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Role of Electrostatic Energy for Distribution of Charges in Case of Earth-Connected Parallel Plate Capacitor

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Abstract: Electrostatic energy plays a fundamental role in driving the distribution of charges in a parallel plate capacitor. When any one plate of a parallel plate capacitor is earthed, then the Earth connection ensures that the system remains in stable equilibrium by influencing the charge accumulation on the plates to minimize the overall energy of the system. This communication explained the role of electrostatic energy for the distribution of charges in the case of an earth-connected parallel plate capacitor.
Keywords: Energy density, parallel plate capacitor, charge distribution, electric field, equilibrium

I. INTRODUCTION

A charged conductor produces an electric field in its surroundings, and the work done in its charging process is stored in the form of field energy of the electric field in space surrounding the conductor. The energy density in an electrostatic field is $\frac{1}{2}\,\epsilon_0\,E^2$. By specific methods, we can use the electrical energy stored in the electric field of a charge for different purposes.

A capacitor is an electrical component designed to store and release electric charge. Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges separated by an insulating material (dielectric).

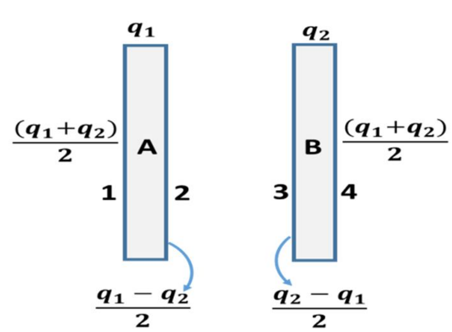
A parallel plate capacitor is a basic type of capacitor consisting of two conductive plates separated by an insulating material (dielectric). It is widely used in electronic circuits for energy storage, filtering, signal processing, etc. In recent years, ultracapacitors and supercapacitors have emerged as promising energy storage solutions. These advanced capacitors are gaining increasing importance as they provide an eco-friendly alternative to batteries and can also work alongside them to handle backup power peaks.

As the energy stored in an electric field is proportional to the square of the electric field strength so regions with stronger electric fields have higher energy density. When a parallel plate capacitor is connected to earth, energy density, which is proportional to the square of the electric field, dictates charge distribution, aiming to minimize the total energy stored to attain the stable equilibrium. Earthing allows charges to flow to the ground, reducing the electric field and, thus, the energy density, leading to a lower energy configuration. When one plate of the capacitor is earthed, charges can move freely to or from the ground. This movement of charge reduces the electric field and, consequently, lowers the energy density in the system. The system adjusts until it reaches a low-energy configuration, ensuring that the distribution of charges satisfies electrostatic equilibrium.

In this paper, we will explore the role of energy density for the distribution of charges in the case of an earth-connected parallel plate capacitor, and by detailed calculation, we will explain how charge is distributed in different plates to attain stable equilibrium.

II. BASIC LAW ABOUT DISTRIBUTION CHARGE ON VARIOUS SURFACES OF A PARALLEL PLATE CAPACITOR

It is a basic principle that when charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges, which is equal to half of the total charges on the plates, and the facing surfaces get equal and opposite charges. So when two conducting plates A and B are placed parallel to each other and A is given a charge q_1 and B a charge q_2 then charges on the two outermost surfaces 1 and 4 are $\frac{q_1+q_2}{2}$.



As total charge on the plate A is q_1, so charge on surface 2 is $\left(q_1 - \frac{q_1+q_2}{2}\right) = \frac{q_1-q_2}{2}$ and charge on surface 3 is $\left\{-\left(\frac{q_1-q_2}{2}\right)\right\} = \frac{q_2-q_1}{2}$

Now we will discuss the cases when the plate/plates are earth connected with and without electric field:

