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Ramanujan: The New Sum of All Natural Numbers

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Abstract: As we know that Sir Ramanujan gave the solution of sum of all natural numbers up to infinity and said that the sum of all natural numbers till infinity is -1/12. I studied on this topic and found that if we try to solve the infinite series in a slightly different way, then we get the answer of its sum different from -1/12, so this is what I have written in this paper that such Ramanujan Sir, what was the mistake in solving the infinite series, which by solving it in a slightly different way from the same concept, we get different answers.
Keywords: Ramanujan the new sum of all natural numbers, Infinite series solution, Gaurav singh patel, Infinite series sum, Natural number sum.

I. INTRODUCTION

As we know Sir Ramanuman gave the sum of all natural numbers as -1/12 so let us first see how he solved it-

We have to find the sum of all natural numbers 1+2+3+4+5+6+……………+∞ = ?

Or $\sum_{n=1}^{\infty} n = \zeta(-1) = -\frac{1}{12}$

Let S_1 = 1+2+3+4+5+6+……………+∞ equation(1)

S_2 = 1-1+1-1+1-1+……………+∞ equation(2)

On writing the terms of equation(2) leaving one space from the beginning

S_2 = 1- (1-1+1-1+1+……………+∞)

S_2 = 1- (S_2)

$S_2 + S_2 = 1$

$2S_2 = 1$

$S_2 = \frac{1}{2}$

Let S_3 = 1-2+3-4+5-6+……………+∞ equation(3)

Subtract equation(3) from equation(2)

S_2 = 1-1+1-1+1-1+……………+∞ equation(2)

S_3 = 1-2+3-4+5-6+……………+∞ equation(3)

($S_2 - S_3$) = 0+1-2+3-4+5……………..+∞

$(S_2 - S_3) = S_3$

$2S_3 = S_2$

$S_3 = S_2/2$

$S_3 = \frac{1}{4}$

A. First Method

The first method to find the sum of all natural numbers (Ramanujan method)

S_1 = 1+2+3+4+5+6+……………+∞ equation(1)

S_3 = 1-2+3-4+5-6+……………+∞ equation(3)

Subtract equation(3) from equation(1)

($S_1 - S_3$) = 4+8+12+16+20+24…………..∞

($S_1 - S_3$) = 4(1+2+3+4+5+6+……………+∞)

$(S_1 - S_3) = 4S_1$

$4S_1 - S_1 = - S_3$

$3S_1 = - S_3$

$3S_1 = -\frac{1}{4}$

$S_1 = -\frac{1}{12}$

$\sum_{n=1}^{\infty} n = \zeta(-1) = -\frac{1}{12}$

Here we are seeing that if we add all the natural numbers till infinity then we will get -1/12, now we adopt another method to solve this problem.

B. Second Method

$S_1 = 1+2+3+4+5+6+\ldots\ldots\ldots+\infty$ equation(1)

On writing the terms of equation(1) leaving one space from the beginning

$S_1 = \quad +1+2+3+4+5+\ldots\ldots\ldots+\infty$ equation(4)

Add equation(1) and equation(4)

$2S_1 = 1+3+5+7+9+11+\ldots\ldots\ldots+\infty$ equation(5)

Subtract equation(2) from equation(5)

$2S_1 = 1+3+5+7+9+11+13+15\ldots\ldots\ldots+\infty$ equation(5)

$S_2 = 1-1+1-1+1-1+1-1+1-1+1\ldots\ldots\ldots+\infty$ equation(2)

$(2S_1 - S_2) = 4+4+8+8+12+12+16+16\ldots\ldots\ldots+\infty$

$(2S_1 - S_2) = 8+16+24+32+\ldots\ldots\ldots+\infty$

$(2S_1 - S_2) = 8(1+2+3+4+\ldots\ldots\ldots+\infty)$

$(2S_1 - S_2) = 8(S_1)$

$6S_1 = -S_2$

$\therefore S_2 = \frac{1}{2}$

$6S_1 = -\frac{1}{2}$

$S_1 = -\frac{1}{12}$

$\sum_{n=1}^{\infty} n = \zeta(-1) = -\frac{1}{12}$

Add equation(2) and equation(5)

$2S_1 = 1+3+5+7+9+11+13+15\ldots\ldots\ldots+\infty$ equation(5)

$S_2 = 1-1+1-1+1-1+1-1+1-1+1\ldots\ldots\ldots+\infty$ equation(2)

$(2S_1 + S_2) = 2+2+6+6+10+10+14+14$

$(2S_1 + S_2) = 4+12+20+28+\ldots\ldots\ldots\infty$

$(2S_1 + S_2) = 4(1+3+5+7+\ldots\ldots\ldots\infty)$

$(2S_1 + S_2) = 4(2S_1)$

$(2S_1 + S_2) = 8S_1$

$6S_1 = S_2$

$\therefore S_2 = \frac{1}{2}$

$6S_1 = \frac{1}{2}$

$S_1 = \frac{1}{12}$

$\sum_{n=1}^{\infty} n = \frac{1}{12}$

Now again add equation(1) and equation(3)

$S_1 = 1+2+3+4+5+6+\ldots\ldots\ldots+\infty$ equation(1)

$S_3 = 1-2+3-4+5-6+\ldots\ldots\ldots+\infty$ equation(3)

