₁) = ₀
φ(b₃) = ₆
φ(b₁) = F₁₁
φ(a₂) = F₄
φ(b₂) = F₃

φ(a₃) = F₆

φ(a₄) = F₈
φ(b₄) = F₇

φ(a₅) = F₁₀
φ(b₅) = F₉
```

3s completed at 12:43 PM

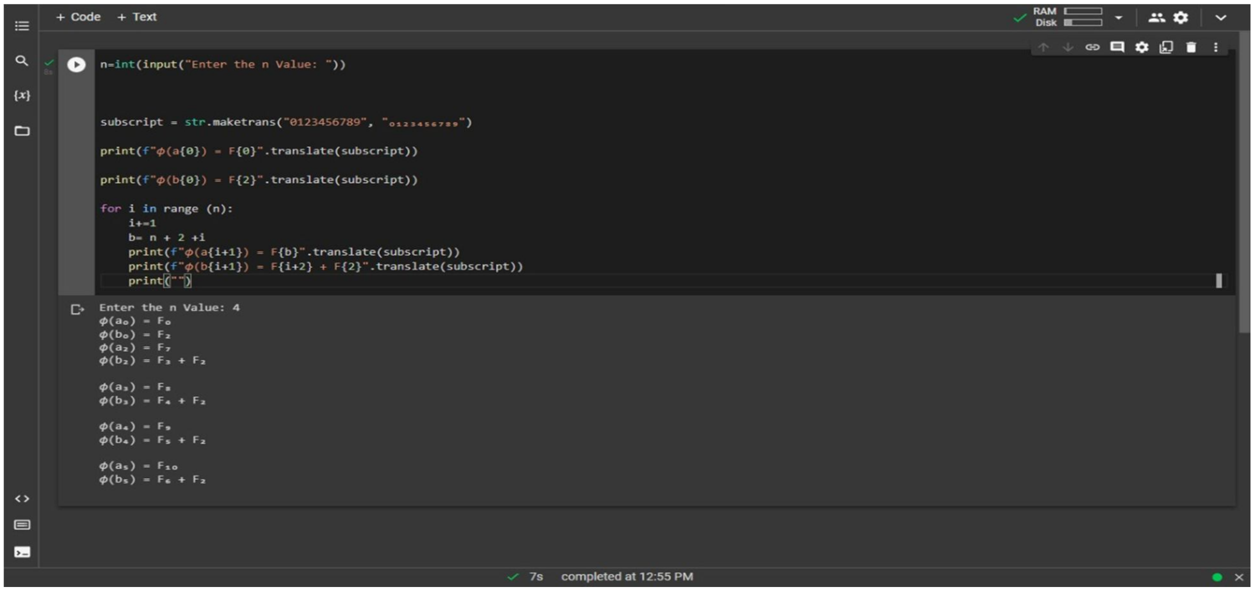
2) Python Coding and output for double star $K_{1,n,n}$

```
n=int(input("Enter the n Value: "))

subscript = str.maketrans("0123456789", "₀₁₂₃₄₅₆₇₈₉")

for i in range (n):
    i+=1
    print(f"φ(a) = F{0}".translate(subscript))

    b=(2*n)+2-i
    print(f"φ(b{i}) = F{b}".translate(subscript))


    d = (n+2-i)
    print(f"φ(c{i}) =F{b} - F{d}".translate(subscript))

    print("")
```

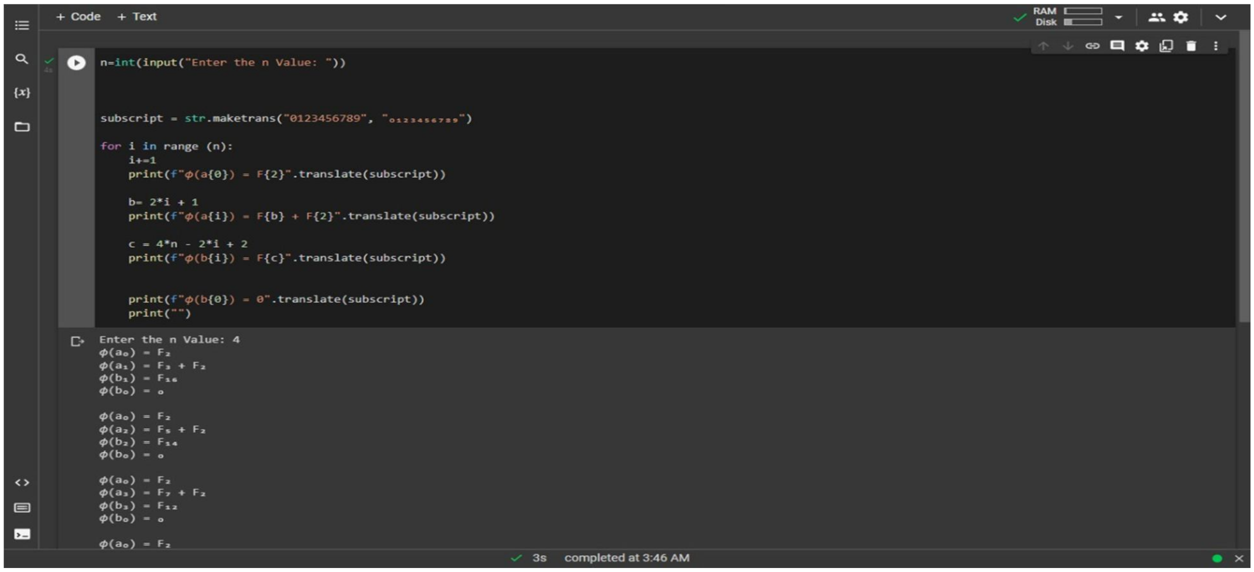
```
Enter the n Value: 5
φ(a) = F₀
φ(b₁) = F₁₁
φ(c₁) =F₁₁ - F₆