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III. CASE I: IF NO EXTERNAL ELECTRIC FIELD IS APPLIED

A. Two Differently Charged Parallel Plates Capacitor Is Taken, One Of Which Is Earth Connected

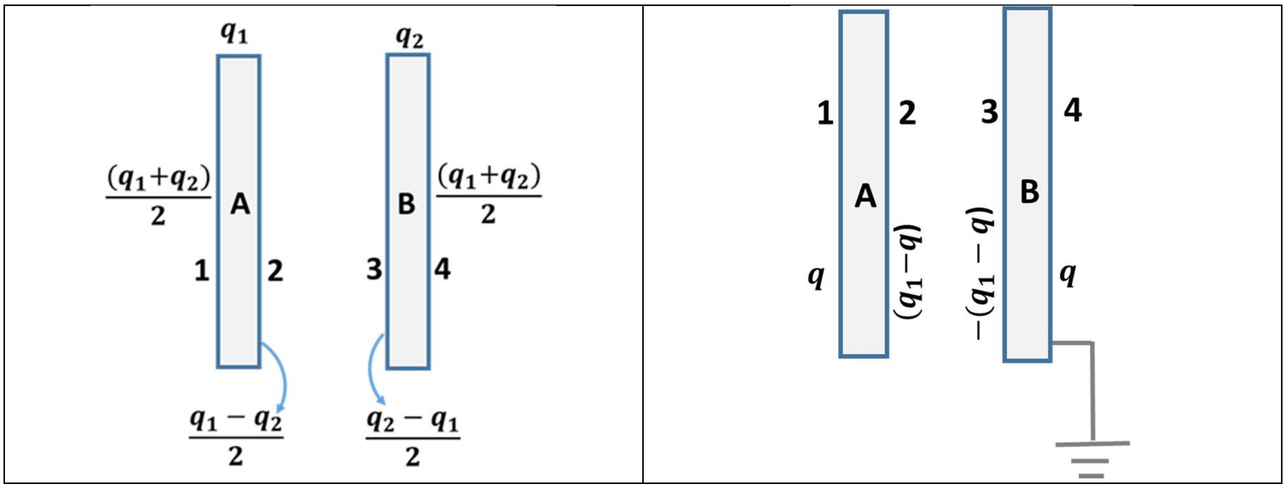
We consider two conducting plates, A and B, with charges q_1 and q_2 as shown in fig. 1, and no plates are connected. So, according to the basic law of distribution of charge, charges on the four surfaces of the two plates are as follows:

i) Charge on the outer surface of plate 1 is $\frac{q_1+q_2}{2}$

ii) Charge on the inner surface of plate 1 is $\frac{q_1-q_2}{2}$

iii) Charge on the inner surface of plate 2 is (- $\frac{q_1-q_2}{2}$)

iv) Charge on the outer surface of plate 2 is $\frac{q_1+q_2}{2}$



Now, we consider the case when one of the plates is earth-connected, and due to the earth connection, the distribution of charges will be different. Let due to the earth connection, the charge appears on the outer surface of plate 1 and 4 is q. So, the Charge on the inner surface of plate 1 is $(q_1 - q)$, and the inner surface of plate 2 is $\{-(q_1 - q)\}$.

We know that the expression for electric energy density is $u = \frac{1}{2}\epsilon_0 E^2$

where ϵ_0 is the permittivity of the medium in which the field exists, and E is the electric field vector.

So the total energy stored in the electric field in a given volume V is therefore U= $u\,V = \frac{1}{2}\epsilon_0 E^2\,V$

As the energy is stored, a mechanical force is produced between the conductors.

It is to be mentioned here that inside the parallel plates, electric field intensity due to charges on the outer surface of the parallel plates is zero. So, to find the electric field inside a parallel plate capacitor, we always consider the inside surface charges.

All charged objects create an electric field that extends outward into the space that surrounds it. So a charged capacitors produce electric field inside the plates as well as outer sides of the capacitor plates.

