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Semi-Analytical Solution of Natural Convection Flow of Non-Newtonian Fluid With Temperature-Dependent Viscosity In Pipe

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Abstract: This paper presents semi-analytical solution of natural convection flow of non-newtonian fluid with temperature-dependent viscosity in pipe. The governing equations were solved using perturbation technique. The results obtained were analyzed for various thermo-solutal parameters involved in the dimensionless equations. Results within the constant viscosity show that increase in these parameters increases the velocity of the fluid flow as well as the temperature of the cylindrical pipe. It is observed that increase in the Reynold's viscosity indices increases the temperature of the cylindrical pipe greatly.
Keywords: Natural convection, non-Newtonian, viscosity, fluid, temperature

I. INTRODUCTION

Natural convection problems involved in non-Newtonian fluids are very significant in many engineering applications. The equations are very complex because of the nature of the fluids. Such fluid include oil, greases etc. Because of the compleity of these fluids, it is difficult to suggest a single model that will handle the problems involved. As such researchers in the past decade have done sme considerable research in the area of non-Newtonian fluid of the differential type. Amongst the earliest researchers are Fosdic and Rajagopal [5]. They examined the thermodynamic stability of fluid of third grade. Massoudi and Christie [7] dealt with the effcts of variable viscosity and viscous dissipation on the flow of third grade fluid. The boundary layer equations of third grade fluid was treated by Pakdemirli [13].

Bejan [4] studied entropy generation in fundamentally convective heat transfer. Johnson *etal* [12] investigated a fluid flow which was infused with solid particles in a pipe, while approximate analytical solutions for flow of third grade fluid was examined by Yurusoy and Pakdemirli [14]. Okedayo *etal* [11] studied the effects of viscous dissipation, constant wall temperature and a periodic field on unsteady flow through a horrizontal channel. Okedayo *etal* [12] analyzed the magnetohydrdynamic (MHD) flow and heat transfer in cylindrical pipe filled with porous media. They applied the Galerkin weighted residual method for the solution of momentum equation and semi- implicit finite differece method for the energy equation. They found that an increase in Darcy number leads to an increase in the velocity profiles, while increase in Brinkman number enhances the temperature of the system.

Obi [9] on approximate analytical solution of natural convection flow of non-Newtonian fluid through parallel plates , solved the coupled momentum and energy equations using the regular perturbation methd. He treated cases of constant and temperature-dependent viscosities in which Reynold's and Vogel's models were considered to account for the temperature-dependent viscosity case, while third grade fluid was introduced to account for the non-Newttonian effects. Obi [10] numerically analyzed the reactive third grade fluid in cylindrical pipe. He observed that the non-Newtonian parameters considered in the analysis: third grade parameter (β), magnetic field parameter (M), Eckert number (E_c) and the Brinkman number (B_r) had psitive effects on the velocity and temperature profiles.

Aksoy and Pakdemirli [1] examined the flow of a non-Newtonian fluid through a porous medium between two parallel plates. They involed Reynold's and Vogel's models viscosity and derived the criteria for validity for the approximate solution. Yurusoy *etal* [15] analyzed the flow of third grade fluid between parallel plates at different temperatures. Narges and Mahmood [8] investigated the effects of thin film flow of third grade fluid for a class of nonlinear second order differential equations. Ayub *etal* [3] examined the exact flow of third grade fluid. They used homotopy perturbation method for the analysis.

The distribution of this paper is in six sections. Section 1 was the intrduction of the research and the literature review. Section 2 was the problem formulation. In section 3, the analytical solutions of the constant viscosity case was presented and velocity and temperature-dependent viscosity considered in Reynold's model. Section 4 was the discussion of the results obtained in section 3. Section 5 was the concluding remarks while section 6 was the reference.

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II. MATHEMATICAL FORMULATION

Considering a steady incompressible thin film flow of third grade fluid, the non-dimesional form of equations of motion as in Aiyesimi *etal* [2] are:

$$\frac{d^2u}{dr^2}+6\beta\left(\frac{du}{dr}\right)^2\left(\frac{d^2u}{dr^2}\right)+Mu=-1 \tag{1}$$

$$\frac{d^2\theta}{dr^2}+\mathrm{B}_r\left(\frac{du}{dr}\right)^2+2\beta\left(\frac{du}{dr}\right)^4+\mathrm{Mu}^2=0 \tag{2}$$

$$u(0)=0,u(1)=1,\theta(0)=0,\theta(1)=1 \tag{3}$$

Where uis the velocity of the fluid, θ is the temperature of the cylinder, The terms are related to the non-dimensional variables

$$r=\frac{\bar{r}}{d},\theta=\frac{T}{T_0},u=\frac{\bar{u}}{u_0},\mu=\frac{\bar{\mu}}{\mu_0}, \tag{4}$$

where r is the radius of the cylinder, T_0 is the reference temperature , u_0 is the reference velocity, μ_0 is the reference viscosity.

