iJRASET
International Journal For Research in Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 13 Issue: II Month of publication: February 2025

DOI: https://doi.org/10.22214/ijraset.2025.67043

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 13 Issue II Feb 2025- Available at www.ijraset.com

Second Type Second Order Slope Rotatable Designs Utilizing Balanced Incomplete Block Designs with Unequal Block Sizes

B. Venkata Ravikumar [1], B. RE. Victorbabu [2]
[1]Research Scholar, Department of Statistics, Acharya Nagarjuna University, Guntur-522510, Andhra Pradesh, India
[1]Assistant Professor, Department of Mathematics, Siddhartha Academy of Higher Education (SAHE) Deemed to be University, Kanuru, Vijayawada-520007, Andhra Pradesh, India
[2]Professor of Statistics (Retired), Department of Statistics, Acharya Nagarjuna University, Guntur-522510, Andhra Pradesh, India

Abstract: Kim and Ko proposed second type second order slope rotatable designs (SOSRD) utilizing central composite designs (CCD) wherein the two digits (a_1, a_2) denotes the position of star points. In this study, we propose SOSRD of second type utilizing balanced incomplete block designs (BIBD) with unequal block sizes. In specific cases, the recommended procedure results in fewer design points than SOSRD of second type acquired through pairwise balanced designs (PBD), symmetrical unequal block arrangements (SUBA) with two unequal block sizes and balanced incomplete block designs (BIBD).
Keywords: Balanced Incomplete block designs, Pairwise balanced designs, Symmetrical unequal block arrangements with two unequal block sizes, Second order slope rotatable designs of second type.

I. INTRODUCTION

A response surfaces are set of statistical and mathematical models used to analyze the issues when several explanatory variables have an impact on a response variable. Box and Hunter [1] suggested the rotatability property for exploring response surface models and developed rotatable central composite designs (CCD). Das and Narasimham [8] studied second order rotatable designs (SORD) utilizing balanced incomplete block designs (BIBD). Das [6] constructed SORD through BIBD with unequal block sizes. Raghavarao [14, 15] developed symmetrical unequal block arrangements (SUBA) with two unequal block sizes and SORD through incomplete block designs (IBD). Tyagi [21] developed SORD utilizing pairwise balanced designs (PBD). Kim [12] developed second type of rotatable CCD where in the two digits (a_1, a_2) are used to denote the star points for $2 \le v \le 8$ (v:factors). Victorbabu and Vasundharadevi [39] developed modified quadratic response surface models utilizing BIBD. Victorbabu and Surekha [37, 38] developed rotatability measure for quadratic response surface models utilizing IBD and BIBD respectively. Jyostna and Victorbabu [10, 11] constructed modified rotatability measure for a quadratic polynomial models utilizing BIBD and CCD. Chiranjeevi et al. [5] developed second type of SORD utilizing CCD for $9 \le v \le 17$. Chiranjeevi and Victorbabu [2, 3, 4] constructed second type of SORD utilizing BIBD, PBD and SUBA with two unequal block sizes respectively.

Slope rotatable central composite designs (SRCCD) were first developed by Hader and Park [9]. Victorbabu and Narasimham [28, 29, 30, 31] studied second order slope rotatable designs (SOSRD) utilizing IBD with different sizes of blocks, BIBD, pair of IBD and PBD respectively. Victorbabu [22] constructed SOSRD utilizing SUBA with two unequal block sizes. Specifically, Kim and Ko [13] introduced the second type of slope rotatability of CCD for the factors $2 \le v \le 5$ by taking $n_a = 1$ (‘n_a’ denotes the number of replications of axial points), where in the two numbers (a_1, a_2) represent the positions of the star points. Victorbabu [23, 24, 25] introduced modified SOSRD utilizing CCD, PBD and BIBD respectively. A review was proposed by Victorbabu [26] on SOSRD. Victorbabu and Surekha [33, 34, 35, 36] constructed SOSRD measure utilizing CCD, BIBD, PBD and SUBA with two unequal block sizes respectively. Victorbabu and Jyostna [27] constructed modified slope rotatability measure for quadratic polynomial models. Ravikumar and Victorbabu [17] extended the work of Kim and Ko [13] and developed second type of SOSRD utilizing CCD for $6 \le v \le 17$ by taking $n_a = 1$. Ravikumar and Victorbabu [18, 19, 20] studied SRCCD of second type for $2 \le v \le 17$ with $2 \le n_a \le 4$, SOSRD of second type utilizing PBD and SUBA with two unequal block sizes respectively. Victorbabu and Ravikumar [32] developed SOSRD of second type using BIBD.

In this study, we suggest SOSRD of second type utilizing BIBD with unequal block sizes. The suggested procedure is found to sometimes result in an SOSRD of second type with fewer design points than the SOSRD of second type obtained through PBD of Ravikumar and Victorbabu [19], SUBA with two unequal block sizes of Ravikumar and Victorbabu [20] and BIBD of Victorbabu and Ravikumar [32] respectively.