So, Energy of this parallel plate system is

U= energy inside the capacitor plates + energy on the two outer sides of the capacitor plates

$$U = \frac{1}{2}\epsilon_0\left(\frac{q_1 - q}{A\,\epsilon_0}\right)^2 A\,d + \frac{1}{2}\epsilon_0\left(\frac{q}{A\,\epsilon_0}\right)^2 \times 2 \times V$$

Here V is the outer volume.

Now,
$$\frac{dU}{dq} = \frac{1}{2\,A^2\,\epsilon_0^2}\,\epsilon_0 \times A\,d \times 2(q_1 - q)\,(-1) + \frac{1}{2\,A^2\,\epsilon_0^2}\,\epsilon_0 \times 2V \times 2\,q$$
$$= -\frac{d}{A\,\epsilon_0}(q_1 - q) + \frac{2\,V}{A^2\,\epsilon_0}\,q$$

Now, for achieving a stable equilibrium of the system, the potential energy of the system should be minimum.

The condition for U minimum is $\frac{dU}{dq} = 0$

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$$\Rightarrow -\frac{d}{A\,\epsilon_0}(q_1 - q) + \frac{2\,V}{A^2\,\epsilon_0}\,q = 0$$
$$\Rightarrow -A\,d\,q_1 + A\,d\,q + 2\,Vq = 0$$
$$\Rightarrow q = \frac{A\,d\,q_1}{Ad + 2\,V}$$
$$\Rightarrow q = \frac{q_1}{1 + \frac{2V}{A\,d}}$$

Here, the volume inside the capacitor $A\,d$ is very small compare to the outside volume V of the capacitor V is very large. So $q \to 0$. So when any plate of the parallel plate capacitor is earth-connected, then for the system to achieve stable equilibrium, charges on the outer surfaces of the parallel plates become zero.
This condition is true not only for the system of two plates parallel plate capacitor but it is true for any number of earth connecting system of parallel plates capacitor.
In the case of a parallel plate capacitor of many parallel plates, if you earth any capacitor then the charge on the outer plate is zero so that the total energy of the system is minimum.

B. Earth Connected System of three Plates

Three large identical metallic plates are placed as shown in the Figure. The middle plate is given a charge Q. All other plates are neutral. Now, plates 1 and 3 are earthed. The area of each plate is A. we will use the above concepts to find the charge appearing on all the sides of all plates.
From the above discussions, we learned that the charge on the outer surface of earth connected plate is zero, so, on the outer surfaces of plates 1 & 3 have no charge.
Let the charge on right face of plate 2 be q. So the charge on the left surface of plate 2 is $(Q - q)$. We know, the opposite faces must have equal and opposite charge. Using this concept, distribution of charges on other faces is shown in the figure. Charge on the inner surface of plate 1 is $-(Q - q)$ and the inner surface of plate 3 is $-q$.