$$\mathrm{B}_r=\frac{\mu_0u_0^2}{kT_0},\beta=\frac{\beta u_0^2}{\mu_0d^2} \tag{5}$$

III. METHOD OF SOLUTION

The semi-analytical solutions for velocity and temperature profiles can be of the form:

$$u(r)=u_0(r)+\beta u_1(r)+\mathrm{O}(\beta^2),\theta(r)=\theta_0(r)+\beta\theta_1(r)+\mathrm{O}(\beta^2),\ \mathrm{M}=\beta\mathrm{M} \tag{6}$$

Substituting eqn (6) into eqns (1) and (2) and separating each order of β , yields

$$\frac{d^2u_0}{dr^2}=-1 \tag{7}$$

$$\frac{d^2u_1}{dr^2}+6\left(\frac{du_0}{dr}\right)^2\frac{d^2u_0}{dr^2}+Mu_0=0 \tag{8}$$

$$\frac{d^2\theta_0}{dr^2}+\mathrm{B}_r\left(\frac{du_0}{dr}\right)^2=0 \tag{9}$$

$$\frac{d^2\theta_1}{dr^2}+2\beta\mathrm{B}_r\frac{du_0}{dr}\frac{du_1}{dr}+Mu_0^2=0 \tag{10}$$

Solving eqns (7-10) with the condition (3), yields

$$u(r)=\frac{3}{2}r-\frac{1}{2}r^2+\beta\left(\frac{27}{4}r^2-3r^3+\frac{1}{2}r^4-M\left(\frac{1}{4}r^3-\frac{1}{24}r^4\right)-\frac{13}{4}r+\frac{5}{24}rM\right) \tag{11}$$

$$\begin{aligned}\theta(r)=&-\mathrm{B}_r\left(-\frac{17}{24}r+\frac{9}{8}r^2-\frac{1}{2}r^3+\frac{1}{12}r^4\right)-r+\beta\Bigg(-\beta\mathrm{B}_r\left(\frac{47}{6}r^3-\frac{9}{2}r^4+\frac{6}{5}r^5-M\left(\frac{5}{16}r^5-\frac{1}{40}r^5\right)\right.\\&\left.-\frac{39}{8}r^2+\frac{5}{16}r^2M-\frac{2}{15}r^6+M\left(\frac{3}{40}r^5-\frac{1}{60}r^6\right)-\frac{5}{24}r^2M\right)-M\left(\frac{3}{16}r^4-\frac{3}{40}r^5+\frac{1}{120}r^6\right)+r\\&+\beta\mathrm{B}_r\left(-\frac{57}{120}r+\frac{78}{240}rM\right)+\frac{29}{240}rM\Bigg)\end{aligned} \tag{12}$$

A. Temperature – Dependent Viscosity

In temperature-dependent viscosity , two models are involved which are Reynold's and Vogel's mdels, but for this present paper, we consider only the Reynold's model.

3.1.1 Reynold's Model

The equations for momentum and energy for this model are

$$\frac{d\mu}{dr}\frac{du}{dr}+\frac{d^2u}{dr^2}+6\beta\left(\frac{du}{dr}\right)^2\left(\frac{d^2u}{dr^2}\right)+Mu=-1 \tag{13}$$

$$\frac{d^2\theta}{dr^2}+\mu\left(\frac{du}{dr}\right)^2+2\beta\left(\frac{du}{dr}\right)^4+\mathrm{M}u^2=0 \tag{14}$$

Defining the perturbation series as in eqn (6),where β is a small parameter. Viscosity depends on temperature in an exponential manner Massoudi and Christie [7] as

$$\mu=\exp(-n\theta) \tag{15}$$

Expanding in Taylor series, eqn (15) and its derivative can be represented respectively as

$$\mu\equiv 1-\beta n\theta \tag{16}$$

$$\frac{d\mu}{dr}\equiv-\beta n\frac{d\theta}{dr} \tag{17}$$

Using eqns (6), (16) and (17) in eqns (13) and (14) yields

$$\frac{d^2u_0}{dr^2}=-1 \tag{18}$$

$$\frac{d^2u_1}{dr^2}-n\frac{d\theta_0}{dr}\frac{du_0}{dr}+6\left(\frac{du_0}{dr}\right)^2\frac{d^2u_0}{dr^2}+Mu_0=0 \tag{19}$$

$$\frac{d^2\theta_0}{dr^2}+\left(\frac{du_0}{dr}\right)^2=0 \tag{20}$$

$$\frac{d^2\theta_1}{dr^2}+2\frac{du_0}{dr}\frac{du_1}{dr}-n\theta_0\left(\frac{du_0}{dr}\right)^2+2\left(\frac{du_0}{dr}\right)^4+Mu_0^2 \tag{21}$$