II. STIPULATIONS FOR SECOND ORDER SLOPE ROTATABLE DESIGNS

A general quadratic polynomial model $D=((X_{sw}))$ for fitting

$$Y_w=\beta_0+\sum_{s=1}^{v}\beta_s X_{sw}+\sum_{s=1}^{v}\beta_{ss}X_{sw}^2+\sum_{s<t}\sum\beta_{st}X_{sw}X_{tw}+\varepsilon_w \tag{1}$$

In which X_{sw} indicates the s[th] factor level in the experiment's w[th] run (w=1,2,…,N) and $\boldsymbol{\varepsilon}_w$'s are uncorrelated random errors having a mean '0' and variation of σ^2. D is then referred to as SOSRD if $V\left(\frac{\partial\hat{Y}}{\partial X_s}\right)$ with regard to every explanatory variable X_s is a $f\left(d^2\right)$ of the point $\left(X_{1w},X_{2w},\ldots,X_{vw}\right)$ from the origin (center) of the design.

The general circumstances for SOSRD are given below (cf. [1, 9, 29]).

All moments of odd order are '0'. In simple terms when minimum of one odd power X equals zero. i.e;

A. $\sum X_{sw}=0,\ \sum X_{sw}X_{tw}=0,\ \sum X_{sw}X_{tw}^2=0,\ \sum X_{sw}^3=0,\ \sum X_{sw}X_{tw}X_{uw}=0,$
$\sum X_{sw}X_{tw}X_{uw}^2=0,\ \sum X_{sw}X_{tw}^3=0,\ \sum X_{sw}X_{tw}X_{uw}X_{lw}=0$, etc. for $s\neq t\neq u\neq l$;

B. (i) $\sum X_{sw}^2=\text{constant}=N\delta_2$

(ii) $\sum X_{sw}^4=\text{constant}=cN\delta_4,\ \forall\ s$

C. $\sum X_{sw}^2X_{tw}^2=\text{constant}=N\delta_4,\ \forall\ s\neq t$ (2)

where c, δ_2 and δ_4 are constants.

The variances and covariances of the estimated parameters are (cf. [7])

$$Var\left(\hat{\beta}_0\right)=\frac{\delta_4(c+v-1)\sigma^2}{N[\delta_4(c+v-1)-v\delta_2^2]}$$

$$Var\left(\hat{\beta}_s\right)=\frac{\sigma^2}{N\delta_2}$$

$$Var\left(\hat{\beta}_{st}\right)=\frac{\sigma^2}{N\delta_4}$$

$$Var\left(\hat{\beta}_{ss}\right)=\frac{\sigma^2}{(c-1)N\delta_4}\left[\frac{(c+v-2)\delta_4-\delta_2^2(v-1)}{(c+v-1)\delta_4-\delta_2^2 v}\right]$$

$$Cov\left(\hat{\beta}_0,\hat{\beta}_{ss}\right)=\frac{\sigma^2}{N}\left[\frac{-\delta_2}{[(c+v-1)\delta_4-\delta_2^2 v]}\right]$$

$$Cov\left(\hat{\beta}_{ss},\hat{\beta}_{tt}\right)=\frac{\sigma^2}{(c-1)N\delta_4}\left[\frac{\left(\delta_2^2-\delta_4\right)}{[(c+v-1)\delta_4-\delta_2^2 v]}\right] \tag{3}$$

and the remaining covariances disappear.

An inspection of $V\left(\hat{\beta}_0\right)$ shows an essential condition for the existence of a non singular second order design is

D. $\frac{\delta_4}{\delta_2^2}>\frac{v}{c+v-1}$ (4)

From (1), we have

$$\frac{\partial \hat{Y}}{\partial X_s}=\hat{\beta}_s+2\hat{\beta}_{ss}X_{sw}+\sum_{s\neq t}\hat{\beta}_{st}X_{tw} \quad (5)$$

$$\mathrm{Var}\left(\frac{\partial \hat{Y}}{\partial X_s}\right)=\mathrm{Var}\left(\hat{\beta}_s\right)+4X_{sw}^2\,\mathrm{Var}\left(\hat{\beta}_{ss}\right)+\sum_{t\neq s}X_{tw}^2\,\mathrm{Var}\left(\hat{\beta}_{st}\right) \quad (6)$$

The criteria for R.H.S of (6) is to be $f\left(d^2\right)$ alone (for slope rotatability) is

$$4\mathrm{Var}\left(\hat{\beta}_{ss}\right)=\mathrm{Var}\left(\hat{\beta}_{st}\right) \qquad \text{(cf. [9])} \quad (7)$$

Simplifying (7) using (3), we get

E. $\left[v(5-c)-(c-3)^2\right]\delta_4+\left[v\left(c-5\right)+4\right]\delta_2^2=0$ (cf. [29]) (8)

Therefore A, B, C of (2), (4) and (8) suggest a set of conditions for slope rotatability in any general quadratic model. (cf. [9, 29]).