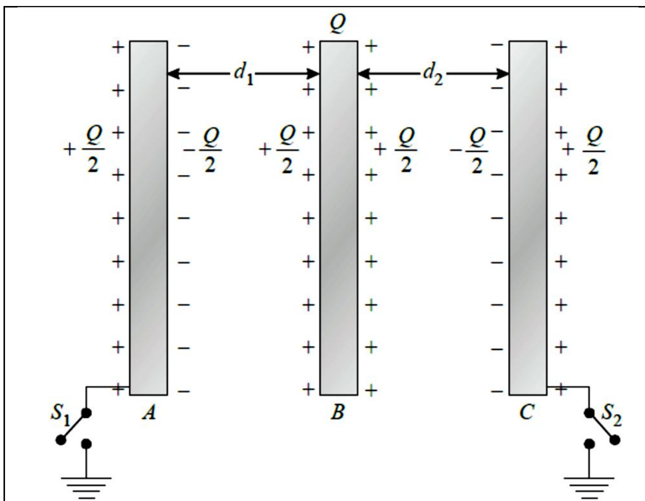

Fig. Distribution of charges when outer plates are not earth-connected

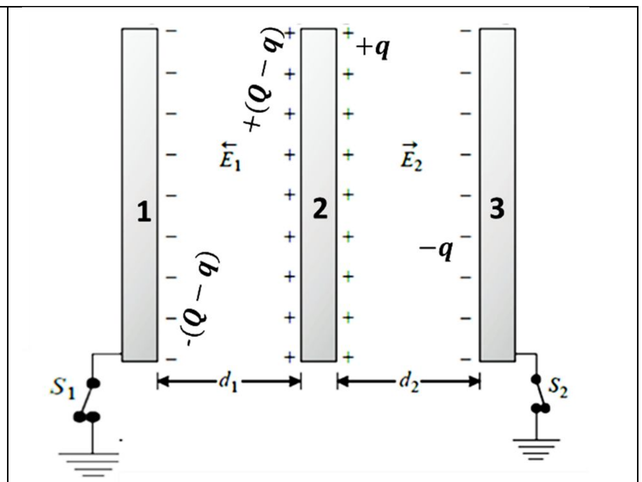

Fig. Distribution of charges when outer plates are earth-connected

Now, the electric field intensity between plate 1 and 2 is $E_1 = \frac{Q-q}{\epsilon_0\,A}$

The electric field intensity between plates 2 and 3 is $E_2 = \frac{q}{\epsilon_0\,A}$

Now, using Kirchhoff's voltage law, we can write
$$V_1 + E_1 d_1 - E_2\,d_2 = V_3$$
As plate 1 and 3 are earth-connected, so $V_1 = V_3 = 0$
$$\Rightarrow E_1 d_1 - E_2\,d_2 = 0$$

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$$\Rightarrow E_1 d_1 - E_2\, d_2 = 0$$
$$\Rightarrow \frac{Q-q}{\epsilon_0\, A}\, d_1 - \frac{q}{\epsilon_0\, A} d_2 = 0$$
$$\Rightarrow Q\, d_1 - q\, d_1 - q\, d_2 = 0$$
$$\Rightarrow q = \frac{Q\, d_1}{d_1 + d_2}$$

Charge on the inner surface of 1 is $-\left(Q - \frac{Q\, d_1}{d_1+d_2}\right) = -\left(\frac{Q\, d_2}{d_1+d_2}\right)$

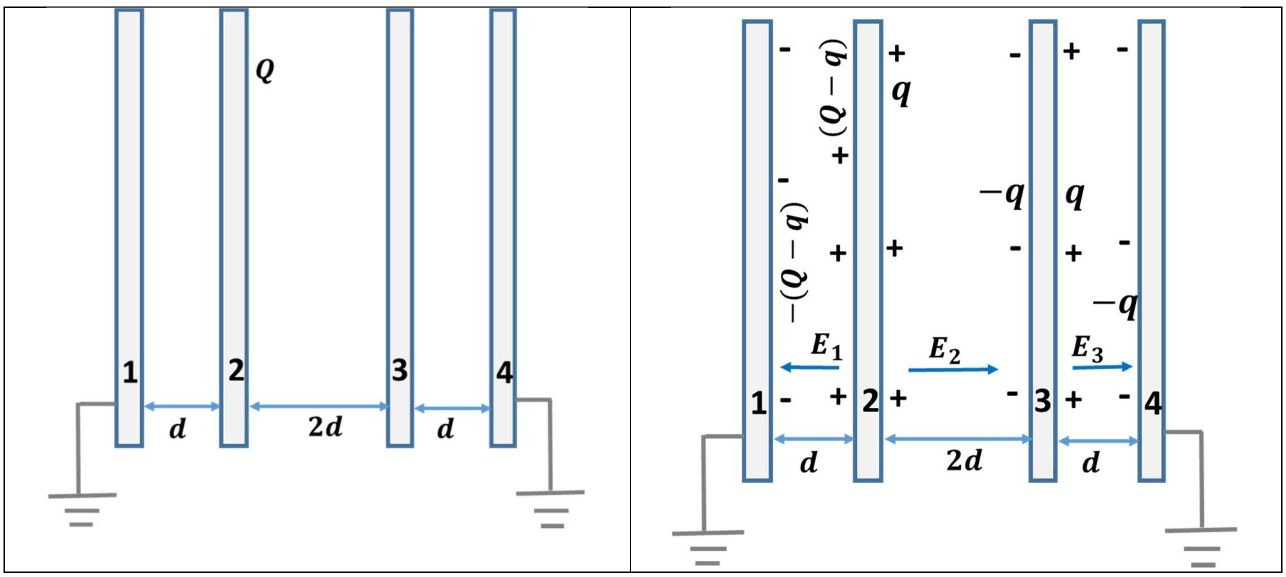
Charge on the left surface of 2 is $+\left(\frac{Q\, d_2}{d_1+d_2}\right)$