φ(a) = F₀
φ(b₂) = F₁₀
φ(c₂) =F₁₀ - F₅

φ(a) = F₀
φ(b₃) = F₉
φ(c₃) =F₉ - F₄

φ(a) = F₀
φ(b₄) = F₈
φ(c₄) =F₈ - F₃

φ(a) = F₀
φ(b₅) = F₇
φ(c₅) =F₇ - F₂
```

5s completed at 12:18 PM

3) Python Coding and output for Bi-star $B_{n,n}$:



4) Python Coding and output for Bi-shell $BS_{n,n}$



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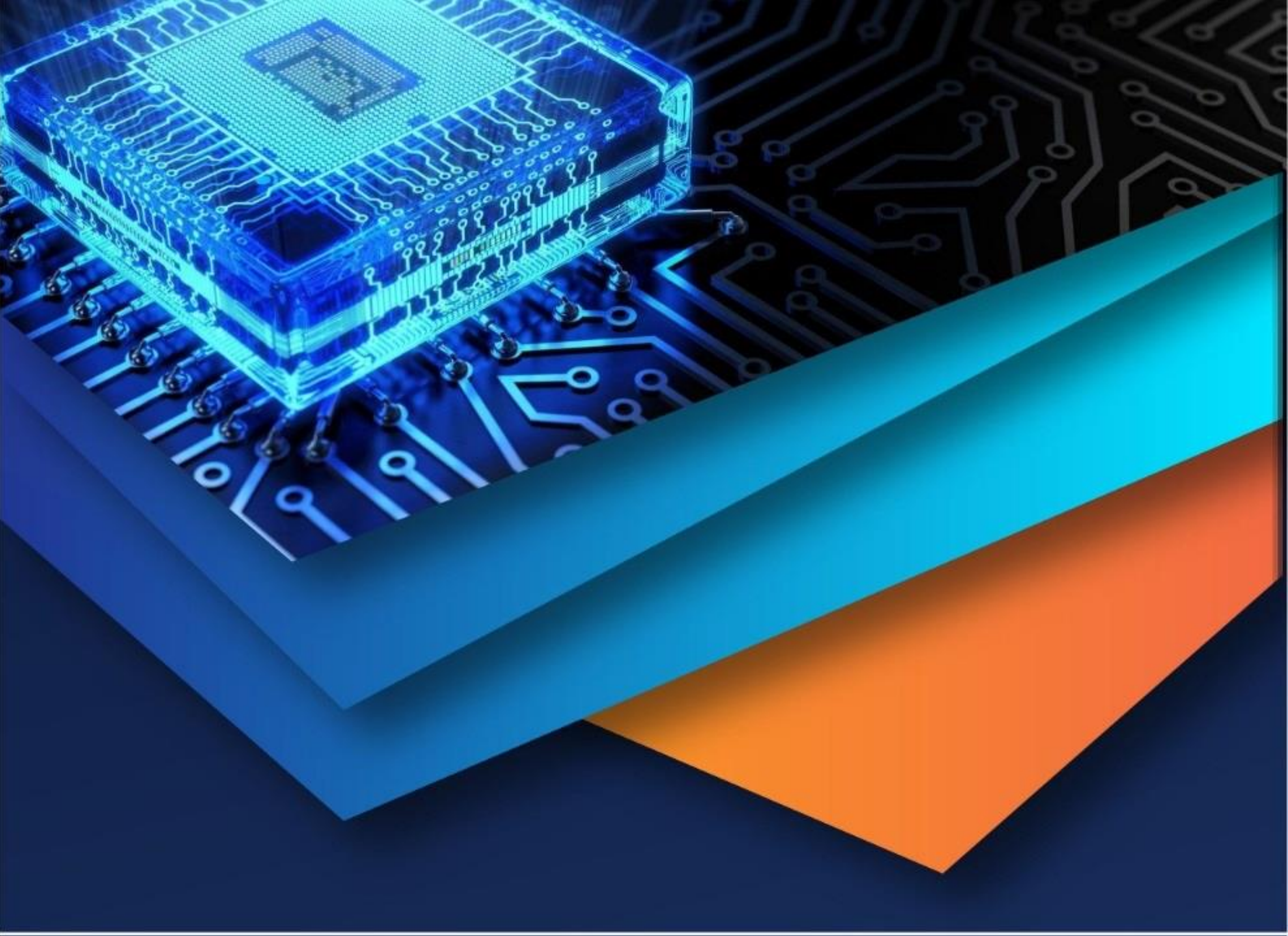

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