$(S_1+S_3) = 2+6+10+14+\ldots\ldots\ldots+\infty$

$(S_1+S_3) = 2(1+3+5+7+\ldots\ldots\ldots+\infty)$

$\therefore 1+3+5+7+9+11+\ldots\ldots\ldots+\infty = 2S_1$

Putting value

$(S_1+S_3) = 2(2S_1)$

$(S_1+S_3) = 4S_1$

$3S_1 = S_3$

$\therefore S_3 = \frac{1}{4}$

$3S_1 = \frac{1}{4}$

$S_1 = \frac{1}{12}$

II. RESULT

So finally

$S_1 = 1+2+3+4+5+6+\ldots\ldots\ldots\ldots\ldots+\infty = \frac{1}{12}$

$\sum_{n=1}^{\infty} n = \frac{1}{12}$

Also

$S_1 = 1+2+3+4+5+6+\ldots\ldots\ldots\ldots\ldots+\infty = -\frac{1}{12}$

$\sum_{n=1}^{\infty} n = \zeta(-1) = -\frac{1}{12}$

III. CONCLUSIONS

Here we're seeing withinside the very last end result that simply extrade the technique of calculating your sum, so we get the solution 1/12 alternatively of -1/12. Now the query arises why is that this happening? So let's apprehend why that is happening. As we see withinside the equation that to get our answer, we've taken into consideration an equation(2) that's incomplete in itself and base 0, therefore, for one of these equation whose base is 0 and which will fill us with further error which is clearly visible to us.

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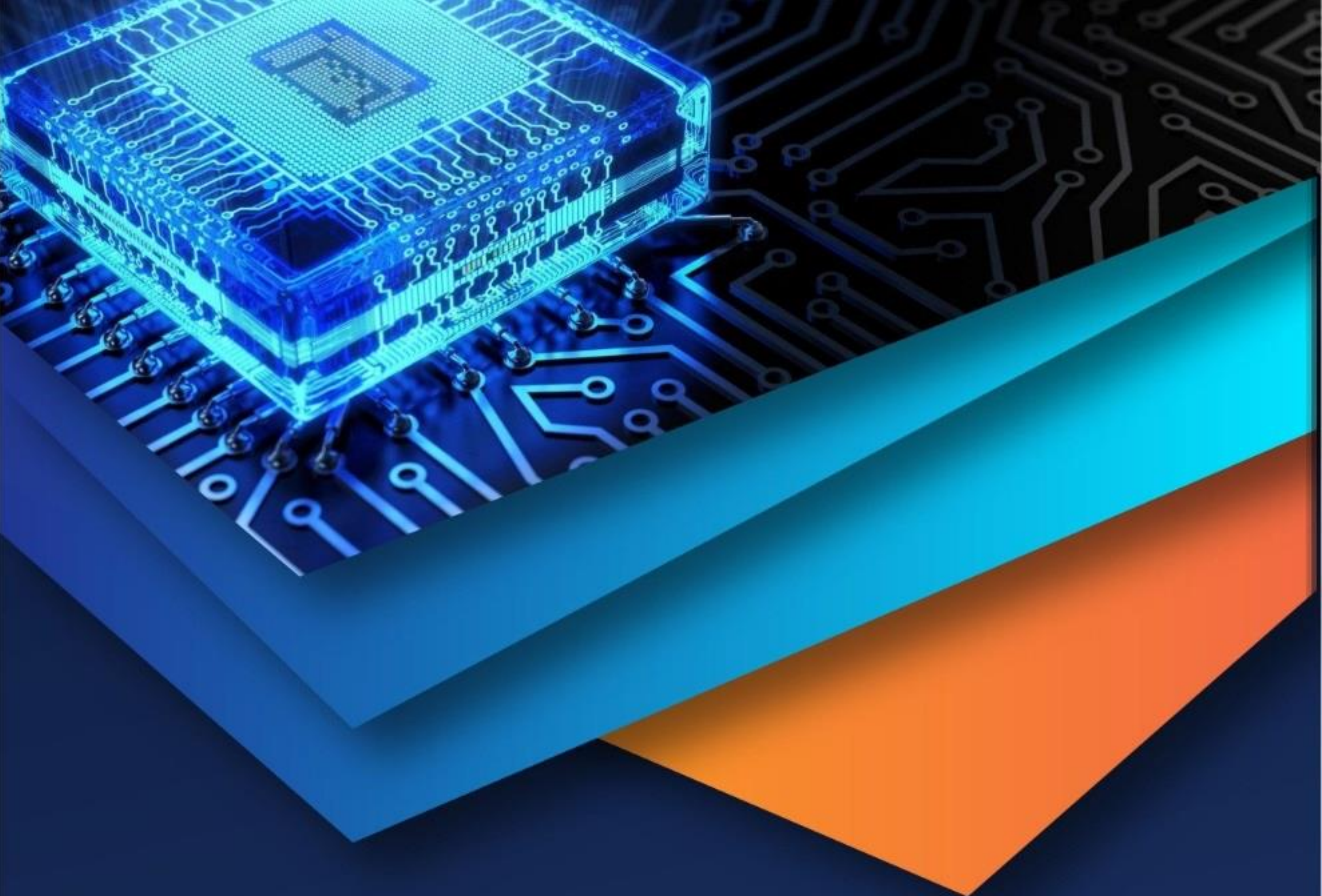

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