Charge on the right surface of 2 is $+(\frac{Q\, d_1}{d_1+d_2})$

Charge on the left surface of 3 is $-(\frac{Q\, d_1}{d_1+d_2})$

C. *Earth Connected System Of Four Plates*

Four large identical metallic plates are placed as shown in the Figure. Plate 2 is given a charge *Q*. All others plates are neutral. Now plates 1 and 4 are earthed. Area of each plate is *A*. we will use the above concepts to find the charge appearing on all the sides of all plates.



From the above discussions, we learned that the charge on the outer surface of earth connected plate is zero, so charge on the outer surface of plate 1 & 4 have no charge.

Let charge on right face of plate 2 be q. We know, the opposite faces must have equal and opposite charge. Using this concepts, distribution of charges on other faces are shown in the figure.

Now, the electric field intensity between plate 1 and 2 is $E_1 = \frac{Q-q}{\epsilon_0\, A}$

Electric field intensity between plate 2 and 3 is $E_2 = \frac{q}{\epsilon_0\, A}$

The electric field intensity between plates 3 and 4 is $E_3 = \frac{q}{\epsilon_0\, A}$

Now, using Kirchhoff's voltage law, we can write

$V_1 + E_1 d - E_2\, 2d - E_3\, d = V_4$

As plate 1 and 4 are earth connected, so $V_1 = V_4 = 0$

$$\Rightarrow E_1 d - E_2\, 2d - E_3 d = 0$$
$$\Rightarrow E_1 - E_2\, 2 - E_3 = 0$$
$$\Rightarrow \frac{Q-q}{\epsilon_0\, A} - 2\frac{q}{\epsilon_0\, A} - \frac{q}{\epsilon_0\, A} = 0$$
$$\Rightarrow Q - q - 2\, q - q = 0$$

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$$\Rightarrow q = Q/4$$

Charge on the inner surface of 1 is $-(Q-q) = -\left(Q-\frac{Q}{4}\right) = (-\frac{3Q}{4})$