III. NEW SECOND TYPE OF SOSRD UTILIZING BALANCED INCOMPLETE BLOCK DESIGNS WITH UNEQUAL BLOCK SIZES

Victorbabu and Narasimham [28] developed SOSRD through IBD with unequal block sizes. Kim [12] developed second type of rotatable CCD, in which the two digits $\left(a_1, a_2\right)$ are used to denote the star points for 2≤v≤8. Chiranjeevi et al. [5] developed second type of SORD utilizing CCD for 9≤v≤17. Chiranjeevi and Victorbabu [2, 3, 4] developed second type of SORD utilizing BIBD, PBD and SUBA with two unequal block sizes respectively. Specifically, Kim and Ko [13] constructed second type of slope rotatability of CCD 2≤v≤5 by taking $n_a=1$. Ravikumar and Victorbabu [17] extended the results of Kim and Ko [13] and developed SOSRD of second type utilizing CCD for $6\leq v\leq 17$ by taking $n_a=1$. Ravikumar and Victorbabu [18, 19, 20] studied SRCCD of second type for 2≤v≤17 with $2\leq n_a\leq 4$ and SOSRD of second type utilizing PBD, SUBA with two unequal block sizes respectively. Victorbabu and Ravikumar [32] developed SOSRD of second type using BIBD.

A BIBD design with unequal block sizes $\left(v, m, r, k_1, k_2, ..., k_n, \lambda\right)$ is defined as the v-treatments are arranged in m blocks of $k_1, k_2, ..., k_n$ sizes so that each treatment takes place exactly in r blocks and any pair of treatments takes place exactly in λ blocks. If m_i denotes the k_i sized blocks, then we have in these designs,

$$vr=\sum_{i=1}^{n}m_ik_i\ ; \qquad \lambda v\left(v-1\right)=\sum_{i=1}^{n}m_ik_i\left(k_i-1\right)$$

For the purpose of constructing SOSRD of second type we take a particular class of these designs wherein the replication of every treatment is a constant r_i in the set of m_i blocks each of size k_i for all i. For this class of designs we must further have $vr_i=m_ik_i$ and $\sum r_i=r$.

By taking a BIBD $\left(v, m, r, k, \lambda\right)$, a specific treatment is discarded, we get another BIBD with unequal block sizes $\left(v-1, m, r, k, k-1, \lambda\right)$ so that every treatment will be replicated (r-λ) times in (m-r) blocks of k size, and λ times in the remaining r blocks of (k-1) size.

Through BIBD with unequal block sizes, SOSRD of second type can be obtained as follows. Let us write the BIBD with unequal block sizes $\left(v-1, m, r, k_1, k_2, ..., k_n, \lambda\right)$ as a mxv matrix with elements zero (when a treatment does not appear in a block) and unity (when the treatment appears) in the set of m_1 blocks of size k_1, zero and α_1 (when the treatment appears) in the set of m_2 blocks of size k_2, zero and α_2 (when the treatment appears) in the set of m_3 blocks of size k_3 and so on.

The design plan of SOSRD of second type utilizing BIBD with unequal block sizes is shown in theorem (1). Let $\left(v, m_1, m_2, r_1, r_2, k_1, k_2, \lambda\right)$ indicate the parameters of BIBD with unequal block sizes, $2^{t(k_i)}$ indicate the fractional replicate of 2^{k_i} in ±1 levels, wherein no interaction is confounded with fewer than five factors. n_0 represents the central points.

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 7.538
Volume 13 Issue II Feb 2025- Available at www.ijraset.com

Let $\left[1\text{-}\left(v,m_1,r_1,k_1,\lambda\right)\right]$ and $\left[\alpha\text{-}\left(v,m_2,r_2,k_2,\lambda\right)\right]$ are indicates the design points produced from transposed incidence matrix of BIBD with unequal block sizes. Let $\left[1-\left(v,m_1,r_1,k_1,\lambda\right)\right]2^{t(k_1)}$ are the $m_1 2^{t(k_1)}$ and $\left[\alpha\text{-}\left(v,m_2,r_2,k_2,\lambda\right)\right]2^{t(k_2)}$ are the $m_2 2^{t(k_2)}$ design points produced from BIBD with unequal sizes of blocks by multiplication (cf. (8)). We employ the extra set of points like $(\pm a_1,0,\dots,0),(0,\pm a_1,0,\dots,0),\dots,(0,0,\dots,\pm a_1)$; $(\pm a_2,0,\dots,0),(0,\pm a_2,0,\dots,0),\dots,(0,0,\dots,\pm a_2)$ are two axial point sets. Here $(a_1,0,\dots,0)2^1 U(a_2,0,\dots,0)2^1$ indicate the 4v axial points produced from $(a_1,0,\dots,0)$ and $(a_2,0,\dots,0)$ point sets. Let U indicate the union of design points produced from various point sets and central points represented by n_0. Following the methods of [19, 20, 32] we suggest a method of construction on second type of SOSRD utilizing BIBD with two unequal block sizes as shown in below theorem.