Solving the second order nonlinear ordinary differential eqns (18-21) with the condition eqn (3) yields

$$u(r)=\frac{3}{2}r-\frac{1}{2}r^2+\beta\left(n\left(\frac{123}{96}r^2-\frac{25}{72}r^3+\frac{3}{8}r^4-\frac{1}{10}r^5+\frac{1}{90}r^6\right)+\frac{27}{4}r^2-3r^3+\frac{3}{4}r^4\right.$$
$$\left.-M\left(\frac{1}{4}r^3-\frac{1}{24}r^4\right)-\frac{7}{2}rn+\frac{5}{24}rM\right) \tag{22}$$

$$\theta(r)=\frac{41}{24}r-\frac{9}{8}r^2+\frac{1}{2}r^3-\frac{1}{12}r^4+\beta\left(\left(-n\left(\frac{123}{192}r^3-\frac{25}{192}r^4+\frac{9}{80}r^5-\frac{1}{40}r^6+\frac{1}{420}r^7\right)\right.\right.$$
$$+\frac{27}{8}r^3-\frac{9}{8}r^4+\frac{9}{40}r^5-M\left(\frac{3}{32}r^4-\frac{1}{80}r^5\right)-\frac{21}{8}r^3+\frac{1037}{1920}r^3n+\frac{5}{32}r^3M+n\left(-\frac{41}{192}r^4\right.$$
$$\left.+\frac{5}{96}r^5-\frac{1}{20}r^6+\frac{1}{84}r^7-\frac{1}{840}r^8\right)-\frac{9}{8}r^4-\frac{9}{20}r^5-\frac{1}{10}r^6+M\left(\frac{3}{80}r^5-\frac{1}{180}r^6\right)+\frac{7}{12}r^3$$
$$\left.-\frac{1037}{8640}r^3n-\frac{5}{144}r^3M\right)+n\left(\frac{123}{192}r^3-\frac{119}{384}r^4+\frac{19}{96}r^5+\frac{7}{160}r^6-\frac{1}{168}r^7-\frac{1}{672}r^8\right)$$
$$-\left(\frac{81}{16}r^2-\frac{9}{2}r^3+\frac{9}{4}r^4-\frac{4}{5}r^5+\frac{1}{15}r^6\right)-M\left(\frac{3}{16}r^4-\frac{3}{40}r^5+\frac{1}{120}r^6\right)-r\left(-\frac{29}{120}+\frac{129241}{120960}n\right.$$
$$\left.\left.-\frac{121}{1440}M\right)-\frac{7593}{13440}n+\frac{319}{240}+\frac{29}{240}M\right)\Bigg) \tag{23}$$

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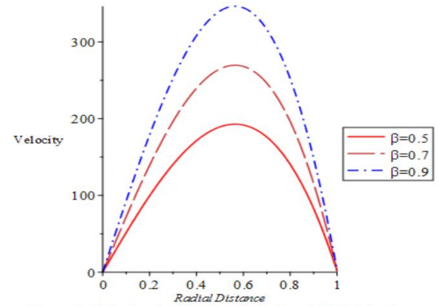

Figure 1: Velocity Profiles For Variation Of Third Grade Parameter (β)

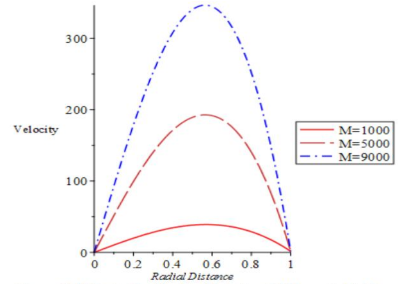

Figure 2: Velocity Profiles For Variation Of Magnetic Field Parameter (M)

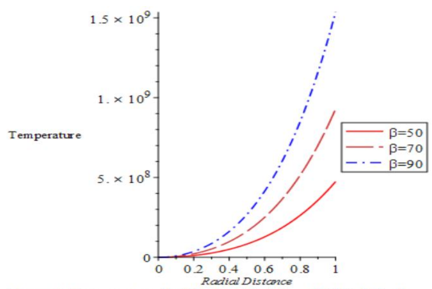

Figure 3: Temperature Profiles For Variation Of Third Grade Parameter (β)