Charge on the left surface of 2 is $(+\frac{3Q}{4})$

Charge on the right surface of 2 is $q = (+\frac{Q}{4})$

Charge on the left surface of 3 is $(-\frac{Q}{4})$

Charge on the right surface of 3 is $(+\frac{Q}{4})$

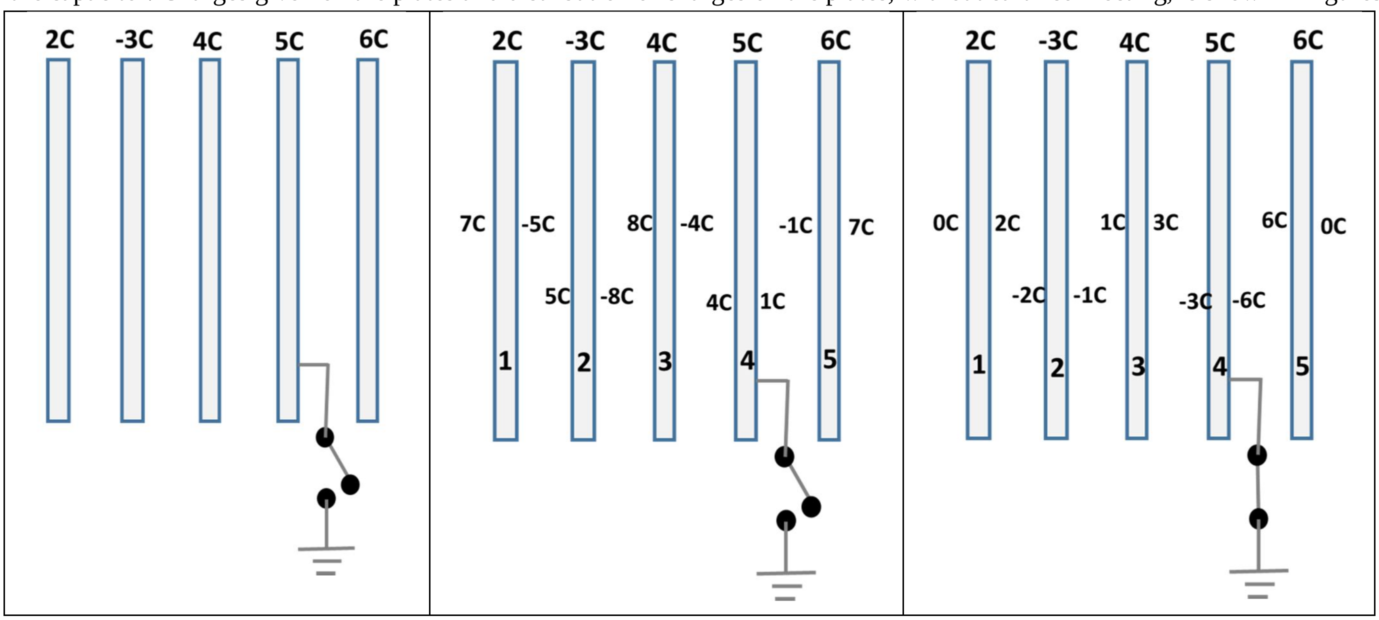
Charge on the inner surface of plate 4 is $(-\frac{Q}{4})$

From the above concepts, we have learned that when any plate/ plates of a parallel plates capacitor is earth-connected, the charge on the outer surface of the outer two plates will be zero.

In all of the above examples, we consider that the outer plate is earth-connected. Now, we will discuss the charge distribution. if any inside plate is earth connected

D. Charge Distribution, When Any Inside Plate Is Earth-Connected

Now we consider the example of five plates and for better understanding, here we use the numerical value of charges on the plates of the capacitor. Charges given on the plates and distribution of charges on the plates, without earth connecting, is shown in figures



The figure indicates the distribution of charge when no plate is earth-connected. There is a shortcut method (Gauss's law can also be used to calculate the distribution of charge) related to the distribution of charge in a combination of charged parallel plate capacitor if no plate is earth connected. Charges on the outer surfaces of the two outer plates (1^{st} and 5^{th} plate) will be the half of total charges i.e. $\frac{2-3+4+5+6}{2} = 7\ C$. Total charge on the first plate is 2 C and so charge on the inner surface of 1^{st} plate is $= (2-7) = -5\ C$. Now due to induction charge on the left surface of 2^{nd} plate is $+5\ C$.

Now we will discuss about distribution of charges on outer and inner surface of each plate.

i) Total charge on first plate is 2 C and charge on the outer surface is 7C, so charge on inner surface of first plate is (2 - 7)= -5 C.

ii) Due to induction charge on the left surface of 2^{nd} plate is +5 C. Now total charge given to 2^{nd} plate is -3 C, so charge on the right surface of 2^{nd} plate is (-3 – 5) = - 8 C

iii) Due to induction charge on the left surface of 3^{rd} plate is +8 C. Now total charge given to 3^{rd} plate is 4 C, so charge on the right surface of 3^{rd} plate is (4 – 8) = - 4 C.

iv) Due to induction charge on the left surface of 4th plate is (+4 C) . Now total charge given to 4th plate is 5 C, so charge on the right surface of 4th plate is {5 – (4)} = 1 C.

v) Due to induction charge on the left surface of 5th plate is (-1 C) . Now total charge given to 4th plate is 6 C, so charge on the right surface of 5th plate is {6 – (-1)} = 7 C.

vi) According to short cut method we already explained that charge on the outer surface of 5th plate is 7 C.