1) Theorem (1)

If $\left(v,m,r,k_1,k_2,m_1,m_2,\lambda\right)$ is a BIBD with two unequal block sizes then the design points,

$[1\text{-}(v,m_1,r_1,k_1,\lambda)]^{*}\times 2^{t(k_1)}U\,[\alpha\text{-}(v,m_2,r_2,k_2,\lambda)]^{*}\times 2^{t(k_2)}U(a_1,0,\dots,0)2^1U(a_2,0,\dots,0)2^1U(n_0)$

will result in a v-dimensional SOSRD of second type utilizing BIBD with two unequal block sizes in $N=\left(m_1 2^{t(k_1)}\right)+\left(m_2 2^{t(k_2)}\right)+\left(4v\right)+n_0$ design points, with the following biquadratic equation

$$\begin{aligned}
&[8v-4N](a_1^8+a_2^8)+[16v-8N]a_1^4a_2^4+16v(a_1^6a_2^2+a_1^2a_2^6)+v\left(r_1 2^{t(k_1)+3}+r_2 2^{t(k_2)+3}\alpha^2\right)\left(a_1^6+a_1^2a_2^4+a_1^4a_2^2+a_2^6\right)\\
&+\left[\begin{array}{l}\left(vr_1-5v\lambda+4\lambda\right)2^{t(k_1)+2}+vr_2 2^{t(k_2)+2}\alpha^4+vr_1^2 2^{t(k_1)+1}+vr_2^2 2^{2t(k_2)+1}\alpha^4+vr_1r_2 2^{t(k_1)+t(k_2)+2}\alpha^2\\ N\left(v\lambda 2^{t(k_1)+1}+r_2 2^{t(k_2)+2}\alpha^4+r_1 2^{t(k_1)+2}-3\lambda 2^{t(k_1)+2}\right)\end{array}\right]\left(a_1^4+a_2^4\right)\\
&+\left[\left(vr_1-5v\lambda+4\lambda\right)2^{t(k_1)+3}+vr_2 2^{t(k_2)+3}\alpha^4\right]a_1^2a_2^2\\
&+\left[\left(vr_1-5v\lambda+4\lambda\right)\left(r_2 2^{t(k_1)+t(k_2)+2}\alpha^2+r_1 2^{t(k_1)+2}\right)+vr_2^2 2^{t(k_2)+2}\alpha^6+vr_1r_2 2^{t(k_1)+t(k_2)+2}\alpha^4\right]\left(a_1^2+a_2^2\right)\\
&+vr_2^3 2^{3t(k_2)}\alpha^8+vr_1r_2 2^{t(k_1)+t(k_2)}\alpha^4\left[r_1 2^{t(k_1)}+r_2 2^{t(k_2)+1}\alpha^2\right]+\left(vr_1-5v\lambda+4\lambda\right)\left[r_1^2 2^{3t(k_1)}+r_2^2 2^{t(k_1)+2t(k_2)}\alpha^4+r_1r_2 2^{2t(k_1)+t(k_2)+1}\alpha^2\right]\\
&+N\left[2^{2t(k_1)}\left\{v\lambda\left(5\lambda-r_1\right)-\left(r_1-3\lambda\right)^2\right\}+2^{t(k_1)+t(k_2)}\alpha^4\left\{r_2\left(6\lambda-2r_1-v\lambda\right)\right\}-r_2^2 2^{2t(k_2)}\alpha^8\right]=0 \qquad (9)
\end{aligned}$$

And $\alpha^4=2^{t(k_1)-t(k_2)}$ (10)

The design exists if the equation mentioned above (9) contains at least one positive real root.

Proof: Regarding the design points produced from second type of SOSRD utilizing BIBD with unequal block sizes, simple symmetry stipulations A, B and C of equation (2) are true. Since condition A of equation (2) is obviously true, condition B and C of (2) are also true as follows.

B. (i) $\sum X_{sw}^2=r_1\,2^{t(k_1)}+r_2 2^{t(k_2)}\alpha^2+2\,a_1^2+2\,a_2^2=N\,\delta_2$

(ii) $\sum X_{sw}^4=r_1 2^{t(k_1)}+r_2 2^{t(k_2)}\alpha^4+2a_1^4+2a_2^4=c\,N\,\delta_4$

C. $\sum X_{sw}^2X_{tw}^2=\lambda 2^{t(k_1)}=\lambda 2^{t(k_2)}\alpha^4=N\,\delta_4$ (11)

From (11) we have $\alpha^4=2^{t(k_1)-t(k_2)}$

From B(ii) and C of (11), we have $c=\dfrac{r_1 2^{t(k_1)}+r_2 2^{t(k_2)}\alpha^4+2\left(a_1^4+a_2^4\right)}{\lambda 2^{t(k_1)}}$.