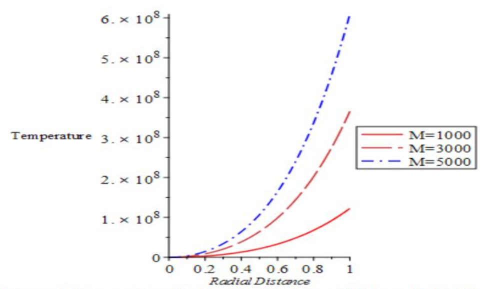

Figure 4: Temperature Profiles For Variation Of Magnetic Field Parameter (M)

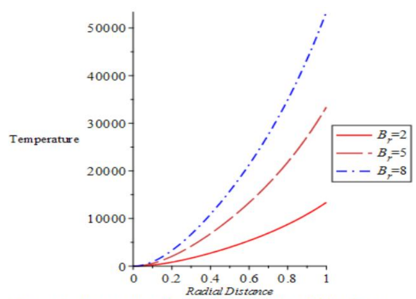

Figure 5: Temperature Profiles For Variation Of Brinkman Parameter (B_r)

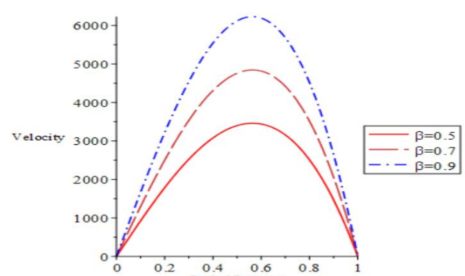

Figure 6: Velocity Profiles For Variation Of Reynold's Viscosity Index (β)

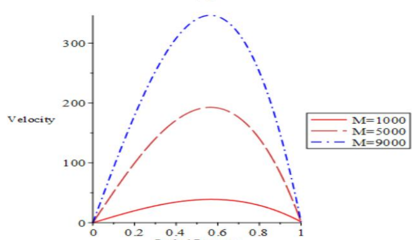

Figure 7: Velocity Profiles For Variation Of Reynold's Viscosity Index (M)

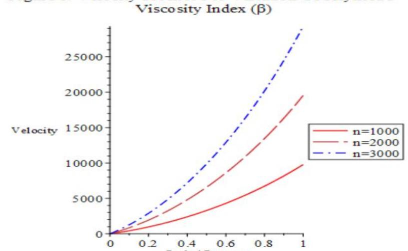

Figure 8: Velocity Profiles For Variation Of Reynold's Viscosity Index (n)

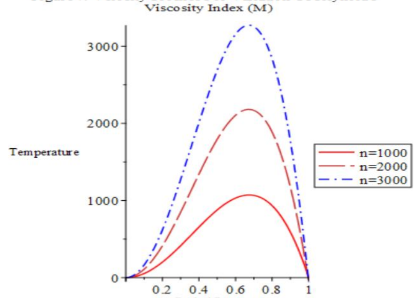

Figure 9: Temperature Profiles For Variation Of Reynold's Viscosity Index (n)

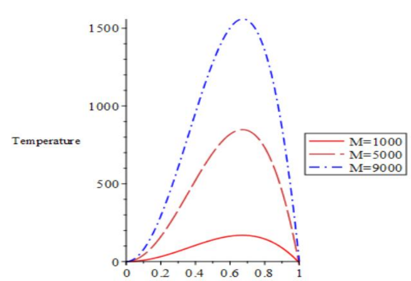

Figure 10: Temperature Profiles For Variation Of Reynold's Viscosity Index (M)

IV. RESULTS AND DISCUSSION

In order to study behaviour of some physical parameters involved in the analysis, graphs are presented in figures (1-10). The solution of momentum and energy equations (1) and (2) with the boundary condion (3) given in equations (22) and (23). Figures 1,2,3 and 4 are the velocity and temperature profiles for different values of the third grade parameter (β) and magnetic field parameter (M). Results show that increase in β and M increases the velocity of the fluid flow as well as the temperature of the cylindrical pipe. Figure 5 shows the temperature profiles for variation of the Brinkman parameter (B_r). Results indicate that as the parameter increases, the temperature of the system is enhanced. Figures 6, 7 and 8 are the velocity profiles for Reynold's viscosity indices β, M and n. Results show that increase in these parameters increases the flow velocity and the temperature profiles. Figures 9 and 10 show the temperature profiles for Reynold's viscosity indices n and M. It is observed that as n and M increases, the temperature of the cylinder is greatly enhanced.