But we observed an interesting phenomenon when any one plate is earth connected not necessary the outer plate. In this case, the system has a provision to minimize its potential energy by taking or giving charge through the earth connecting plates. We know that energy density in an electrostatic field is

$$\frac{dU}{dV} = \frac{1}{2}\epsilon_0 E^2$$

So potential energy $= \frac{1}{2}\epsilon_0 E^2 \times$ volume

The outside volume in the system of parallel plates capacitors is much greater compared to the inner volume, so to minimize the potential energy, the system tries to bring the intensity to zero in the outside region of the system. This is possible only if the charges on the outer surfaces of the outer plates are zero. So we can say that charges on the outer surfaces of the outer plates will be zero if any plate of the combination is earth-connected; it is not necessary to connect the outer surface to earth.

So charges on the outer surface of plates 1 and 5 is zero. Now we will discuss about distribution of charges on outer and inner surface of each plate.

vii) Total charge on first plate is 2 C and as charge on the outer surface is zero, so charge on inner surface of first plate is +2 C.

viii) Due to induction charge on the left surface of 2nd plate is -2 C. Now total charge given to 2nd plate is -3 C, so charge on the right surface of 2nd plate is {-3 – (-2)}= -1 C

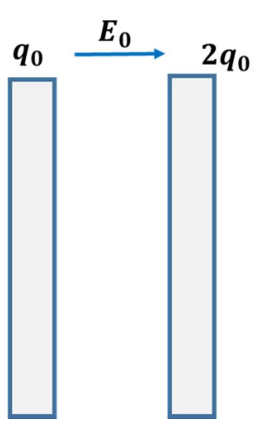
ix) Due to induction charge on the left surface of 3rd plate is +1 C. Now total charge given to 3rd plate is 4 C, so charge on the right surface of 3rd plate is 4 –(1)= 3 C.

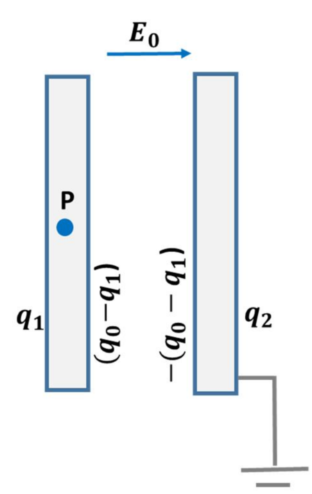
x) Total charge on fifth plate is 6 C and as charge on the outer surface is zero, so charge on inner surface of fifth plate is +6 C.

xi) Due to induction charge on the left surface of 4th plate is -3 C and on right surface is -6 C. Though the total charge on 4th plate is 5C, but here due to the induction charge on this plate, it becomes -9 C. As 4th plate is earth-connected, it has a provision to accept or donate electrons to earth. So to bring the system in equilibrium, this plate accepts (-14 C) charge from the earth.

Now we will study the case of what will happen if the external electric field is applied to an earth connecting parallel plate capacitor.