The result of simplifying equation [8] by substituting c, δ_2 and δ_4 is

* $[1\text{-}(v,m_1,r_1,k_1,\lambda)]^{*}$ and $[\alpha\text{-}(v,m_2,r_2,k_2,\lambda)]^{*}$ are the design points produced from the blocks of sizes k_1 and k_2 respectively from the given BIBD with unequal block sizes.

$$\frac{\lambda 2^{t(k_1)}}{N}\left[v\left(5-\frac{r_1 2^{t(k_1)}+r_2 2^{t(k_2)}\alpha^4+2\left(a_1^4+a_2^4\right)}{\lambda 2^{t(k_1)}}\right)-\left(\frac{r_1 2^{t(k_1)}+r_2 2^{t(k_2)}\alpha^4+2\left(a_1^4+a_2^4\right)}{\lambda 2^{t(k_1)}}-3\right)^2\right]+\left(\frac{r_1 2^{t(k_1)}+r_2 2^{t(k_2)}\alpha^2+2\left(a_1^2+a_2^2\right)}{N}\right)^2$$

$$\left[v\left(\frac{r_1 2^{t(k_1)}+r_2 2^{t(k_2)}\alpha^4+2\left(a_1^4+a_2^4\right)}{\lambda 2^{t(k_1)}}-5\right)+4\right]=0 \qquad (12)$$

The biquadratic equation shown in (9) is obtained by simplifying (12).

2) Example (1)

Construction on SOSRD of second type for 6-factors utilizing BIBD with two unequal block sizes.

BIBD with two unequal block sizes for v=6 obtained by deleting a treatment from the BIBD$\left(v=7, m=7, r=3, k=3, \lambda=1\right)$ then the design points,

$$[1-(v=6,m_1=4,r_1=2,k_1=3,\lambda=1)]2^{t(3)}U[\alpha-(v=6,m_2=3,r_2=1,k_2=2,\lambda=1)]2^{t(2)}U$$
$$(a_1,0,...,0)2^1U(a_2,0,...,0)2^1U(n_0=1)$$

will result in a second type of SOSRD utilizing BIBD with two unequal block sizes in N=69 design points with $n_0=1$ and $a_1=1$.

For the design points produced from second type SOSRD utilizing BIBD with two unequal block sizes, simple symmetry stipulations A of equation (2) are true.

Here B and C of equation (11) are

B. (i) $\sum X_{sw}^2=16+4\alpha^2+2a_1^2+2a_2^2=N\delta_2$

(ii) $\sum X_{sw}^4=16+4\alpha^4+2a_1^4+2a_2^4=cN\delta_4$

C. $\sum X_{sw}^2 X_{tw}^2=8=4\alpha^4=N\delta_4$ (13)

From C of equation (13), we have $\alpha^4=2$

From B (ii) and C of equation (13), we have $c=\frac{24+2a_1^4+2a_2^4}{8}$; $\left[\because \alpha^4=2\right]$

Substitute c, δ_2 and δ_4 in equation (8) and after simplifying, we obtain the biquadratic equation that follows.

$$228(a_1^8+a_2^8)-96(a_1^6a_2^2+a_1^2a_2^6)+456a_1^4a_2^4-1039.5(a_1^6+a_1^4a_2^2+a_1^2a_2^4+a_2^6)+1251.8(a_1^4+a_2^4)+5544.2(a_1^2+a_2^2)$$
$$+512a_1^2a_2^2-22974.8=0$$

Substitute $a_1=1$ in the above equation and on simplification, we get

$$228a_2^8-1135.5a_2^6+668.3a_2^4+4920.7a_2^2-16990.3=0 \qquad (14)$$

Only a single positive real root exists in equation (14) $a_2^2=4.0627 \Rightarrow a_2=2.0156$. The non-singularity criterion D of (4} is fulfilled.

In this case, it might be noted that SOSRD of second type utilizing BIBD with two unequal block sizes contains 69 design points for 6 factors, while the corresponding SOSRD of second type obtained utilizing PBD of Ravikumar and Victorbabu [19], SUBA with two unequal block sizes of Ravikumar and Victorbabu [20], BIBD of Victorbabu and Ravikumar [32] for 6 factors need 81, 81 and 85 design points respectively.

3) Example (2)

Construction on SOSRD of second type for 8-factors using BIBD with two unequal block sizes.

BIBD with two unequal block sizes for v=8 obtained by deleting a treatment from the BIBD$\left(v=9, m=12, r=4, k=3, \lambda=1\right)$ then the design points,

$$[1-(v=8,m_1=8,r_1=3,k_1=3,\lambda=1)]2^{t(3)}U[\alpha-(v=8,m_2=4,r_2=1,k_2=2,\lambda=1)]2^{t(2)}U(a_1,0,...,0)2^1$$
$$U(a_2,0,...,0)2^1U(n_0=1)$$

will result in a second type of SOSRD utilizing BIBD with two unequal block sizes in N=113 design points with $n_0=1$ and $a_1=1$. For the design points produced from second type of SOSRD utilizing BIBD with two unequal block sizes, simple symmetry stipulations A of equation (2) are true.
Here B and C of equation (11) are

B. (i) $\sum X_{sw}^2 = 24+4\alpha^2+2a_1^2+2a_2^2 = N\delta_2$

(ii) $\sum X_{sw}^4 = 24+4\alpha^4+2a_1^4+2a_2^4 = cN\delta_4$

C. $\sum X_{sw}^2 X_{tw}^2 = 8 = 4\alpha^4 = N\delta_4$ (15)

From C of equation (15), we have $\alpha^4 = 2$

From B (ii) and C of equation (15), we have $c=\dfrac{32+2a_1^4+2a_2^4}{8}$; $\left[\because \alpha^4 = 2\right]$

