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# Result Obtained in the learn of Generalized Derivations of Prime Rings

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**Abstract:** In this subsection, we will assume that  $R$  is a prime ring that  $H$  is a generalization of  $R$ , that  $L$  is a noncentral Lie idyllic of  $R$ , and that  $0 \neq \alpha \in R$ . If  $s, t \geq 0$  and  $n > 0$ , then for all  $u$  in  $L$ ,  $\text{aus}(H(u)^n)u(t)$  must equal 0. All of them fit neatly inside the specified ranges of integers.  $H(x) = bx$  for every  $x \in R$  if and only if  $s = 0$  and  $b = 0$ ; this is not the case if  $R$  satisfy  $S_4$ , the normal uniqueness in four variables. In this context,  $b$  stands for the Utumi quotient ring of the right side of  $R$ . The set  $R$  is said to meet the  $S_4$  condition if and only if the function  $H(x)$  is equal to  $bx$ . Once  $s$  is equal to zero,  $H(x)$  equals  $bx$  for every  $x$  in  $R$ . It follows that  $H = 0$  if and only if  $s > 0$  and  $R$  does not meet  $S_4$  the requirement.

**Keywords:** Derivation, Generalized derivation, Utumi quotient ring, Extended Centroid, Prime ring.

## I. INTRODUCTION

In order to progress in ring theory, a solid understanding of additive mappings is required. In mathematics, derivations are represented by additive maps. Many authors have attempted to widen the meaning of the word "derivation" by giving it new names; Some common types of derivations include the generalized derivation,  $(\alpha; \beta)$ -derivation, multiplicative derivation, Jordan derivation, and multiplicative generalized derivation. Extensive investigation of extended derivations on prime rings has led us to the conclusion that, if nothing else, these derivations must satisfy an identity, it must have a certain shape. See, here is where our study of prime ring theory truly comes into its own. If we loosen the restrictions on  $d$  and remove  $F$ 's additivity, we have a multiplicative (generalized) derivation  $d : R \rightarrow R$  does not need to be additive and does not need to be a derivation). The definition of generalized derivation requires that  $F$  be additive. That's why it was held up as an example.

The ring will be referred to as  $R$  henceforth, and its symbol will be  $d : R$ . The map  $R$  is assumed to be additive on  $R$ . We claim that if and only if the equation  $d(xy) = d(y)x + yd(x)$  holds for all  $x$  and  $y$  variables inside  $R$ , then  $d$  represents a repeal derivation on  $R$ .

Derivation and derivatization in reverse are equal for a commutative  $R$ . The original inversion derivation was provided by Herstein back in 1957. We additionally take into account ring geometry and mapping dynamics, and present an inverse derivation that is multiplicatively defined.

As used above, a derivation is defined as an preservative map  $d$  on a ring  $R$  such that for any members  $x, y$  of  $R$ ,  $d(x, y) = d(x)y + xd(y)$ . Ring derivatives play a significant role in ring theory. An important result in the study of ring derivations was demonstrated by Richard Posner in 1957. He also investigated the inner workings of maps and the building of rings. Posner's result on appropriate subsets of a ring has been elaborated upon by a plethora of authors, who have introduced concepts like the ideal, the left ideal, the right ideal, the Lie ideal, the Jordan ideal, etc. Wong rewrote Posner's work on multilinear polynomials after realizing All commutators of a non-zero ideal of a non-zero feature ring are included in a non-central Lie perfect, even if this ideal is not central. This is because a non-central Lie ideal may include a non-zero ideal of a ring with a non-2 property, which is not the case for central Lie ideals, which is the reason for the conclusion. Scientists from several teams investigated many different mappings on multilinear polynomials and found many different identities. Using extended derivations using multilinear polynomials on prime rings, we investigate identities in this thesis, which allows us to learn more about the relationship between mappings and ring structures.