V. CONCLUSION

This paper presents semi-analytical solution of natural convection flow of non-newtonian fluid with temperature-dependent viscosity in pipe. The governing equations were solved using perturbation technique. The results obtained were analyzed for various thermo-solutal parameters involved in the dimensionless equations. Results within the constant viscosity show that increase in β and M increases the velocity of the fluid flow as well as the temperature of the cylindrical pipe. Results further indicate that increase in Brinkman number increases the temperature of the system. It is observed that increase in the Reynold's viscosity indices n and M increases the temperature of the cylinder greatly.

A. *Declarations*

1) Funding: Not applicable
2) Informed Consent Statement: Not applicable
3) Data Availability: Not applicable
4) Conflict of Interest Statement: No conflict of interest

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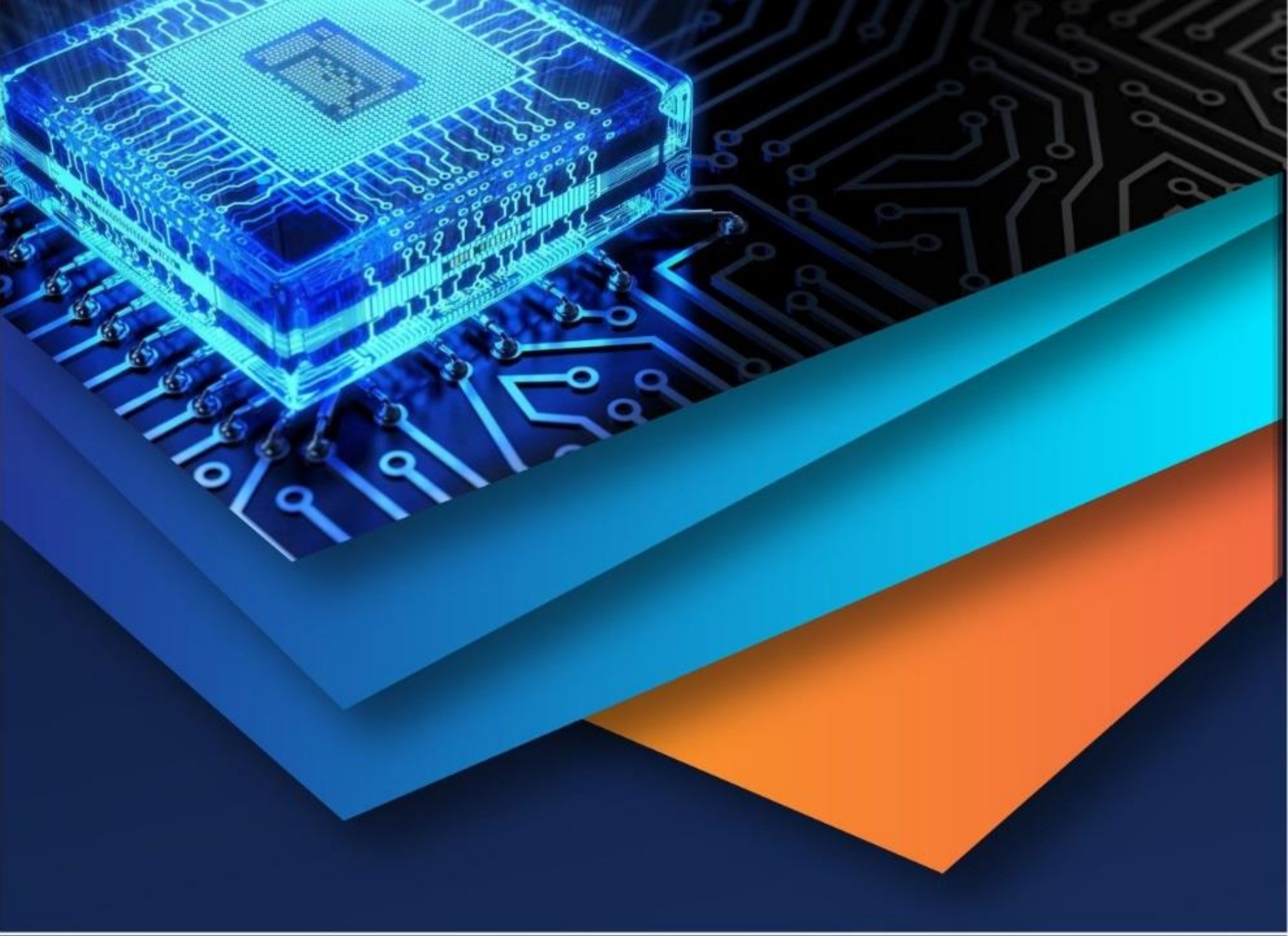

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