Substitute c, δ_2 and δ_4 in equation [8] and after simplifying, we obtain the biquadratic equation that follows.

$$388(a_1^8+a_2^8)-128(a_1^6a_2^2+a_1^2a_2^6)+776a_1^4a_2^4-1898.04(a_1^6+a_1^4a_2^2+a_1^2a_2^4+a_2^6)+4135.54(a_1^4+a_2^4)+3796.08(a_1^2+a_2^2)$$
$$+256a_1^2a_2^2-22479.06=0$$

Substitute $a_1=1$ in the above equation and on simplification, we get

$$388a_2^8-2026.04a_2^6+3013.5a_2^4+2026.04a_2^2-16057.5=0 \quad (16)$$

Only a single positive real root exists in equation [16] $a_2^2=3.5442 \Rightarrow a_2=1.8826$. The non-singularity criteria D of (4) is fulfilled.

In the context of 8-factors, this new approach contains 113 design points, while the corresponding SOSRD of second type obtained utilizing PBD of Ravikumar and Victorbabu [19], SUBA with two unequal block sizes of Ravikumar and Victorbabu [20], BIBD of Victorbabu and Ravikumar [32] for 8 factors need 273, 129 and 145 design points respectively.

The Appendix table gives the appropriate SOSRD of second type values of a_2 for designs utilizing BIBD with unequal block sizes for $3 \le v \le 16$.

IV. CONCLUSION

In this paper, second type of SOSRD utilizing BIBD with unequal block sizes is suggested. It is observed that the proposed procedure can generate designs with fewer design points than SOSRD of second type acquired utilizing PBD, SUBA with two unequal block sizes and BIBD.

V. ACKNOWLEDGEMENT

The authors are thankful to the editorial board.

VI. APPENDIX

Table: Values of a_2 taking $a_1=1$ for second type of SOSRD utilizing BIBD with unequal block sizes for $3 \le v \le 16$. [These are SOSRDs of second type with design points

$[1-(v, m_1, r_1, k_1, \lambda)]^* \times 2^{t(k_1)} U[\alpha-(v, m_2, r_2, k_2, \lambda)]^* \times 2^{t(k_2)} U(a_1,0,...,0)2^1 U(a_2,0,...,0)2^1 U(n_0)]$

BIBD design (v,m,r,k,λ)	BIBD with unequal block sizes $(v,m_1,r_1,k_1,\lambda),(v,m_2,r_2,k_2,\lambda)$	n_0	N	a_2
(4,6,3,2,1)	(3,3,2,2,1) (3,3,1,1,1)	1	31	1.8450
(5,10,4,2,1)	(4,6,3,2,1) (4,4,1,1,1)	1	49	1.7757
(6,15,5,2,1)	(5,10,4,2,1)(5,5,1,1,1)	1	71	1.7129
(7,7,3,3,1)	(6,4,2,3,1) (6,3,1,2,1)	1	69	2.0156
(8,28,7,2,1)	(7,21,6,2,1) (7,7,1,1,1)	1	127	1.5938