Deletion situations are only one of several types of identity that have been studied from a derived viewpoint when Posner established that either  $R$  or  $d$  must be prime if  $R$  is a prime ring with  $d$  is a derivation on  $R$  such to  $ad(x) = 0$  for every  $x$  in  $R$ , he did so by proving that either  $R$  or  $d$  must be prime. In this field, Posner has published the seminal work. Several authors have provided further explanations of this finding. We use extended derivation on  $R$  under the annihilating condition to explore the identities, which we then use to show the commutativity of the prime ring  $R$ . Here, we concentrate on generalizable derivations of identities on the prime ring  $R$ . We also consider the use of automorphism on prime rings to investigate the prospect of an identity destroying itself.

It was in Jacobson's "Structure of Rings" when the term "(s1; s2) derivation" first emerged. It is likely that the word " $(\alpha; \beta)$ -derivation" will come to be used more often to denote this concept in the near future. We investigate the connection among the commutativity of the prime ring  $R$  with the features of generalized  $(\alpha; \beta)$  -derivation on appropriate subsets of  $R$  in this dissertation. By doing so, we will have a clearer appreciation for the relationship between the two ideas at hand. When the additivity of  $G$  is removed, the resulting generalized  $(\alpha; \beta)$  -derivation is called a multiplicative generalized  $(\alpha; \beta)$  -derivation. When the map  $d$  associated with  $G$  is entirely unbounded, i.e., it may be substituted by any map, we refer to  $G$  as a multiplicative (generalized)  $(\alpha; \beta)$  -derivation. We show in this dissertation that every multiplicatively generalized  $(\alpha; \beta)$  -derivation on  $R$  may be converted into an additive form under certain conditions if and only if close by exist a nontrivial idempotent ingredient in  $R$ . For this development, we examine the identities by performing multiplicative (generalized)  $(\alpha; \beta)$ -derivation on left principles of prime rings.

A value of zero for either  $a$  or  $d$  is guaranteed if  $\text{char } R = 2$  and  $R$  satisfies  $s_4$ . We'll refer to this noncommutative Lie ideal of  $R$  as  $L$  if it exists. In this definition,  $L$  is a noncumulative Lie ideal of  $R$ ,  $H$  is a generalization of  $R$ ,  $s$  and  $t \geq 0$  are constant integers, and  $H = 0$  if not  $\text{char } R = 2$  and  $R$  satisfy  $s_4$ . This rule holds if and only if  $\text{char } R$  equals 2 and  $R$  satisfies  $s_4$ . This idea is supported by the observation that  $uH(u)u^t = 0$  for all  $u$  values. In his most recent paper, the author investigates what happens when author impose the constraint  $uH(u)u^t = 0$  on every  $u$  in  $L$ , everywhere  $L$  denotes a noncentral Lie perfect of  $R$ . In this part, we will briefly summarize the grades described in the preceding section.

Our hope is that this example will help convey the following key result from our study.

#### A. Theorem

$H$  is a generalization of  $R$ ,  $L$  is a noncentral Lie equal of  $R$ , and  $0$  is not a subring of  $R$  if and just if  $R$  is a prime ring. If we presume that  $s; t \geq 0$  and  $n > 0$  are constant integers, then we may write  $uH(u)u^t = 0$  for every  $u$  in the range  $L$ . Unless  $R$  satisfies  $s_4$ ,  $H(x) = bx$  for every  $x \in R$ , and then  $b \in U$  and  $ab = 0$ . It follows that if  $s$  is not zero,  $H=0$  except  $R$  satisfy the conditions of  $S_4$ .

The Proof which suggests that the main conclusion is valid.

The following result serves as our jumping off point and will be used to show our major discovery. Lemma, The prime ring  $R$  is said to have a  $\dim \text{CRC} \geq 4$  if and only if the number of primes in the ring is more than or equal to four. In order to do this, Let  $0 \neq \alpha \in R$  and  $b \in U$  such that

$$a[x, y]^s (b[x, y])^n [x, y]^t = 0$$

Any pair  $(x, y) \in R$  satisfying  $(s, t) > 0$  plus  $(n, s) > 0$  are xed integers. If  $s = 0$ , then the negation of  $ab$  also has the same outcome. It follows that  $b$  must be zero if  $s$  is not zero. Proof. Assuming  $b$  is bigger than  $C$ , we may get the conclusion