IV. CASE-II WHEN EXTERNAL ELECTRIC FIELD IS APPLIED TO AN EARTH CONNECTING PARALLEL PLATE CAPACITOR:





We consider a parallel plate capacitor with charges on the plates q_0 and $2\,q_0$ and plate 2 is earthed. An external electric field E_0 is applied as shown in fig. ( ). Charge distribution on the different surfaces of the plates is shown in the figure.
Now we consider a point 'P' inside the first plate. Since point P is inside the conductor, so intensity at P will be zero.
So we can write,

$$\frac{q_1}{2\,A\,\epsilon_0} + E_0 = \frac{q_2}{2\,A\,\epsilon_0}$$
$$q_1 = q_2 - 2\,A\,\epsilon_0\,E_0 \ldots\ldots\ldots (i)$$

Now total energy of the system,

$$U = \frac{1}{2}\,\epsilon_0\left(\frac{q_0-q_1}{A\,\epsilon_0}\right)^2 A\,d + \frac{1}{2}\,\epsilon_0\left(\frac{q_1}{A\,\epsilon_0}\right)^2 \times V + \frac{1}{2}\,\epsilon_0\left(\frac{q_2}{A\,\epsilon_0}\right)^2 \times V$$
$$\Rightarrow U = \frac{1}{2}\,\epsilon_0\left(\frac{q_0-q_1}{A\,\epsilon_0}\right)^2 A\,d + \frac{1}{2}\,\epsilon_0\left(\frac{q_1}{A\,\epsilon_0}\right)^2 \times V + \frac{1}{2}\,\epsilon_0\left(\frac{q_1+2\,A\,\epsilon_0\,E_0}{A\,\epsilon_0}\right)^2 \times V$$

Here V is the outer volume

$$\frac{dU}{dq_1} = \frac{1}{2\,A^2\,\epsilon_0^2}\,\epsilon_0 \times A\,d \times 2(q_0-q_1)\,(-1) + \frac{1}{2\,A^2\,\epsilon_0^2}\,\epsilon_0 \times V \times 2\,q_1 + \frac{1}{2\,A^2\,\epsilon_0^2}\,\epsilon_0\,2(q_1+2\,A\,\epsilon_0\,E_0) \times V$$
$$\Rightarrow \frac{dU}{dq_1} = -\frac{d}{A\,\epsilon_0}\,(q_0-q_1) + \frac{V}{A^2\,\epsilon_0}q_1 + \frac{1}{A^2\,\epsilon_0}\,(q_1+2\,A\,\epsilon_0\,E_0) \times V$$

Condition for U minimum is $\frac{dU}{dq_1} = 0$

$$-\frac{d}{A\,\epsilon_0}\,(q_0-q_1) + \frac{V}{A^2\,\epsilon_0}q_1 + \frac{1}{A^2\,\epsilon_0}\,(q_1+2\,A\,\epsilon_0\,E_0) \times V = 0$$
$$\Rightarrow\quad -A\,dq_0 + A\,d\,q_1 + V\,q_1 + V\,q_1 + 2\,A\,V\,\epsilon_0\,E_0 = 0$$
$$\Rightarrow\; q_1 = \frac{A\,d\,q_0 - 2\,A\,V\,\epsilon_0\,E_0}{A\,d + 2V} = \frac{\frac{A\,d\,q_0}{V} - 2\,A\,\epsilon_0\,E_0}{\frac{A\,d}{V}+2}$$

Here, the volume inside the capacitor $A\,d$ is very small but volume outside the capacitor V is very large. So the term $\frac{A\,d\,q_0}{V} \to 0$ and $\frac{A\,d}{V} \to 0$

So the value of $q_1 = -A\,\epsilon_0\,E_0$
The value of $q_2 = 2\,A\,\epsilon_0\,E_0 + q_1$
$$= 2\,A\,\epsilon_0\,E_0 - A\,\epsilon_0\,E_0 = A\,\epsilon_0\,E_0$$

So we see that the charge induced in the two outer surfaces is equal and opposite. Since the electric field is in the positive X-direction, the charge induced in the outer left surface of the first plate is negative, and the outer surface of the second plate is positive.

V. CONCLUSION

This paper will help to understand about the distribution of charge in an earth-connected parallel plates capacitor. With example, it has been shown that the energy of an electric field plays a pivotal/significant role for the distribution of charges in different sides of an earth-connected parallel plate capacitor or a system of parallel plates.

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