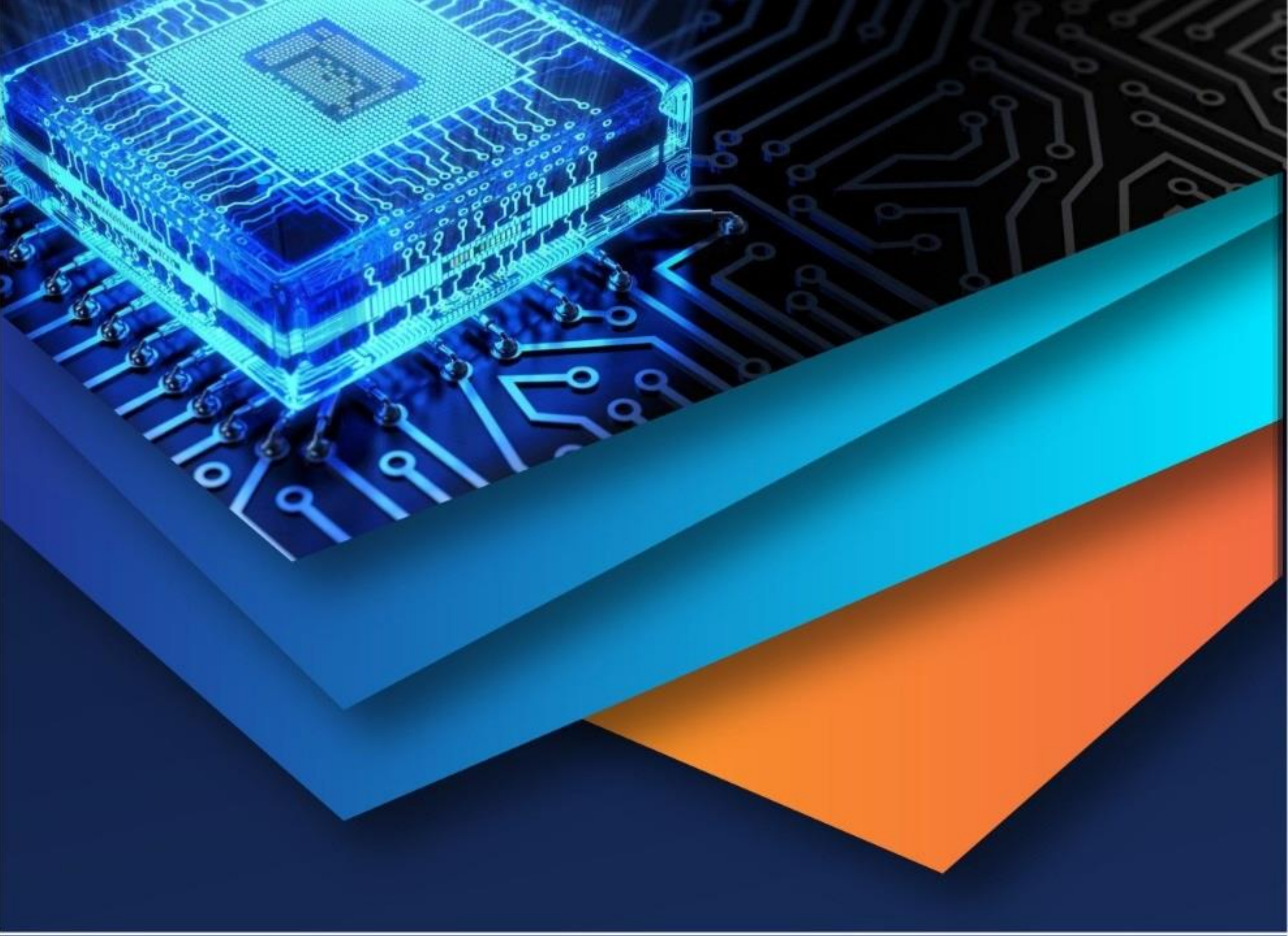
(9,12,4,3,1)	(8,8,3,3,1) (8,4,1,2,1)	1	113	1.8826
(10,45,9,2,1)	(9,36,8,2,1) (9,9,1,1,1)	1	199	1.4412
(11,11,5,5,2)	(10,6,3,5,2) (10,5,2,4,2)	1	217	2.7115
(12,44,11,3,2)	(11,33,9,3,2) (11,11,2,2,2)	1	353	2.0764
(13,13,4,4,1)	(12,9,3,4,1) (12,4,1,3,1)	1	225	2.0240
(15,15,7,7,3)	(14,8,4,7,3) (14,7,3,6,3)	1	793	4.1759
(16,20,5,4,1)	(15,15,4,4,1) (15,5,1,3,1)	1	341	1.3220

* $[1\text{- }(v, m_1, r_1, k_1, \lambda)]^*$ and $[\alpha\text{- }(v, m_2, r_2, k_2, \lambda)]^*$ are the design points produced from the blocks of sizes k_1 and k_2 respectively from the given BIBD with unequal block sizes.