Proof. reason First to  $b \in C$ , by hypothesis include

$$ab^n [x, y]^{s+n+t} = 0$$

Every time  $x$  occurs,  $y \in R$ . Either  $abn^2 = 0$  or the commutativity of  $R$  can be shown with little effort. The value of  $b$  is zero because  $0$  and  $\dim \text{CRC} > 4$ . Here, let's imagine  $b$  is bigger than  $C$  as an example. Inferring that because  $R$  and  $U$  both gratify the same extended polynomial characteristics,

$$a[x, y]^s (b[x, y])^n [x, y]^t = 0$$

$U$  consists of  $y$  elements for each  $x$  in the set. If and only if  $C$  is infinite, then  $U \otimes C$ , everywhere  $C$  is the numerical conclusion of  $C$ , fulfills the GPI. Since  $U$  as well as  $U \otimes C$  are both prime as well as centrally stopped, we may interchange them and use either one in place of  $R$ , depending on whether or not  $C$  is finite. It is reasonable to suppose that  $R$  is centrally stopped over  $C$  if and only if  $C$  has a limited number of components or is algebraically closed.

$$a[x, y]^s (b[x, y])^n [x, y]^t = 0 \text{ for all } x, y \in R.$$



Assuming If  $s = 0$  and  $ab \neq 0$ , then  $\alpha(b\{X, Y\}^n \{X, Y\}^t)$  is a nonzero GPI on  $R$  due to the presence of a nonzero monomial.  $Ab$  is negative if and only if  $s$  is zero. Since  $D$  is central partition algebra of finite dimension over  $C$  if and only if  $R$  contains a nonzero socle, this proves Martindale's theorem. This is due to the commutative nature of division by  $D$ . If  $C$  is fixed or algebraically stopped, then  $D$  must concur with  $C$ .

In this way, for every vector space over  $C$ ,  $R$  is isomorphic to a opaque subring of  $\text{End}_C V$ .  $\dim_C V$  must be less than 3 while  $\dim_C R$  is more than 4. In this paper, we will show that  $v$  and  $bv$  are both ward on  $C$  for any value of  $v$  with regard to  $V$ . If it helps you to calculate  $W$ , you may pretend that  $v$  and  $bv$  have no connection to  $C$  and substitute  $C$  and  $Cb$  instead. There exists a value of  $u$  smaller than  $V$  such to  $v$ ,  $bv$ , and  $u$  are all sovereign of  $C$ , provided that  $\dim_C V$  is less than 3. The density of  $R$  in  $\text{End}_C V$  allows us to prove that if  $abv$  is negative, then there are two elements in  $R$ ,  $r_1$  and  $r_2$ , whose positions satisfy the condition.

$$r_1 v = 0, \quad r_1 b v = 0, \quad r_1 u = v; \quad r_2 v = u, \quad r_2 b v = u, \quad r_2 u = 0$$

and so

$$[r_1, r_2]v = v \quad \text{and} \quad [r_1, r_2]bv = v.$$

Hence,

$$0 = a(b[r_1, r_2])^n [r_1, r_2]^t v = abv,$$

contradictory assertion. For the sake of argument, let's say that  $abv = 0$ . It follows that  $ab(v-w) > 0$  since there is a non-zero value of  $w$  in  $V$  such that  $abw > 0$ . From our previous inferences, we learn that for certain values of  $C$ ,

$$bw = \beta w \quad \text{and} \quad b(w - v) = \gamma(w - v).$$

This yields that  $(\beta - \gamma)w \in W$ . Now  $\beta = \gamma$  implies the contradiction that  $bv = \beta v$ , we may now say that " $bv = v$ ." Since  $w = w$ ,  $w \in W$ . Assuming  $u \in V$  and  $au = 0$ , we have  $Ab(w + u) > 0$ . Thus,  $u$  must be  $W$  because  $w + u$  equals  $W$ . Since  $V = W$ , we may deduce that  $\dim_C V = 2$ , which is clearly not the case. Therefore, both  $v$  and  $bv$  depend linearly on  $C$ , independent of the value of  $v$  relative to  $V$ . The conventional wisdom, however, is that  $b$  is smaller than  $C$ , which is at odds with our hypothesis.

There is conclusive proof that success was achieved.

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