REFERENCES

[1] Box GEP, Hunter JS. Multifactor experimental designs for exploring response surfaces. Annals of Mathematical Statistics. 1957; 28:195-241.
[2] Chiranjeevi P, Victorbabu BRe. Second order rotatable designs of second type using balanced incomplete block designs. Journal of Mathematical and Computational Science. 2021a; 11:2341-2361.
[3] Chiranjeevi P, Victorbabu BRe. Second order rotatable designs of second type using pairwise balanced designs. Journal of Mathematical and Computational Science. 2021b ; 17:20-33.
[4] Chiranjeevi P, Victorbabu BRe. Second order rotatable designs of second type using symmetrical unequal block arrangements with two unequal block sizes. International Journal for Research in Applied Science & Engineering Technology. 2021c;9: 515-525.
[5] Chiranjeevi P, John Benhur K, Victorbabu BRe. Second order rotatable designs of second type using central composite designs. Asian Journal of Probability and Statistics. 2021;11:30-41.
[6] Das MN. Construction of second order rotatable designs through balanced incomplete block designs with blocks of unequal sizes. Calcutta statistical Association Bulletin. 1963; 12:31-46.
[7] Das MN, Giri NC. Design and Analysis of Experiments. New Age International (P) Limited. New Delhi, India, (1999).
[8] Das MN, Narasimham VL. Construction of rotatable designs through balanced incomplete block designs. Annals of Mathematical Statistics. 1962; 33:1421-1439.
[9] Hader RJ, Park SH. Slope rotatable central composite designs. Technometrics. 1978; 20:413-417.
[10] Jyostna P, Victorbabu BRe. Evaluating measure of modified rotatability for second degree polynomial using balanced incomplete block designs. Asian Journal of Probability and Statistics. 2021; 10:47-59.
[11] Jyostna P, Sulochana B, Victorbabu BRe. Measure of modified rotatability for second order response surface designs using central composite designs. Journal Mathematical and Computational Science. 2021; 11:494-519.
[12] Kim Hyuk Joo. Extended central composite designs with the axial points indicated by two numbers. The Korean Communications in Statistics. 2002; 9:595-605.
[13] Kim Hyuk Joo, Ko Yun Mi. On slope rotatability of central composite designs of second type. The Korean Communications in Statistics. 2004; 11:121-137.
[14] Raghavarao D. Symmetrical unequal block arrangements with two unequal block sizes. Annals of Mathematical Statistics. 1962; 33:620-633.
[15] Raghavarao D. Construction of second order rotatable designs through incomplete block designs. Journal of Indian Statistical Association. 1963; 1:221-225.
[16] Raghavarao D. Constructions and combinatorial problems in design of experiments. 1971; John Wilkey, New York.
[17] Ravikumar BV, Victorbabu BRe. Second order slope rotatable designs of second type using central composite designs. Journal of the Oriental Institute. 2022a; 71:12-25.
[18] Ravikumar BV, Victorbabu BRe. Slope rotatable central composite designs of second type. Asian Journal of Probability and Statistics. 2022b; 19:1-12.
[19] Ravikumar BV, Victorbabu BRe. Second order slope rotatable designs of second type using pairwise balanced designs. HTL Journal. 2022c; 28:409-422.
[20] Ravikumar BV, Victorbabu BRe. Second order slope rotatable designs of second type using symmetrical unequal block arrangements with two unequal block sizes. International journal of scientific research updates. 2024; 8(1): 54-64.
[21] Tyagi BN. On the construction of second order and third order rotatable designs through pairwise balanced designs and doubly balanced designs. Calcutta Statistical Association Bulletin. 1964; 13:150-162.
[22] Victorbabu BRe. Construction of second order slope rotatable designs using symmetrical unequal block arrangements with two unequal block sizes. Journal of the Korean Statistical Society. 2002; 31:153-161.
[23] Victorbabu BRe. Modified slope rotatable central composite designs. Journal of the Korean Statistical Society. 2005a; 34:153-160.
[24] Victorbabu BRe. Modified second order slope rotatable designs using pairwise balanced designs. Proceedings of Andhra Pradesh Akademi of Sciences. 2005b; 9:19-23.
[25] Victorbabu BRe. Modified second order slope rotatable designs using BIBD. Journal of the Korean Statistical Society. 2006; 35:179-192.
[26] Victorbabu BRe. On second order slope rotatable designs - A Review. Journal of the Korean Statistical Society. 2007; 33:373-386.
[27] Victorbabu BRe, Jyostna P. Measure of modified slope rotatability for second order response surface designs. Thailand Statistician. 2021; 19:195-207.
[28] Victorbabu BRe, Narasimham VL. Construction of second order slope rotatable designs through incomplete block designs with unequal block sizes. Pakistan Journal of Statistics. 1990; 6:65-73.

[29] Victorbabu BRe, Narasimham VL. Construction of second order slope rotatable designs through balanced incomplete block designs. Communications in Statistics-Theory and Methods. 1991a; 20:2467-2478.
[30] Victorbabu BRe, Narasimham VL. Construction of second order slope rotatable designs through a pair of incomplete block designs. Journal of the Indian Society of Agricultural Statistics. 1991b; 43:291-295.
[31] Victorbabu BRe, Narasimham VL. Construction of second order slope rotatable designs using pairwise balanced designs. Journal of the Indian Society of Agricultural Statistics. 1993; 45:200-205.
[32] Victorbabu BRe, Ravikumar BV. Second order slope rotatable designs of second type using balanced incomplete block designs. Paper presented in International conference on knowledge discoveries on statistical innovations & recent advances in optimization, organized by Department of Statistics, Andhra University, Visakhapatnam, India during 29th -30th Dec, 2022.
[33] Victorbabu BRe, Surekha ChVVS. Construction of measure of second order slope rotatable designs using central composite designs. International Journal of Agricultural and Statistical Sciences. 2011; 7:351-360.
[34] Victorbabu BRe, Surekha ChVVS. Construction of measure of second order slope rotatable designs using balanced incomplete block designs. Journal of Agricultural and Statistics. 2012a; 19:1-10.
[35] Victorbabu BRe, Surekha ChVVS. Construction of measure of second order slope rotatable designs using pairwise balanced designs. International Journal of Statistics and Analysis. 2012b; 2:97-106.
[36] Victorbabu BRe, Surekha ChVVS. Construction of measure of second order slope rotatable designs using symmetrical unequal block arrangements with two unequal block sizes. Journal of Statistics and Management Systems. 2012c; 15:569-579.
[37] Victorbabu BRe, Surekha ChVVS. A note on measure of rotatability for second order response surface designs using incomplete block designs. Journal of Statistics: Advances in Theory and Applications. 2013; 10:137-151.
[38] Victorbabu BRe, Surekha ChVVS. A note on measure of rotatability for second order response surface designs using balanced incomplete block designs. Thailand Statistician Journal. 2015; 13:97-100.
[39] Victorbabu BRe, Vasundharadevi V. Modified second order response surface designs using balanced incomplete block designs. Sri Lankan Journal of Applied Statistics.2005;6:1-11.



10.22214/IJRASET

45.98

IMPACT FACTOR: 7.129

IMPACT FACTOR: 7.429

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY